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20.GEM GEM4 Summer School: Cell and Molecular Biomechanics in Medicine: Cancer
Summer 2007

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**GEM⁴ Summer School on Cell and Molecular
Mechanics in Biomedicine**

June 25 - July 6 2007

Molecular mechanics

Lecture 1

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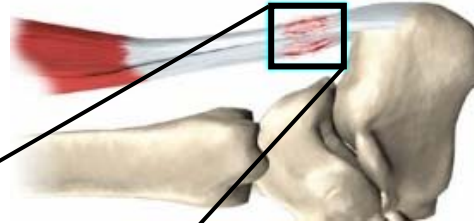


Molecular mechanics: Definition of scales



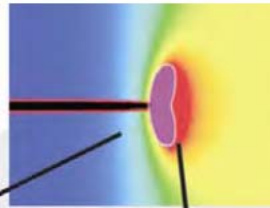
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1..100 mm
Macroscale/tissue



Physiological role

100..1000 μm
Continuum scale



*Fracture properties
Elasticity, yield*

Tissues

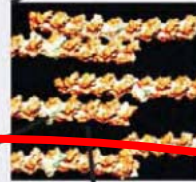
10..100 μm
Mesoscale



Formation of yield regions

Cells

10 nm .. 10 μm
Molecular scale



Intermolecular adhesion

Sub-cellular scale

1 nm .. 10 nm
Molecular scale

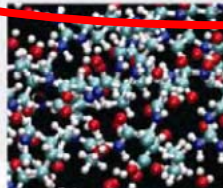


*Molecular properties
(single molecule)*

Focus of these lectures

Molecules: building blocks

1 \AA .. 1 nm
Atomistic scale
Chemistry, physics



*Chemical interactions
Explicit solvents*



Protein (molecules) are crucial for cellular functions

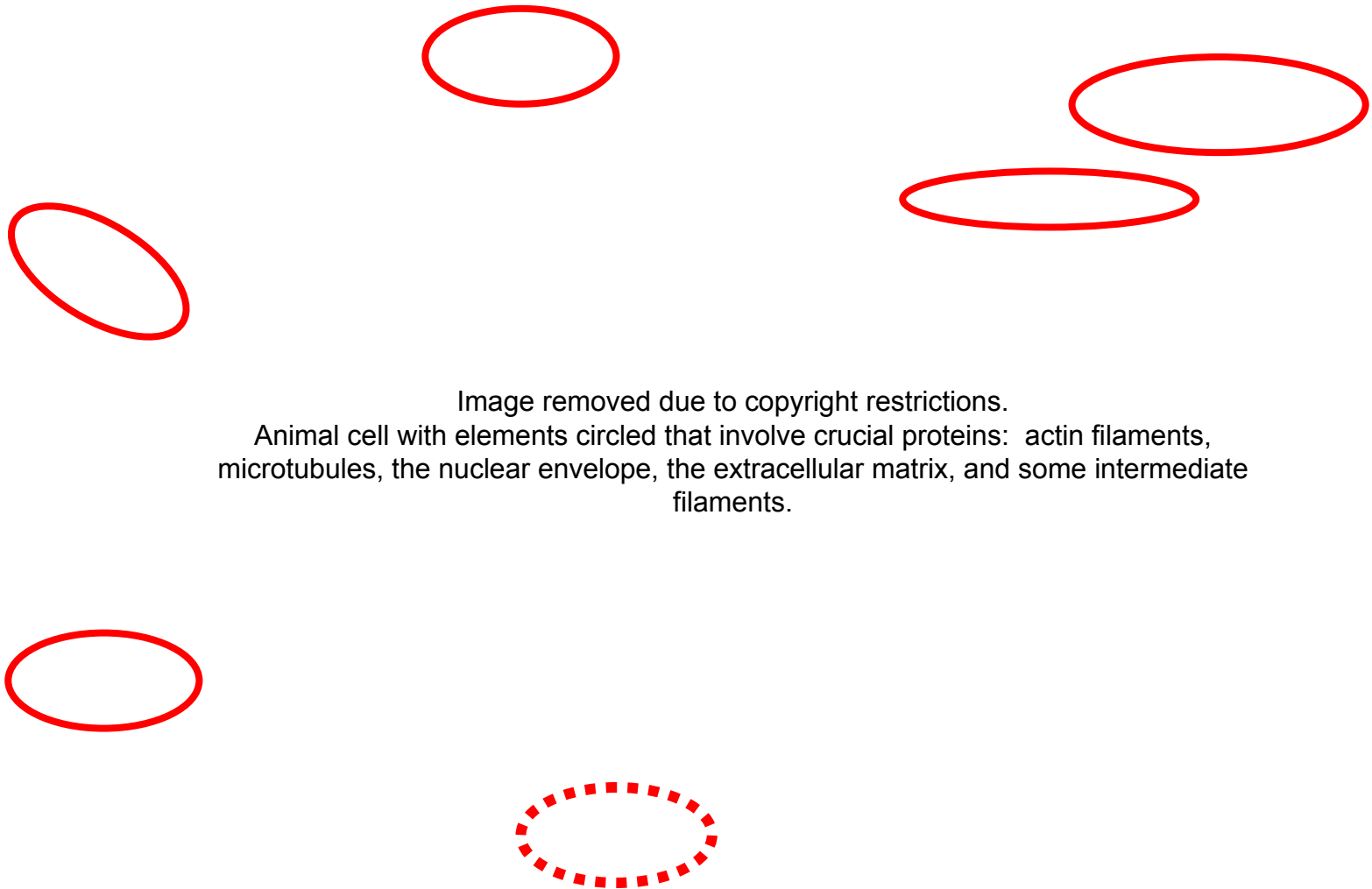
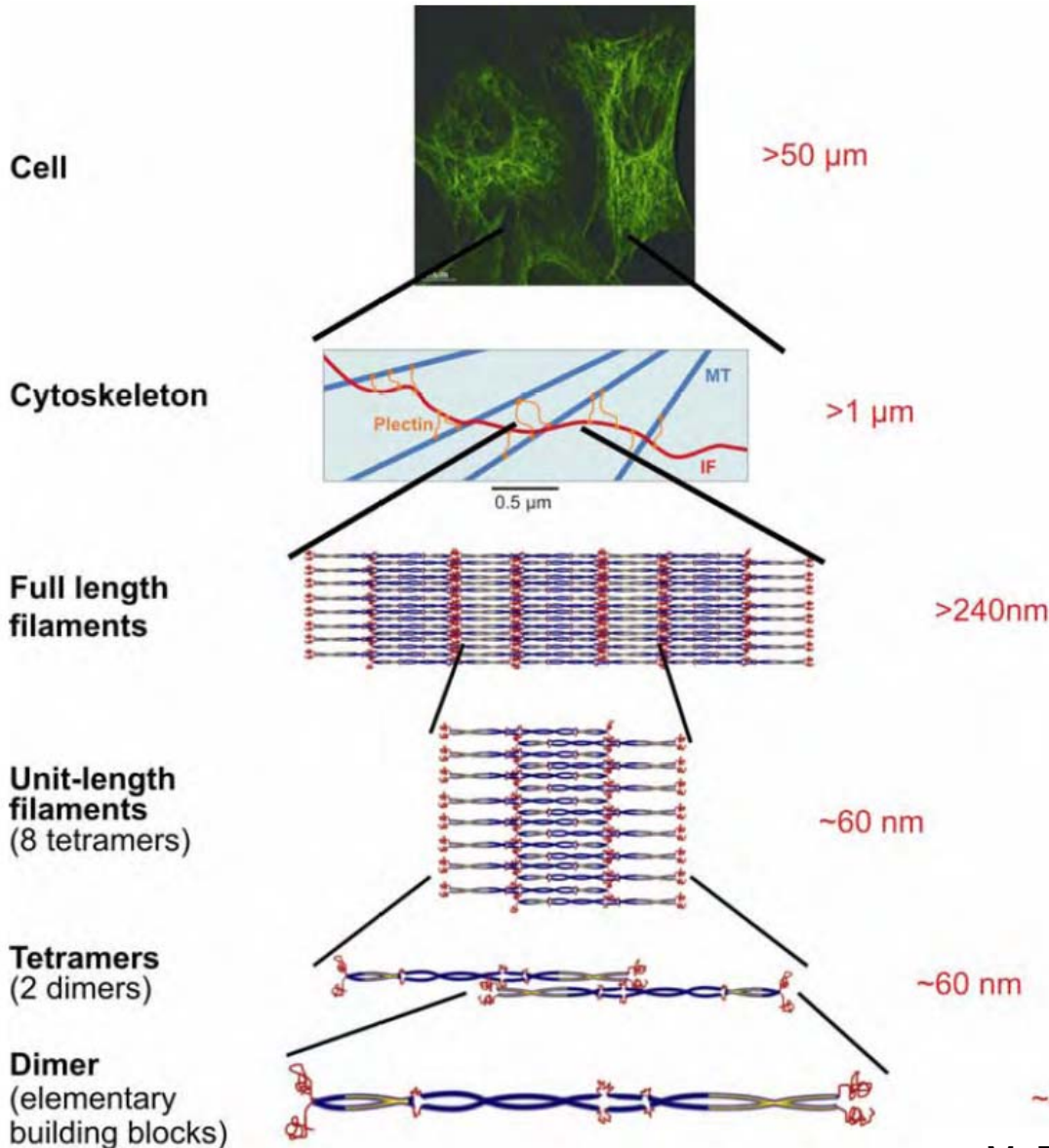


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Animal cell with elements circled that involve crucial proteins: actin filaments, microtubules, the nuclear envelope, the extracellular matrix, and some intermediate filaments.

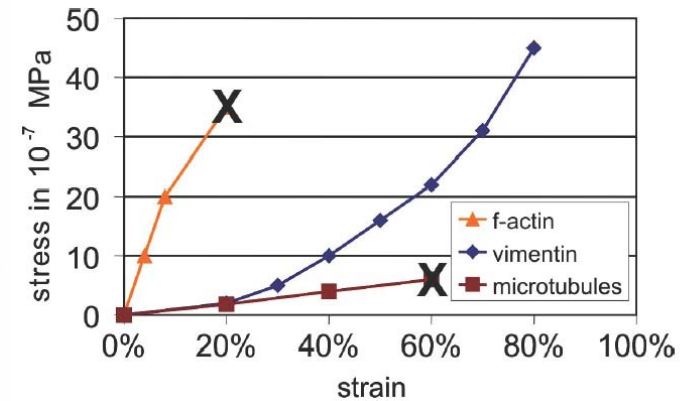


Example: Intermediate filament (IF) proteins



Intermediate filaments (IFs)

Structure significant under large deformation (stability, mechanosensation)



Janmey PA, Euteneuer U, Traub P, Schliwa M (1991) *J Cell Biol* 113:4 113:155

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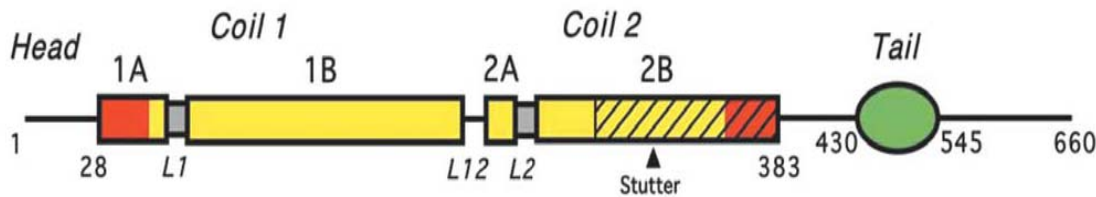


~45 nm Courtesy Elsevier, Inc., <http://www.sciencedirect.com>. Used with permission.

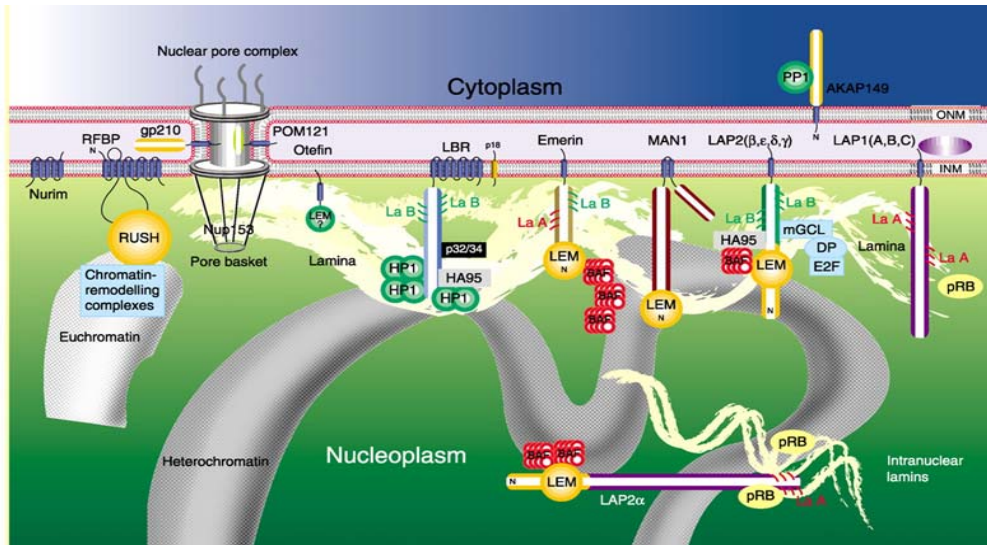
M. Buehler & T. Ackbarow, *Materials Today*, in press



Example: Nuclear envelope



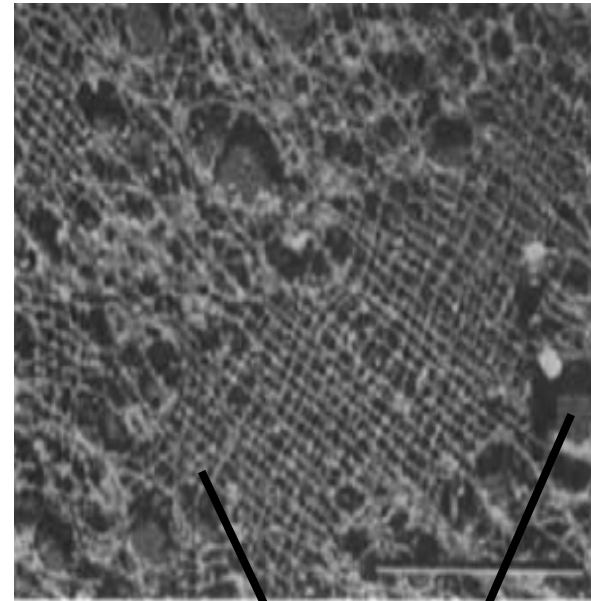
Lamin filament



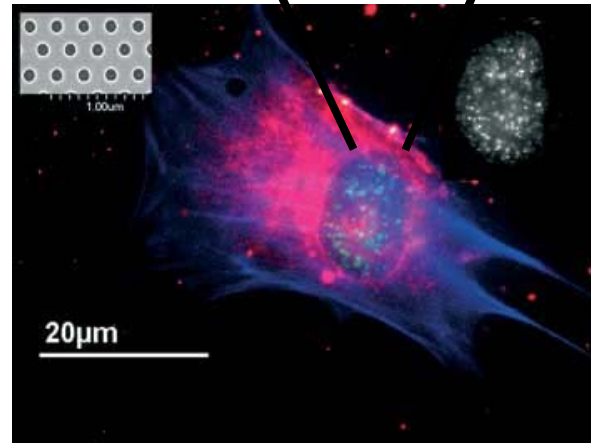
Foisner et al., 2001

Courtesy of the Company of Biologists. Used with permission.

Lamina - a mesh network
U. Aebi et al.



Courtesy of the Company of Biologists. Used with permission.



Structural: Maintaining Nuclear Shape, Absorbing Mechanical Shock, Organizing Chromatin, etc
Biological: Regulating Cell Cycle, Controlling DNA replication, Determining Apoptosis, etc



Structural change in protein molecules can lead to fatal diseases



- Single point mutations in IF structure causes severe diseases such as **rapid aging disease progeria – HGPS** (*Nature*, 2003; *Nature*, 2006, *PNAS*, 2006)
- Cell nucleus loses stability under cyclic loading
- Failure occurs at heart (fatigue)

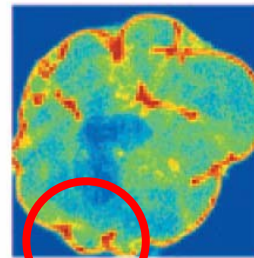
Courtesy of National Academy of Sciences, U. S. A. Used with permission.
Source: Dahl, et al. "Distinct Structural and Mechanical Properties of the Nuclear Lamina in Hutchinson–Gilford Progeria Syndrome." *PNAS* 103 (2006): 10271-10276. Copyright 2006 National Academy of Sciences, U.S.A.

Substitution of a single DNA base: Amino acid guanine is switched to adenine

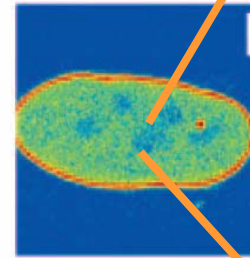
Experiment suggests that mechanical properties of nucleus change (Dahl *et al.*, *PNAS*, 2006)



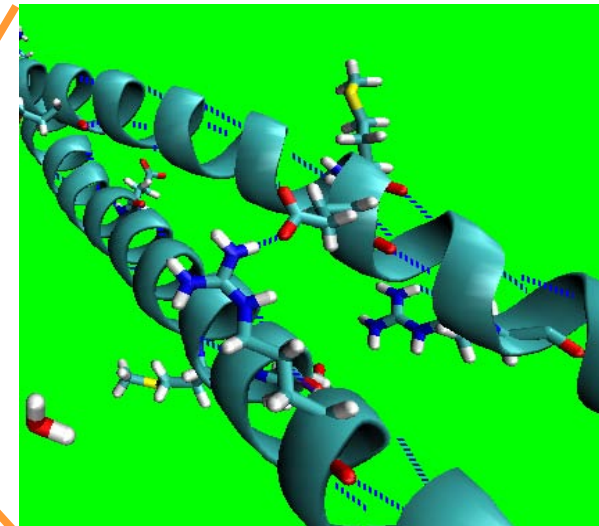
HGPS cells
wt LA



wt cells
wt LA



Fractures





- Many cancer cells have abnormal expression of lamin: For example, HL-60 cells (type of leukemic cells) shows a change expression level of lamin A/C and B and changes in nuclear shape.
 - Changes in lamin structure can lead to **abnormal expression of genes** in cancer cells
 - A key step of **apoptosis** is the disassembly of lamina and detachment of chromatin: Mutation in Lamin A/C can make the cleavage site of lamina uncleavable, thus hinder apoptosis
- Understanding of molecular mechanics – e.g. assembly, stability, for instance under chemical / enzymatic signals – is vital to understand these processes rigorously



Why is mechanics important in biology?



- **Mechanics** describes the relationship between the deformation state of a material or system as a function of the applied load and boundary conditions
- **Mechanical properties are crucial for biological processes:**
 - **Functioning of biological systems – passive role**
(structural materials such as bone, skin, tendon,...)
 - Mechanical stimulation is utilized to facilitate **signaling in biological processes** (in cells, e.g. via mechanosensation) – **passive role**
 - **Active mechanical stimulation** (protein motors) – **active role**
- **Change in mechanical properties** may interfere with biological processes and **lead to diseases**
- **Pathological condition may in turn change mechanical behavior**, e.g. cells become stiffer (RBCs in malaria)



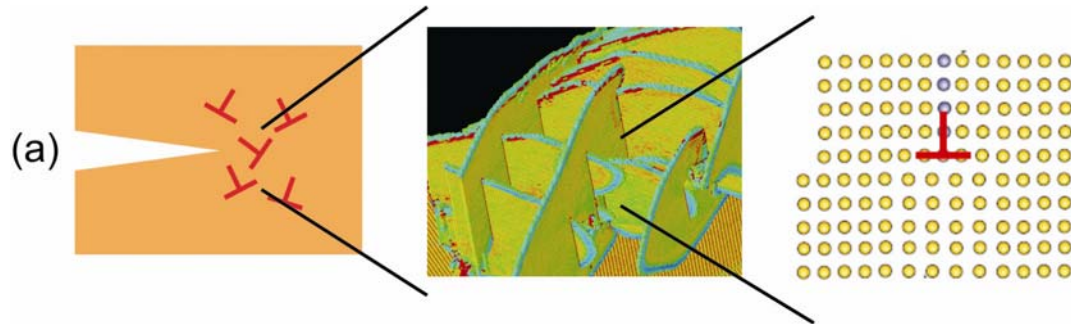
Goal of these lectures



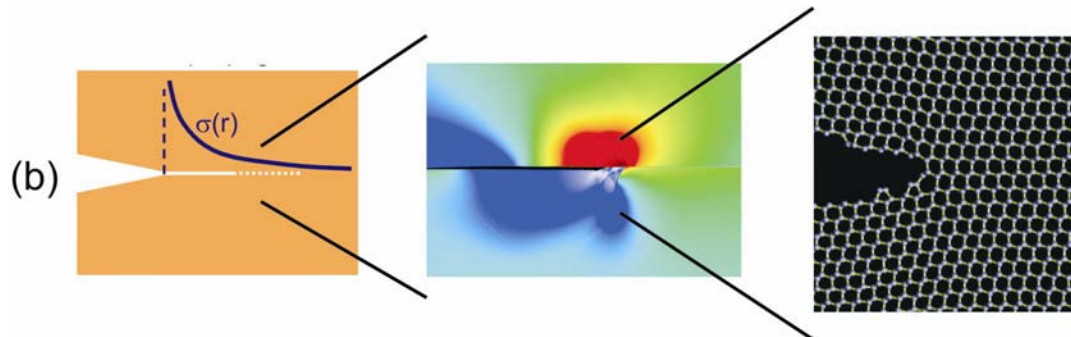
- Molecular mechanics with focus on biomolecules (proteins)
- Topics covered:
 - Chemical bonding in molecules & theory
 - Characterization of molecular properties (tensile stiffness, persistence length, adhesion..)
 - Application of continuum mechanical concepts to molecular mechanics
 - Link of molecular properties to tissue properties (collective behavior of many molecules, elasticity, fracture,..)
 - Molecular defects and consequence for diseases: Certain mutation may induce changes in mechanical properties, e.g. molecular defects, mutations; leads to pathological consequence (too soft, too stiff,..)
- Multi-scale modeling
- Emphasis on developing a sensitivity for the significance of molecular mechanics in biology and how atomistic and continuum viewpoints can be coupled



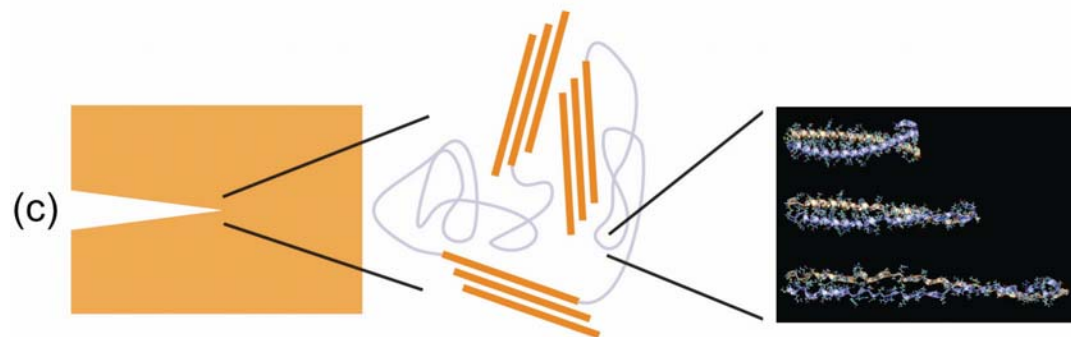
Significance of molecular mechanics from a material scientist's perspective



Metals - dislocation



Brittle materials – crack extension



Protein materials – **unfolding?**
Sliding?

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From electrons to molecules

A brief review of chemical concepts



Atomic scale



- Atoms are composed of electrons, protons, and neutrons. Electron and protons are negative and positive charges of the same magnitude, 1.6×10^{-19} Coulombs
- Chemical bonds between atoms by interactions of the electrons of different atoms (e.g. sharing of electrons)

“Point” representation

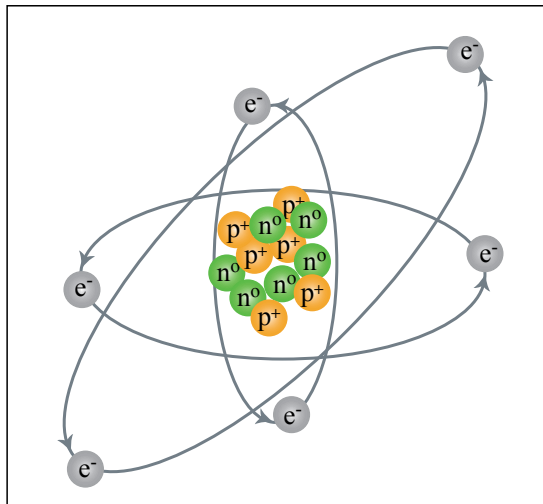


Figure by MIT OpenCourseWare.

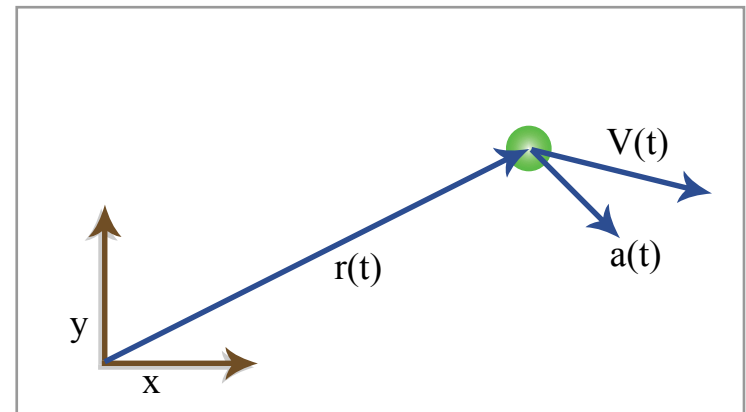


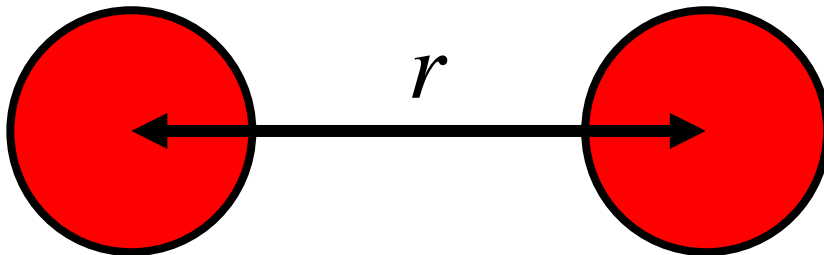
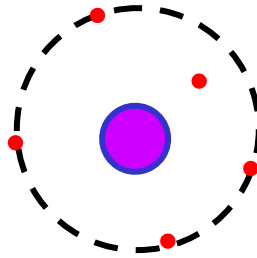
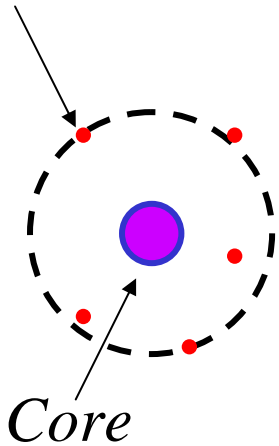
Figure by MIT OpenCourseWare. After Buehler.



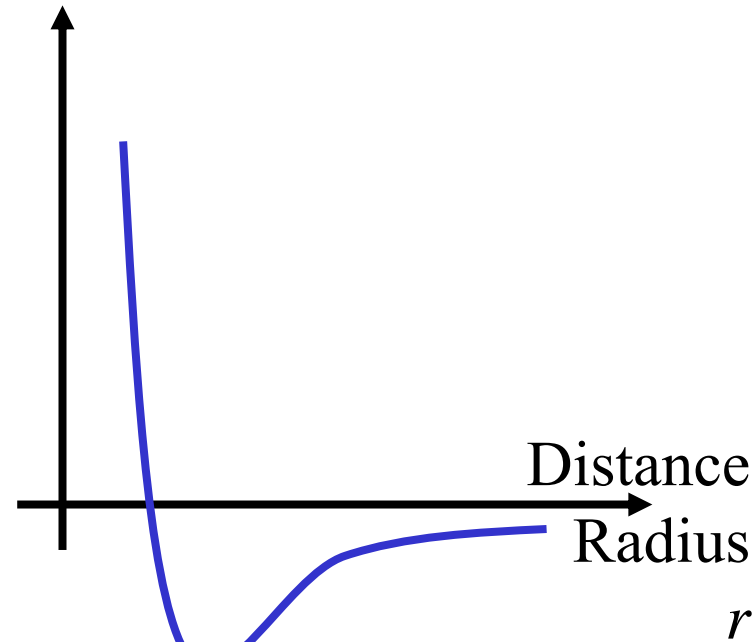
From electrons to atoms



Electrons



Energy



Governed by laws of quantum mechanics: Numerical solution by Density Functional Theory (DFT), for example



- **Primary bonds (“strong”)**

- Ionic,
- Covalent,
- Metallic (high melting point, 1000-5000K)

Strength: several nN

- **Secondary bonds (“weak”)**

- Van der Waals,
- Hydrogen bonds
(melting point 100-500K)

Strength: 10..100 pN

- Ionic: Non-directional
- Covalent: Directional (angles, torsions)
- Metallic: Non-directional



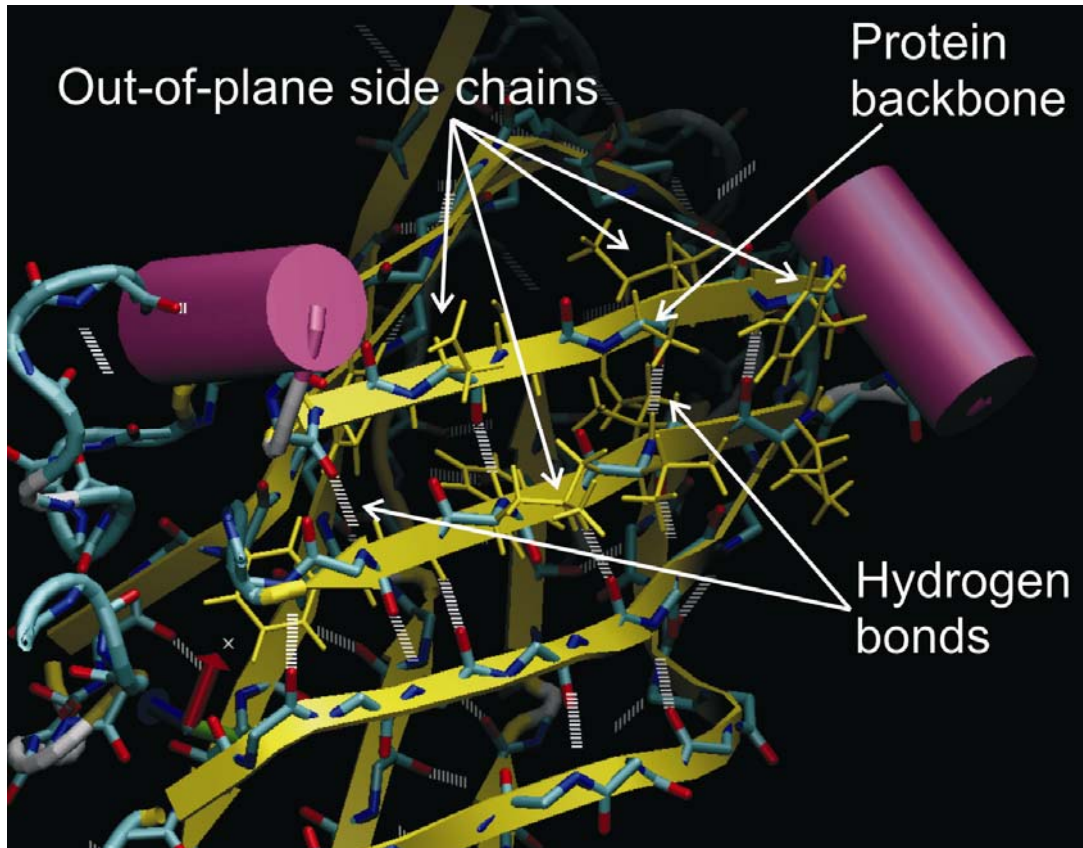
Chemistry: A fundamental language



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Proteins often include a variety of atomic interactions



Covalent bonds

Between differently charged species:

Electrostatic interactions

Hydrogen bonds

vdW interactions



Optical tweezer experiment

Protein unfolding

Image removed due to copyright restrictions.

Please see Figure 3(a) in Marszalek, Piotr E., et al. "Mechanical unfolding intermediates in titin modules." *Nature* 402 (1999): 100-103.

Images removed due to copyright restrictions.

Please see Figures 1(f) and 2 in Tskhovrebova, L., J. Trinick, J. A. Sleep and R. M. Simmons. "Elasticity and unfolding of single molecules of the giant muscle protein titin." *Nature* 387 (1997): 308-312.



Stretching of tropocollagen molecules

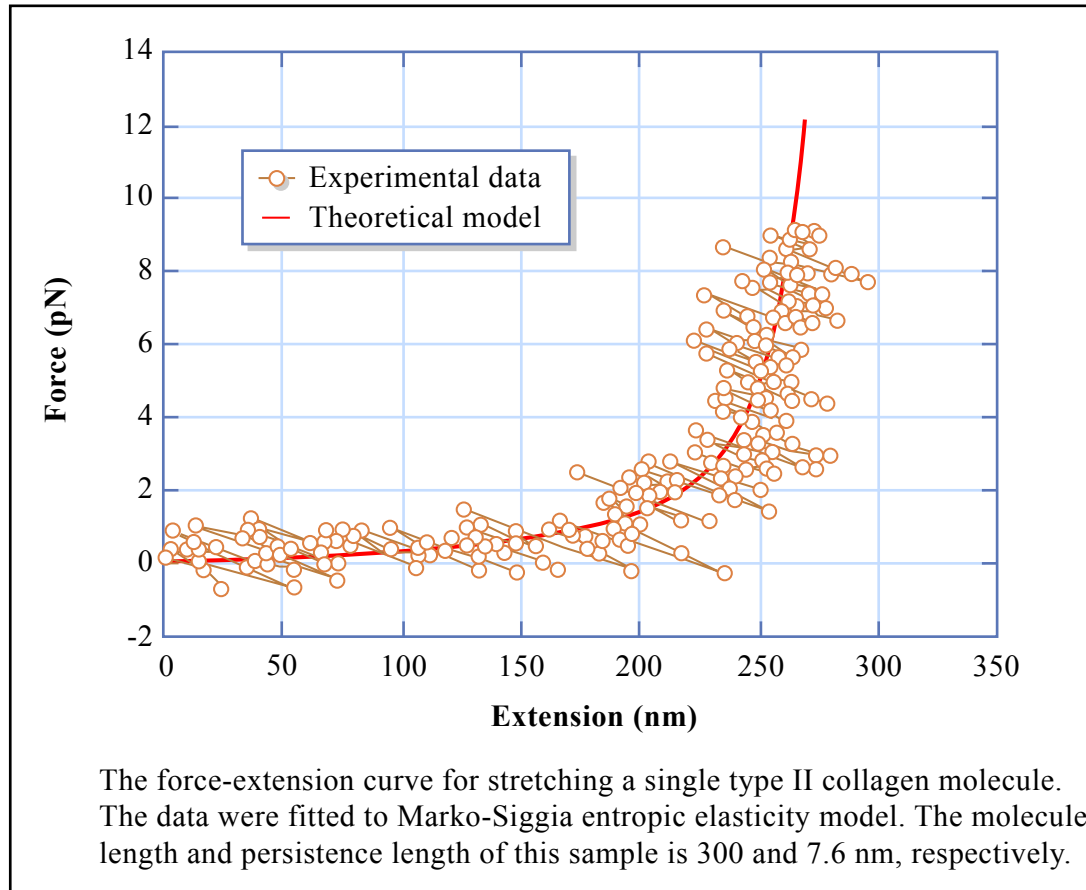


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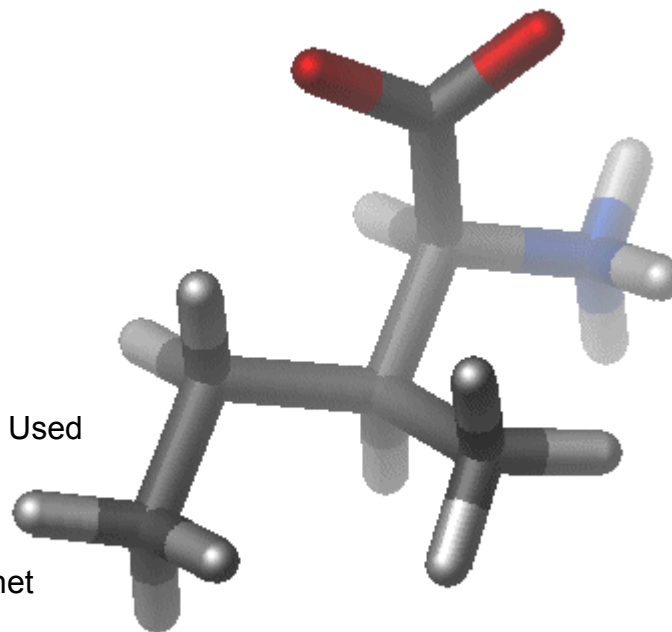
$$V(R) = E_{\text{bonded}} + E_{\text{non-bonded}}$$

$$E_{\text{bonded}} = E_{\text{bond-stretch}} + E_{\text{angle-bend}} + E_{\text{rotate-along-bond}}$$

Bonding between atoms described as combination of various terms, describing the angular, stretching etc. contributions

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Images created for the CHARMM tutorial by Dr. Dmitry Kuznetsov (Swiss Institute of Bioinformatics) for the EMBnet Education & Training committee (<http://www.embnet.org>)

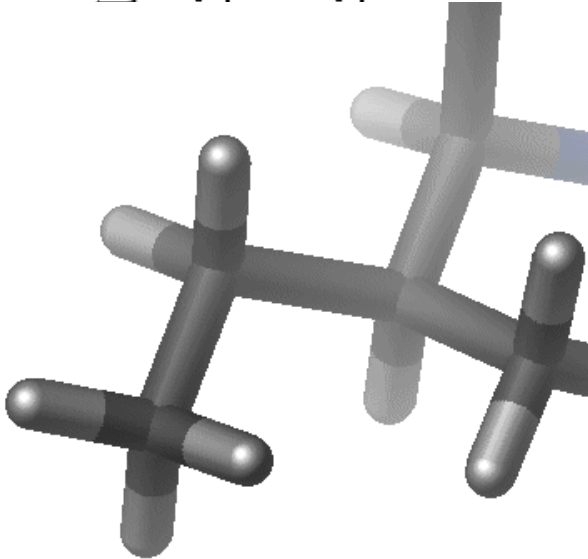




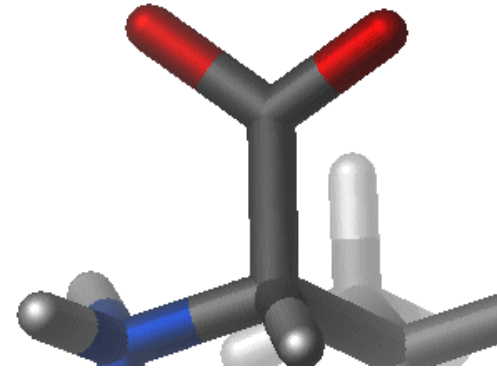
Model for covalent bonds



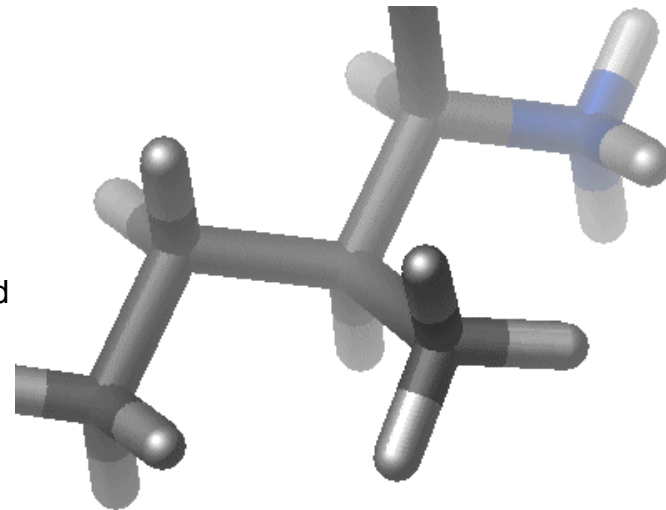
$$E_{bond-stretch} = \sum K_b (b - b_0)^2$$



$$E_{bond-bend} = \sum_{angles} K_\theta (\theta - \theta_0)^2$$



$$E_{rotate-along-bond} = \sum_{1,4\ pairs} K_\phi (1 - \cos(n\phi))$$



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Review: CHARMM potential

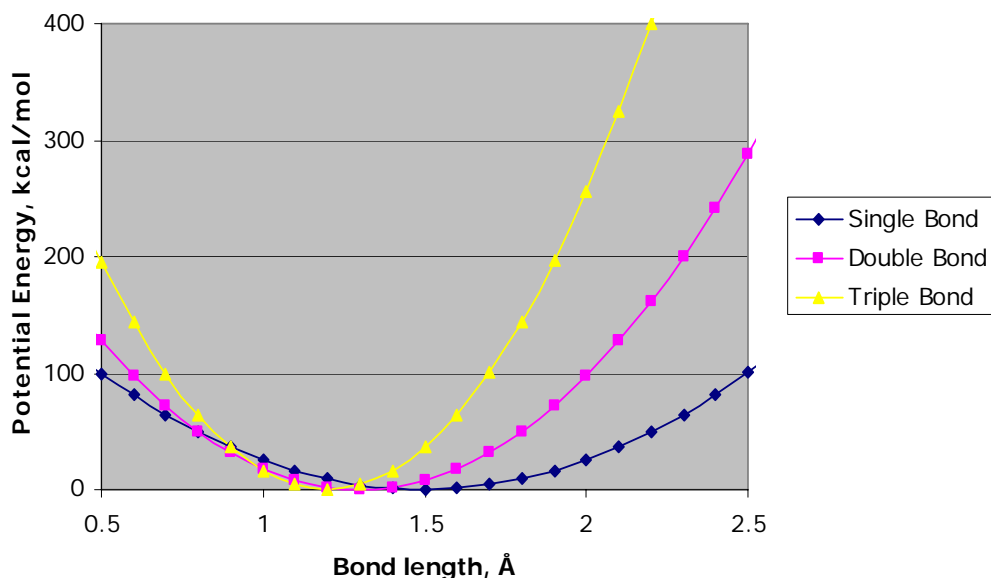


Chemical type	K_{bond}	b_0
C-C	100 kcal/mole/Å ²	1.5 Å
C=C	200 kcal/mole/Å ²	1.3 Å
C≡C	400 kcal/mole/Å ²	1.2 Å

Different types of C-C bonding represented by different choices of b_0 and k_b ;

Need to retype when chemical environment changes

Bond Energy versus Bond length



$$V_{\text{bond}} = K_b (b - b_0)^2$$

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http://www.ch.embnet.org/MD_tutorial/pages/MD.Part2.html

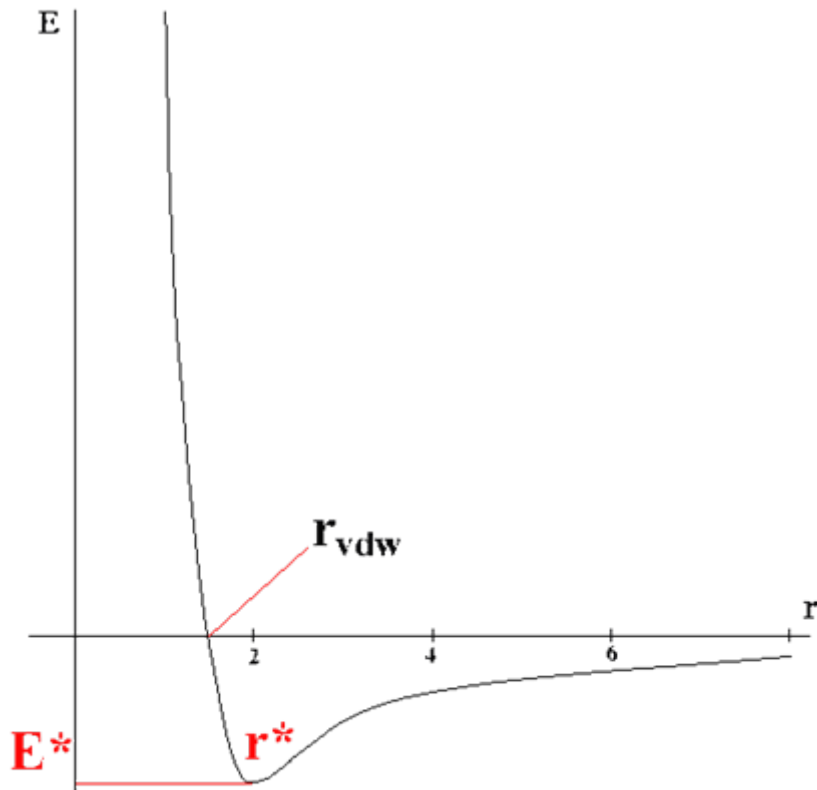
http://www.pharmacy.umaryland.edu/faculty/amackere/force_fields.htm



Review: CHARMM potential



$$E_{non-bonded} = E_{van-der-Waals} + E_{electrostatic}$$



$$E_{van-der-Waals} = \sum_{nonbonded\ pairs} \left(\frac{A_{ik}}{r_{ik}^{12}} - \frac{C_{ik}}{r_{ik}^6} \right)$$

$$E_{electrostatic} = \sum_{nonbonded\ pairs} \frac{q_i q_k}{D r_{ik}}$$

Nonbonding interactions

vdW (dispersive)

Coulomb (electrostatic)

H-bonding

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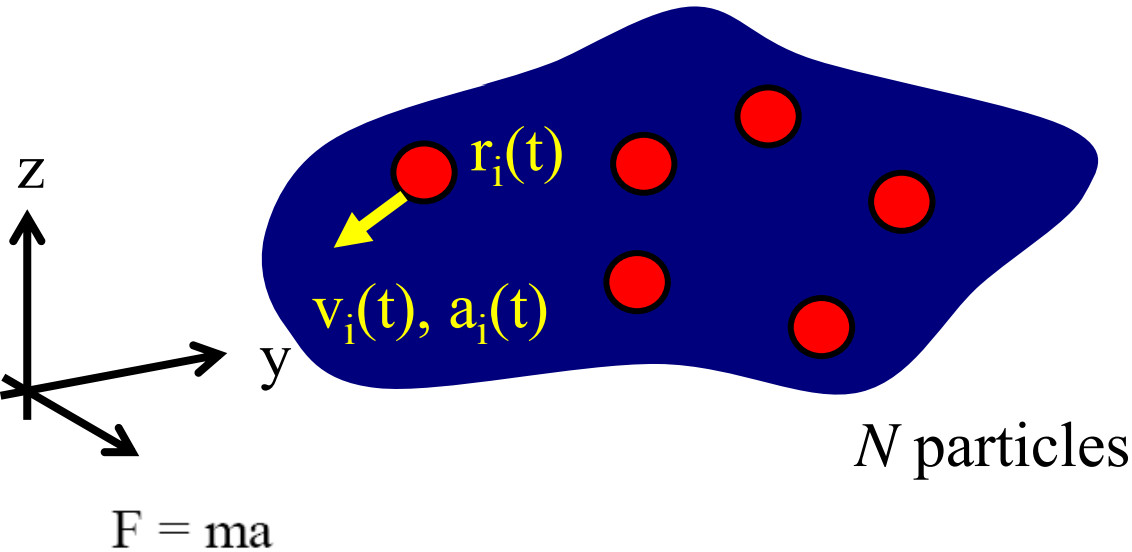
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Molecular dynamics

Numerical simulation of stretching experiments



$$m \frac{d^2 r_j}{dt^2} = -\nabla_{r_j} U(r_j) \quad j = 1..N$$

Total energy of system

$$E = K + U$$

$$K = \frac{1}{2} m \sum_{j=1}^N v_j^2$$

$$U = U(r_j)$$

Coupled system N-body problem, no exact solution for $N > 2$

System of coupled 2nd order nonlinear differential equations

Solve by discretizing in time (spatial discretization given by “atom size”)



Solving the equations



Solve those equations: Discretize in time (n steps), Δt time step:

$$r_i(t_0) \rightarrow r_i(t_0 + \Delta t) \rightarrow r_i(t_0 + 2\Delta t) \rightarrow r_i(t_0 + 3\Delta t) \rightarrow \dots \rightarrow r_i(t_0 + n\Delta t)$$

Taylor series expansion

$$r_i(t_0 + \Delta t) = r_i(t_0) + v_i(t_0)\Delta t + \frac{1}{2}a_i(t_0)(\Delta t)^2 + \dots$$

Adding this expansion together with one for $r_i(t_0 - \Delta t)$:

$$r_i(t_0 - \Delta t) = r_i(t_0) - v_i(t_0)\Delta t + \frac{1}{2}a_i(t_0)(\Delta t)^2 + \dots$$



Steered molecular dynamics



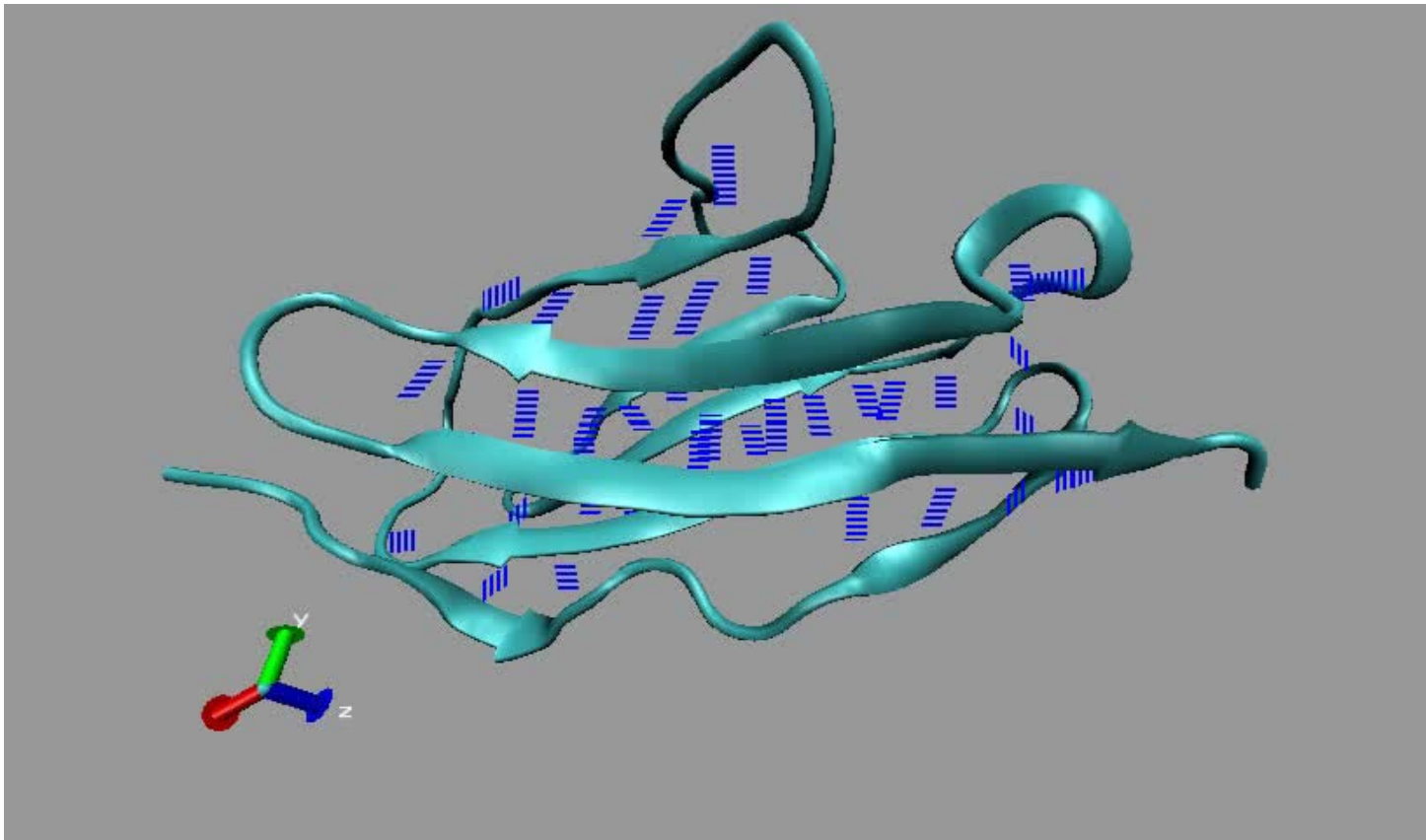
$$F = k(v \cdot t - x)$$

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Please see Fig. 6 in Buehler, M., and T. Ackbarow. "Fracture Mechanics of Protein Materials." *Materials Today* 10 (2007): 46-58.



Steered molecular dynamics



Unfolding of a titin molecule



Multi-scale simulation and experiment



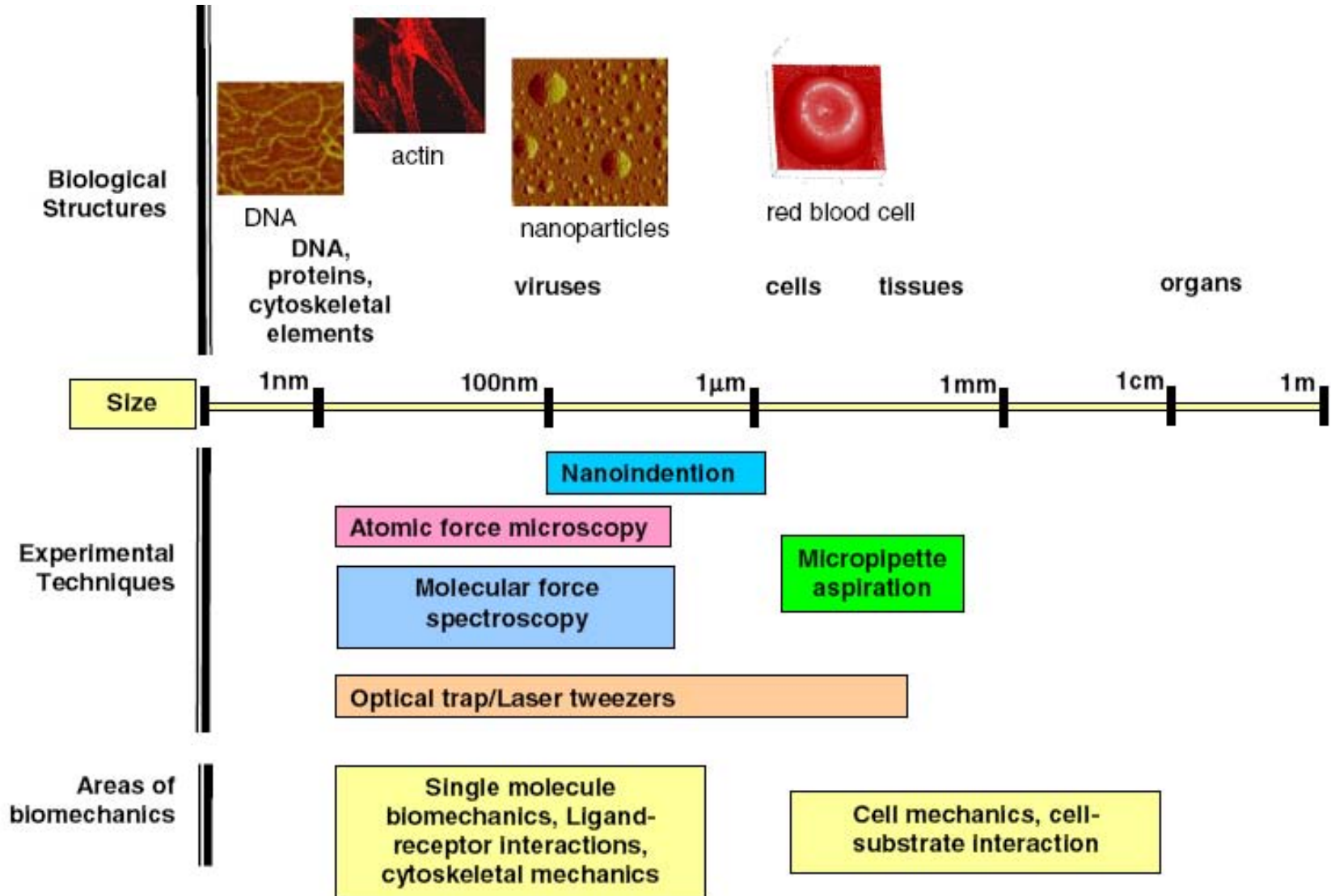
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Experimental techniques have progressed very much over the past decade – covering more time- and length-scales ...

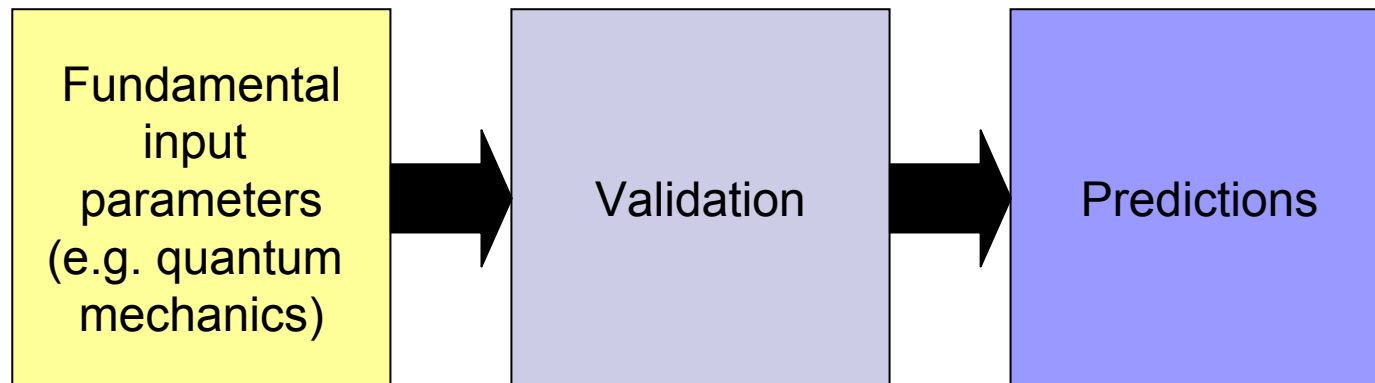


Experimental techniques





- Development of quantitative, predictive theoretical models
 - utilizing large-scale molecular dynamics simulation & multi-scale integration is in reach
- Predict behavior of complex biological materials across many hierarchies and many time- and length scales





Energy approach to elasticity
Linking atomistic scales to 'meso'-scale

Elasticity: Reversible deformation



Stretching of tropocollagen molecules

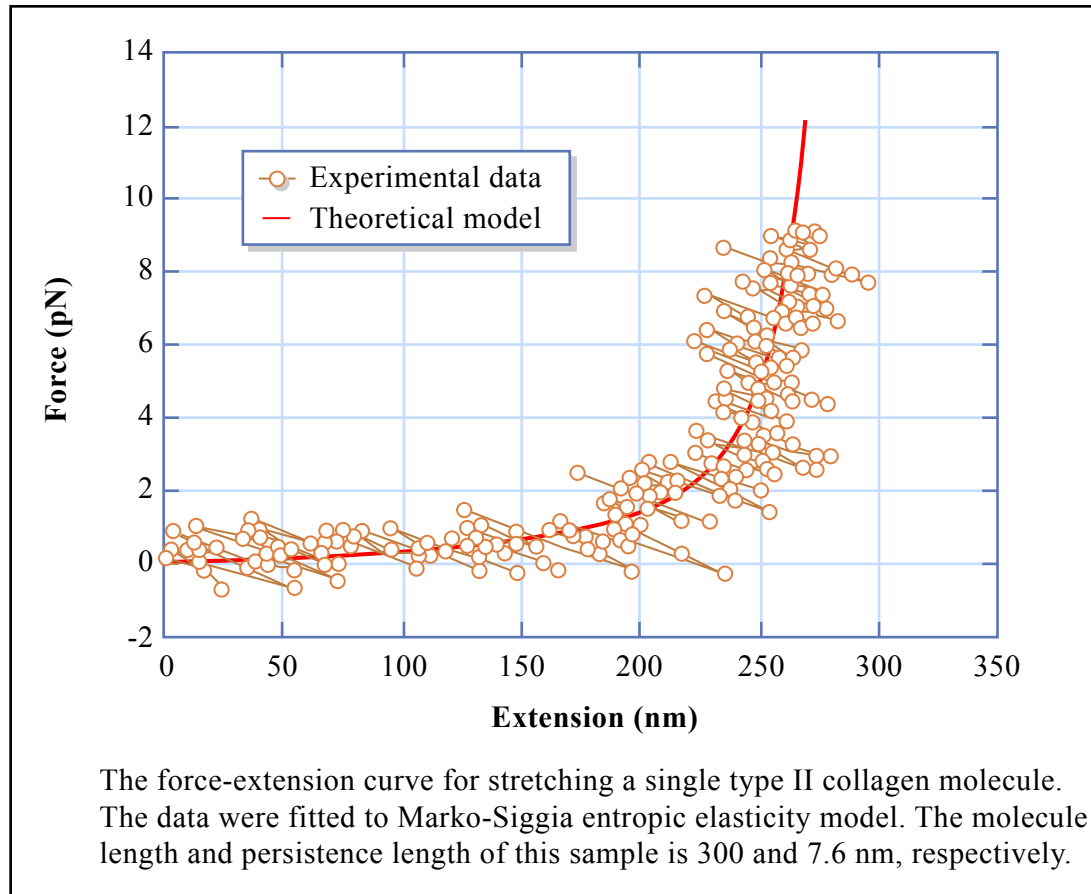
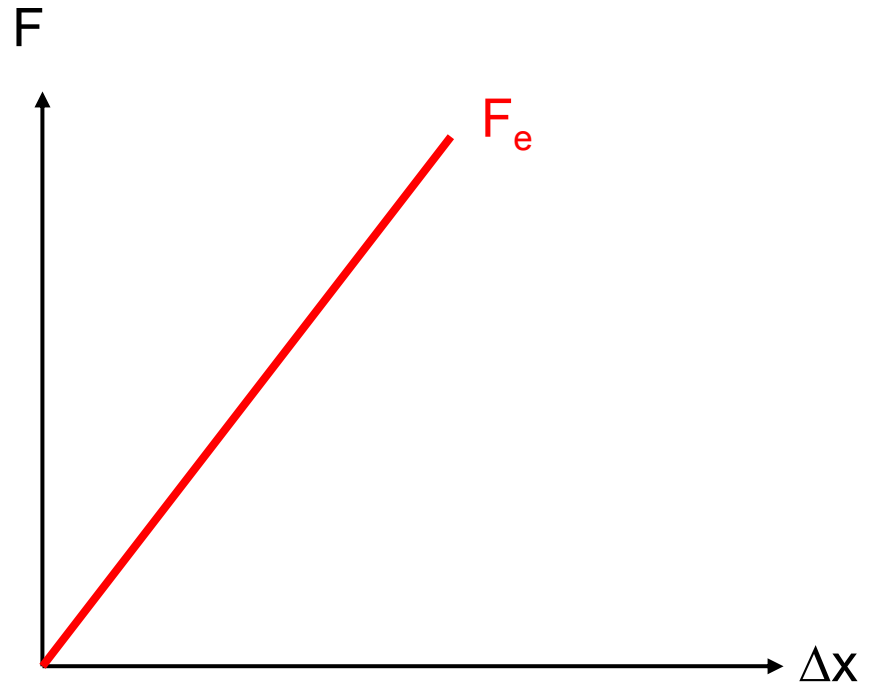
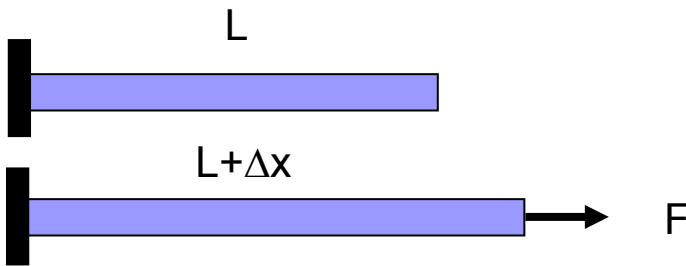


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Link between elasticity and energy state



External work rate

$$\delta W = \dot{x} F_e$$

Applied force



1st law of TD

$$\frac{dU}{dt} = \delta W + \delta Q$$

External work rate

$$\delta W = \dot{x}F_e$$

Applied force

2nd law

$$\frac{dS}{dt} \geq \frac{\delta Q}{T}$$

Change in entropy is always greater or equal than the **entropy supplied in form of heat**; difference is due to internal dissipation

$$\frac{dD}{dt} = T \frac{dS}{dt} - \delta Q \geq 0$$

Dissipation rate

$$\delta Q = dU/dt - \delta W$$

Dissipation rate after consider 1st law of TD:

$$\frac{dD}{dt} = \delta W - \frac{d}{dt} (U - TS)$$

Dissipation rate=External work rate
-change in usable energy U-TS

$F = U - TS$ is defined as free energy or Helmholtz energy,



Energy approach to elasticity



Elastic deformation (no dissipation by definition):

$$\frac{dD}{dt} = 0$$

$$\delta W - \frac{dF}{dt} = 0$$

Assume only internal energy change

$$\delta W = \dot{x}F_e$$

Expand equation

$$dU/dt = dU/dx \, dx/dt \quad \dot{x}F_e - \frac{dF}{dx} \frac{dx}{dt} = 0$$

$$\dot{x} \left(F_e - \frac{dF}{dx} \right) = 0.$$

Therefore: If applied force equals change in free energy of the system, have elastic deformation

$$F_e = \frac{dF}{dx} \quad F_e = \frac{dU}{dx}$$

With strain energy density:

$$\Psi = F/V \quad \Psi = U/V$$

$$\sigma_{ij} = \frac{d\Psi}{d\varepsilon_{ij}} \quad c_{ijkl} = \frac{d^2\Psi}{d\varepsilon_{ij}d\varepsilon_{kl}}$$



- The equations

$$\sigma_{ij} = \frac{d\Psi}{d\varepsilon_{ij}} \quad c_{ijkl} = \frac{d^2\Psi}{d\varepsilon_{ij}d\varepsilon_{kl}}$$

are significant since they provide a direct link between energy state and elasticity (stress & moduli)

- Physics of deformation:

How does free energy state change when the deformation changes?

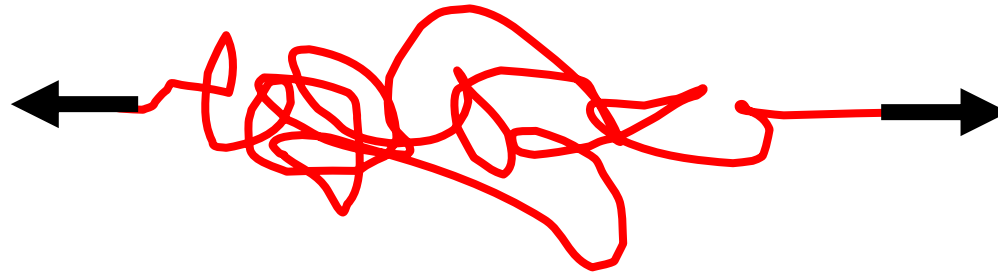
$$**F = U - TS**$$



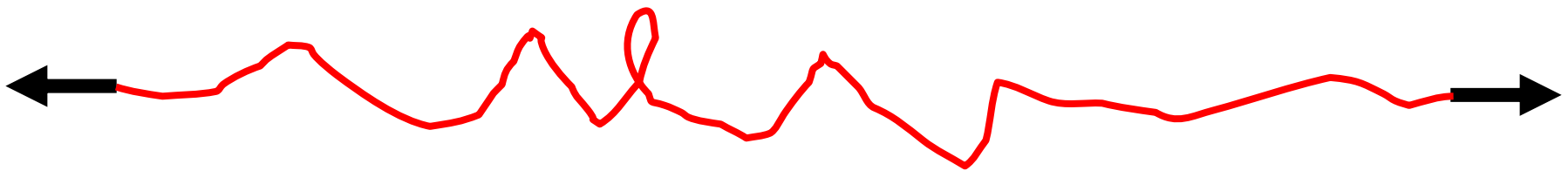
Change of free energy as a function of deformation provides an intimate link between atomistic/molecular scales and ‘continuum’ (average)



Entropic change as a function of stretch



High entropy



Low entropy

$$F = U - TS$$

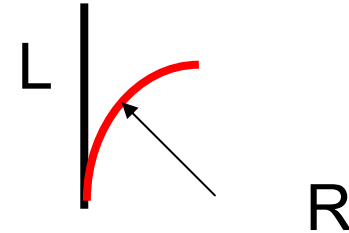


Why are molecules convoluted?



- Bending deformation (R=radius, EI=flexural rigidity of the rod)
- energy

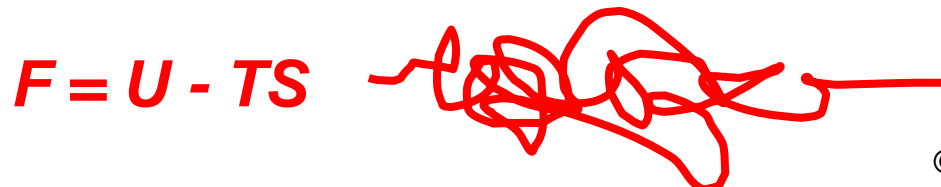
$$E_{bend} = EI \frac{L}{2R^2}$$



- Thermal (kinetic) energy per molecule (kinetic theory of gases)
- energy

$$E_{kin,mol} = \frac{3}{2} kT$$

- Example: $kT \sim 4 \times 10^{-21}$ J at body temperature
- If EI is very small, thermal energy may be enough to bend the molecule





Persistence length



$$\langle \mathbf{t}(s) \cdot \mathbf{t}(s') \rangle = e^{-|s-s'| / \xi_p}$$

$\mathbf{t}(s)$ tangent slope

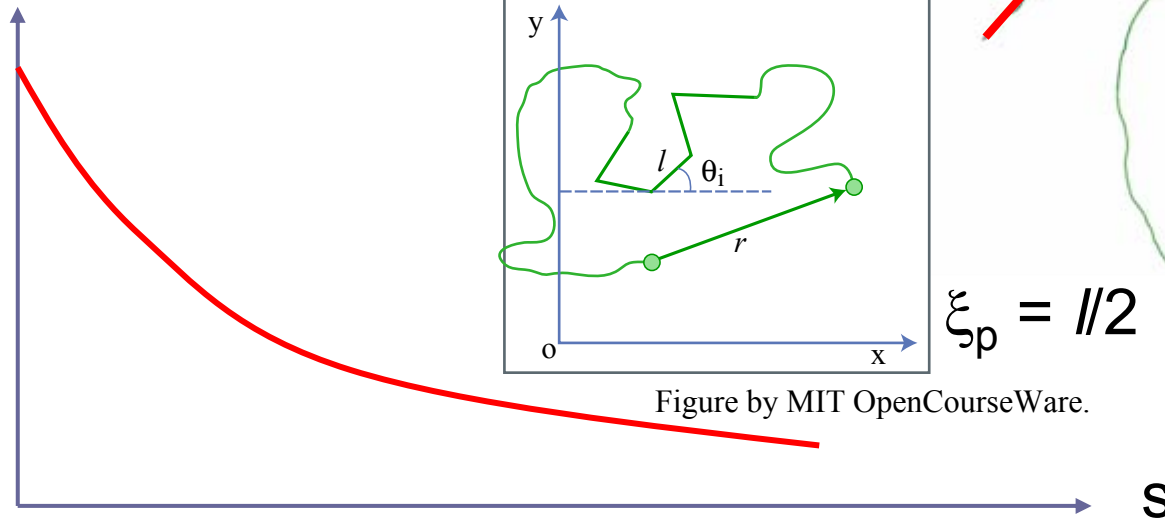


Figure by MIT OpenCourseWare.

$$\xi_p = \frac{EI}{k_B T}$$

The length at which a filament is capable of bending significantly in independent directions, at a given temperature.

This is defined by a autocorrelation function which gives the characteristic distance along the contour over which the tangent vectors $\mathbf{t}(s)$ become uncorrelated



Bending of tropocollagen molecule



Bending stiffness given by

$$EI = \frac{F_{appl} L^3}{48d}$$

$$EI = 9.71 \times 10^{-29} \text{ Nm}^2$$

(5,5) CNT: ~ 1,000 times stiffer

$$EI = 6.65 \times 10^{-26} \text{ Nm}^2$$

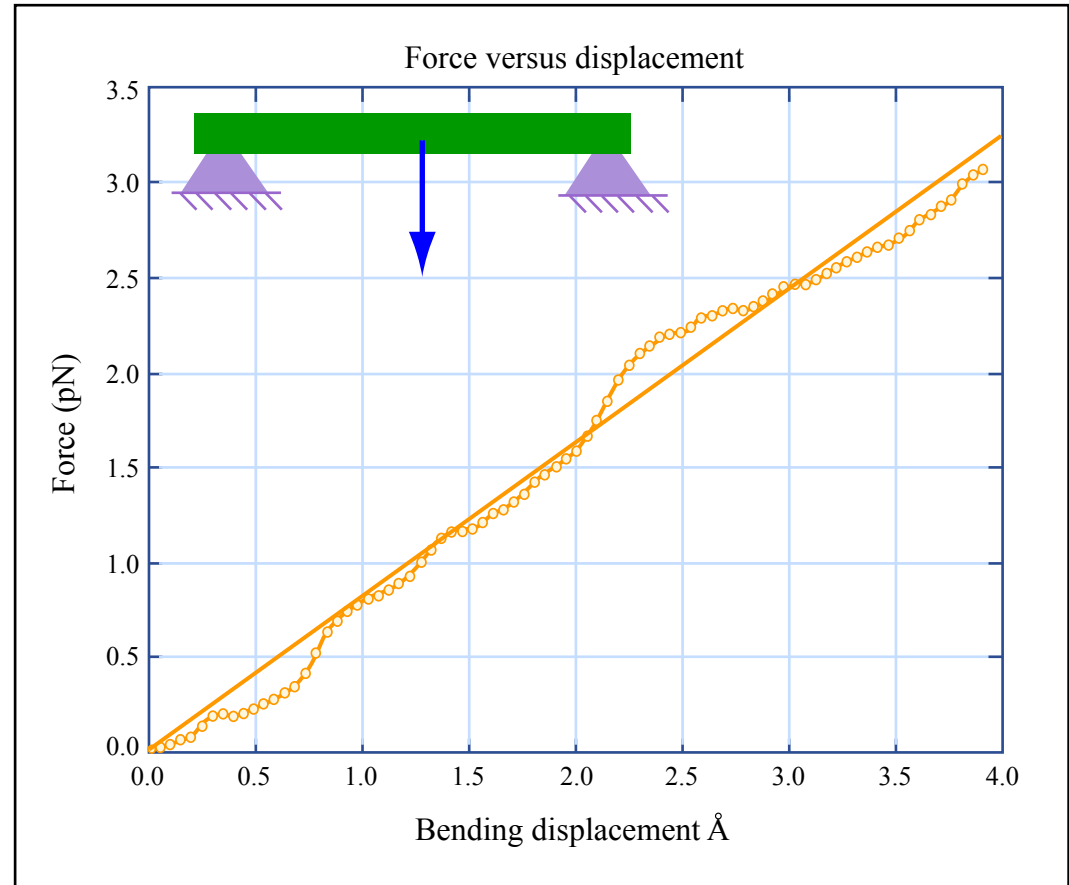


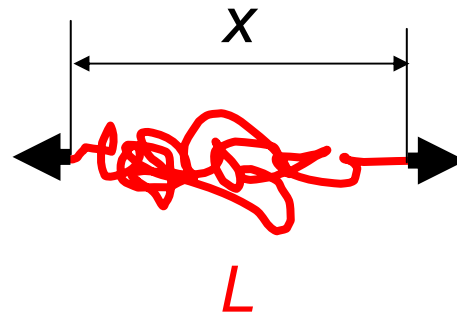
Figure by MIT OpenCourseWare.



Contour length of molecules



- The contour length of a molecule is the total length in the stretched configuration, denoted as L
- When $L \ll \xi_p$
a filament appears relatively straight.
- When $L \gg \xi_p$
a filament adopts more convoluted shapes





Entropic spring (single freely jointed chain)



- To pull a highly convoluted molecule apart, a force is necessary; define effective spring constant

No energetic interactions!

$$L \gg \xi_p$$

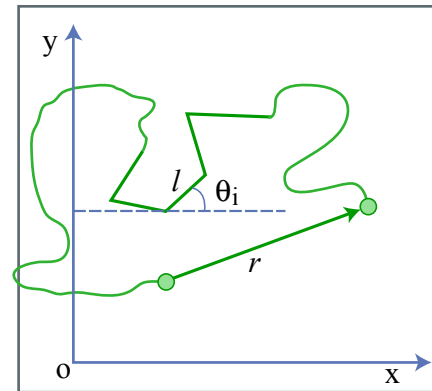
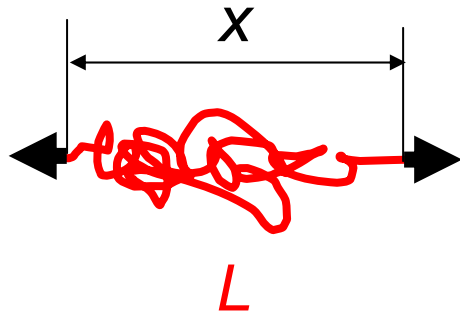


Figure by MIT OpenCourseWare.

$$k_{sp} = \frac{3kT}{2L\xi_p}$$

$$F \sim k_{sp}x \quad x \ll L$$



Note: No change in elastic energy of molecules

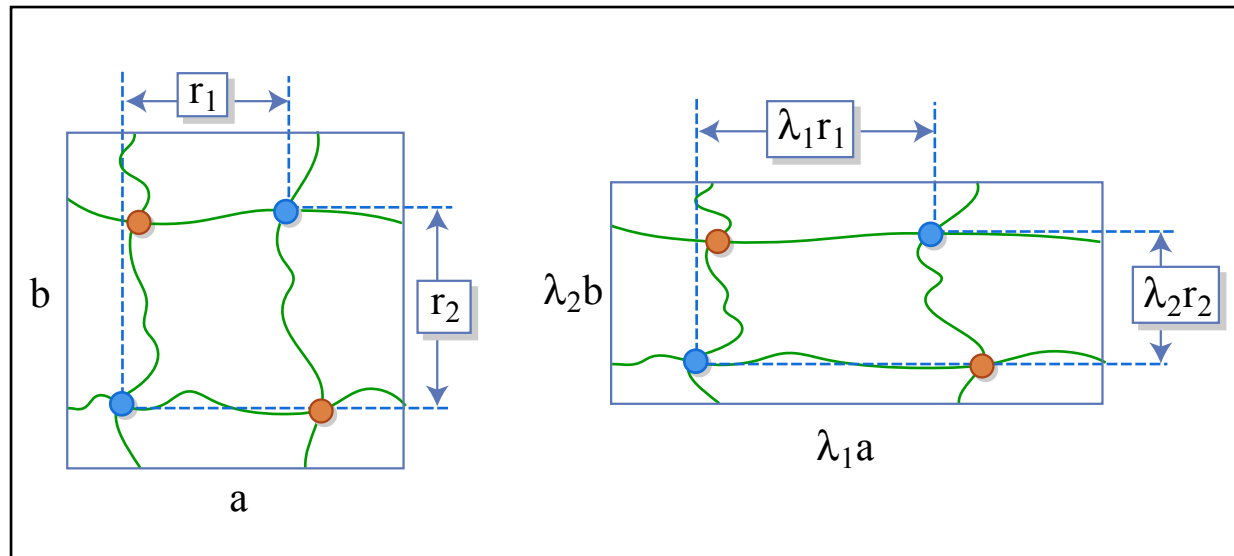


Figure by MIT OpenCourseWare.

Needed to understand elasticity: Expression of free energy as a function of the applied strain!

Entropic elasticity – therefore change in entropy



Entropic elasticity: Derivation



Freely jointed Gaussian chain with n links and length l each
(same for all chains in rubber)

$$S = c - k_B b^2 r^2 \quad \text{where} \quad b^2 = \frac{3}{2nl^2} \quad r \quad \begin{array}{l} \text{end-to-end} \\ \text{distance of} \\ \text{chain} \end{array}$$

$$\Delta S = -kb^2 \sum_{N_b} (\lambda_1^2 - 1)x^2 + (\lambda_2^2 - 1)y^2 + (\lambda_3^2 - 1)z^2$$

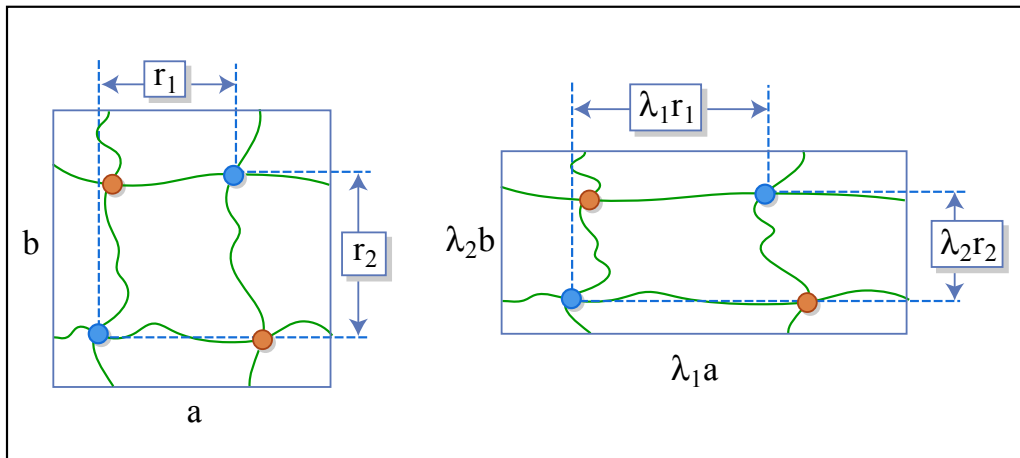


Figure by MIT OpenCourseWare.

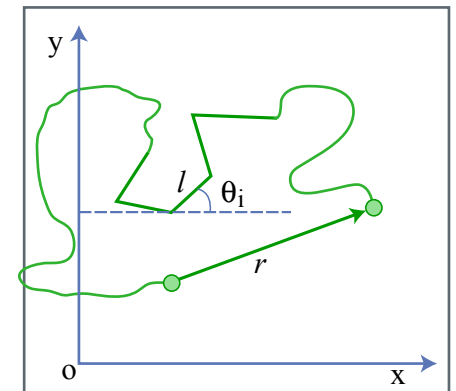


Figure by MIT OpenCourseWare.

$$F = U - TS$$



For SED: *Free energy*

$$F = -T\Delta S = \underbrace{\frac{1}{2} N_b kT}_{C = E / 6} (\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3)$$

Predictions:

$$E = 3N^* kT \quad N^* = N_b / V$$

Stiffness is proportional to temperature

$$E \sim T$$

Stiffness is proportional to degree of cross-linking (for ideal network, N^* equals twice the cross-link density)

$$E \sim N^*$$



Entropic change as a function of stretch

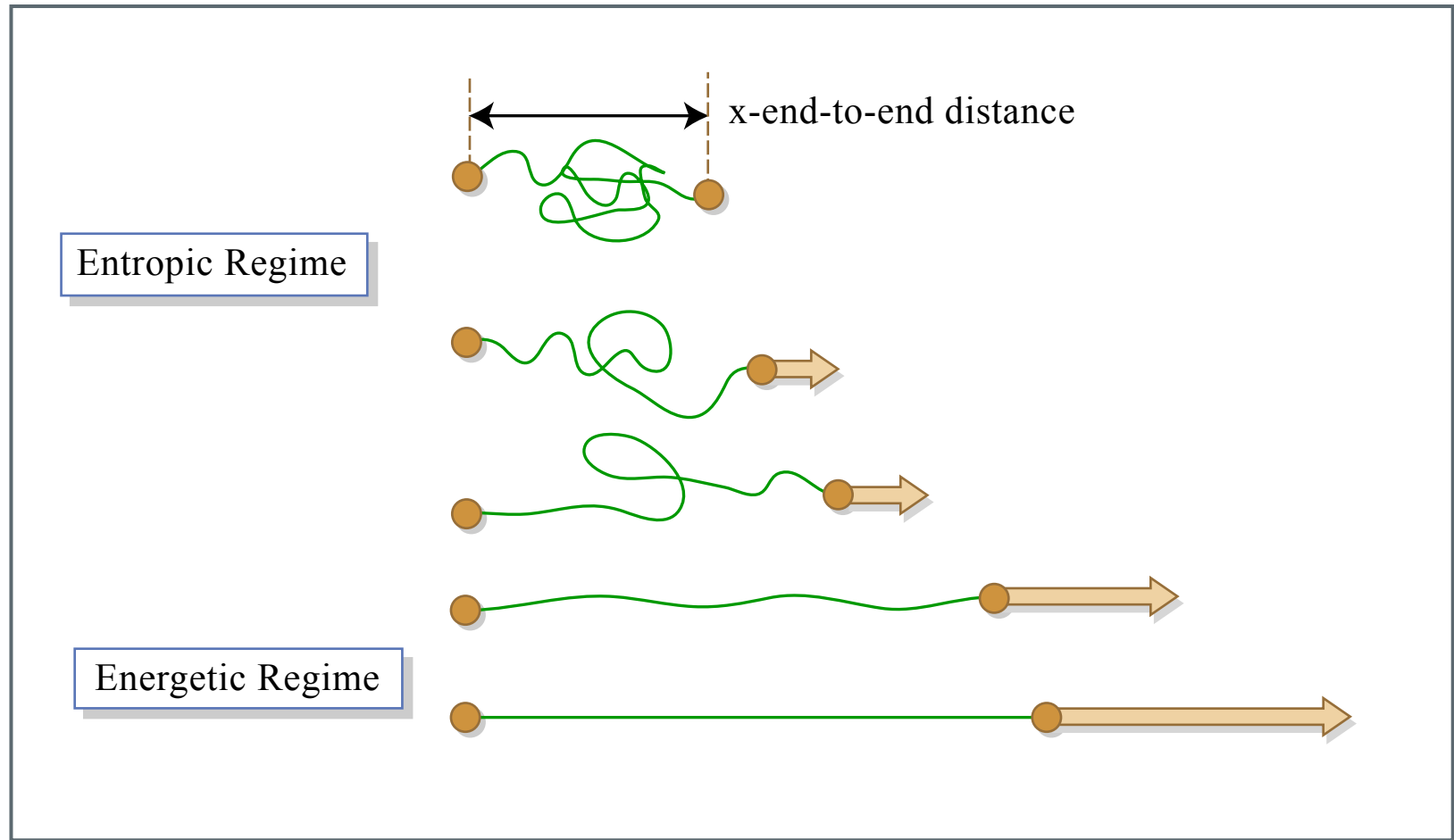


Figure by MIT OpenCourseWare.



Worm-like chain model



Freely-jointed rigid rods

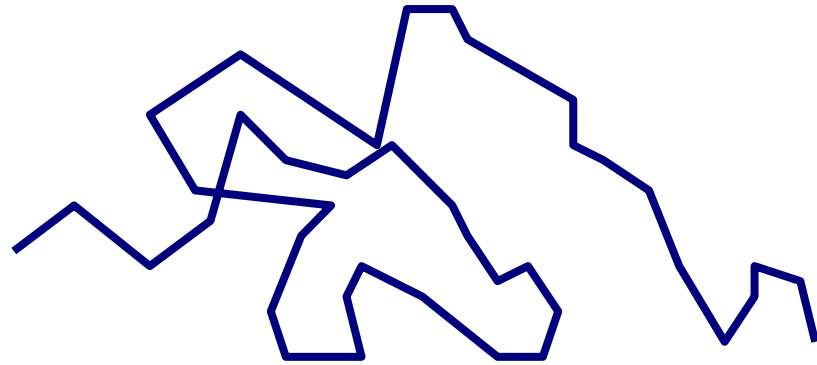
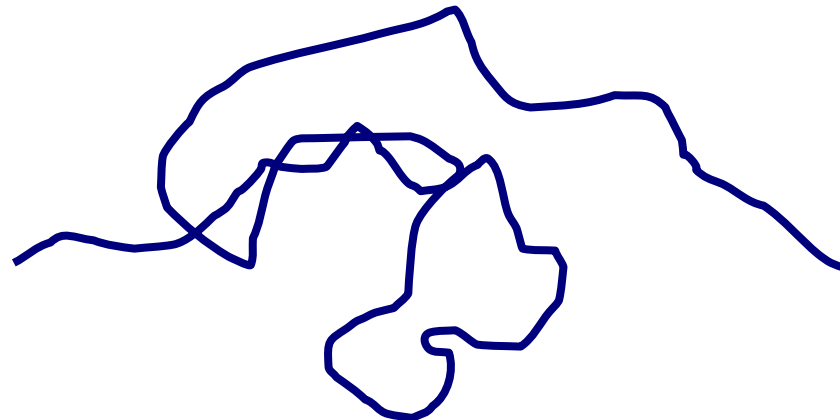


Image removed due to copyright restrictions.

DNA 4-plat electron micrograph
(Cozzarelli; Berkeley)



Continuously flexible ropes



Worm like chain model



- This spring constant is only valid for small deformations from a highly convoluted molecule, with length far from its contour length

$$x \ll L$$

- A more accurate model (without derivation) is the Worm-like chain model (WLC) that can be derived from the Kratky-Porod energy expression
- A numerical, approximate solution of the WLC model:

$$F = \frac{kT}{\xi_p} \left(\frac{1}{4} \frac{1}{(1 - x/L)^2} - \frac{1}{4} + x/L \right)$$



Applicability of WLC model

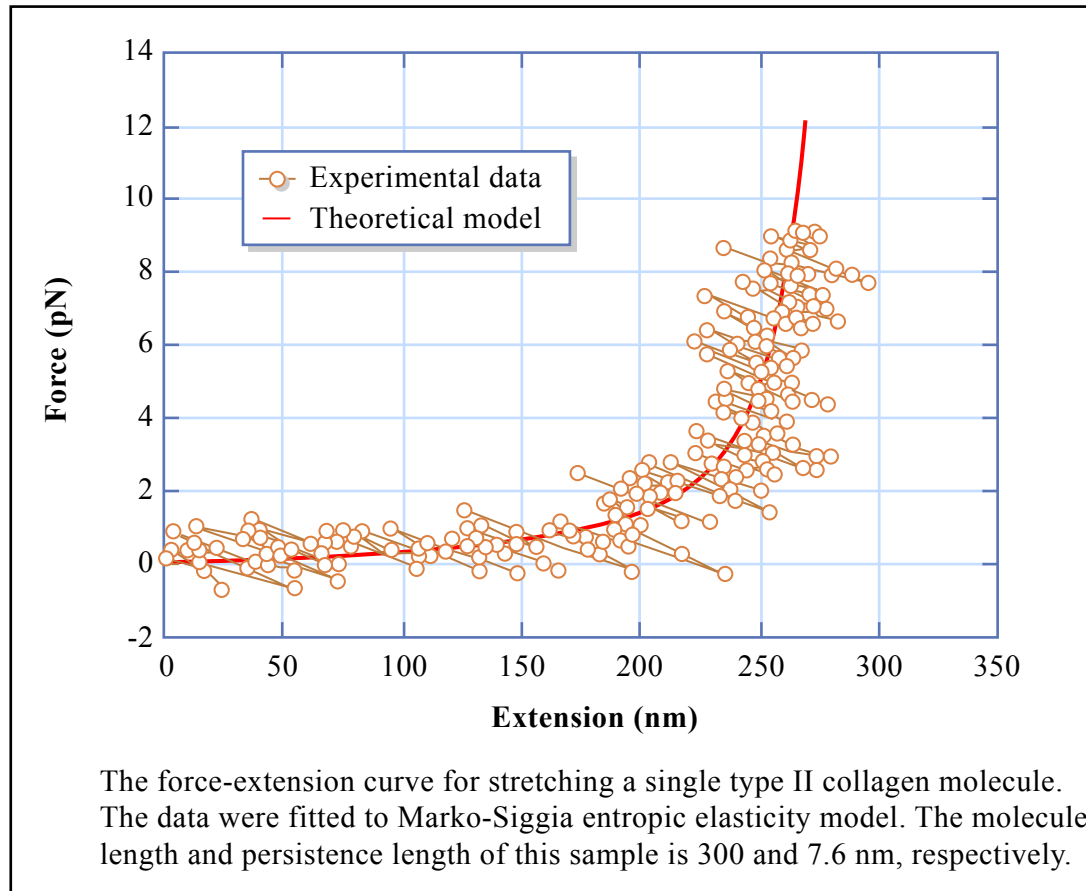


Figure by MIT OpenCourseWare. After Sun, 2004.



Summary and review

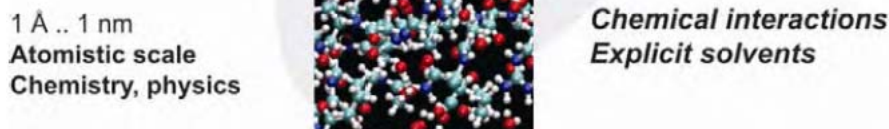
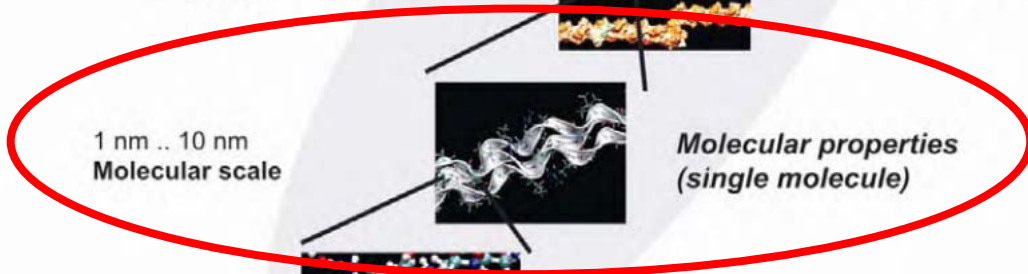
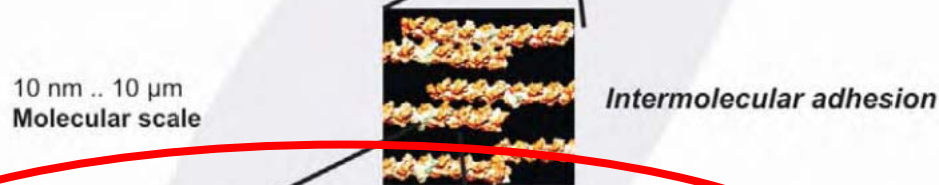
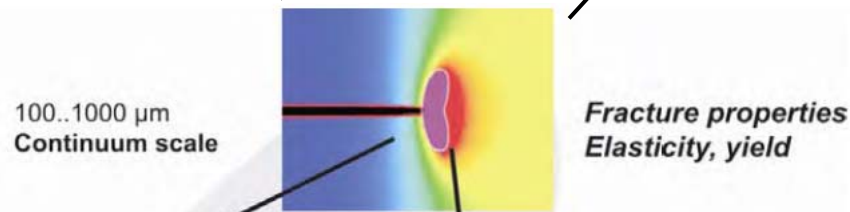
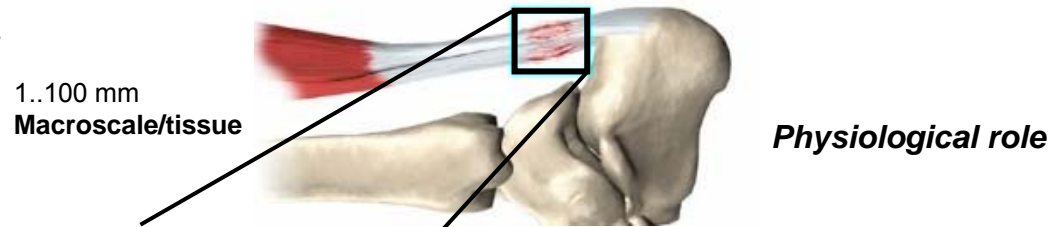
Lecture 1



Molecular mechanics: Definition of scales



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Focus of these lectures



Goal of these lectures



- Molecular mechanics with focus on biomolecules (proteins)
- Topics covered:
 - Chemical bonding in molecules & theory
 - Characterization of molecular properties (tensile stiffness, persistence length, adhesion..)
 - Application of continuum mechanical concepts to molecular mechanics
 - Link of molecular properties to tissue properties (collective behavior of many molecules, elasticity, fracture,..)
 - Molecular defects and consequence for diseases: Certain mutation may induce changes in mechanical properties, e.g. molecular defects, mutations; leads to pathological consequence (too soft, too stiff,..)
- Multi-scale modeling
- Emphasis on developing a sensitivity for the significance of molecular mechanics in biology and how atomistic and continuum viewpoints can be coupled



**GEM⁴ Summer School on Cell and Molecular
Mechanics in Biomedicine**

June 25 - July 6 2007

Molecular mechanics

Lecture 2

Markus J. Buehler

**Massachusetts Institute of Technology
Laboratory for Atomistic and Molecular Mechanics**





- Proteins are made up of amino acids
- 20 amino acids carrying different side groups (R)
- Amino acids linked by the amide bond via condensation; formation of proteins controlled by genes
- Proteins have four levels of structural organization: primary, secondary, tertiary and quaternary

Fascinating for a material scientist:

*Understand structure-function relationship for protein materials
20 building blocks; machinery of material synthesis from DNA
molecular scale control of structure, multi-functionality, ...*



20 natural amino acids



Images removed due to copyright restrictions.

Table of amino acid chemical structures.

See similar image:

<http://web.mit.edu/esgbio/www/lm/proteins/aa/aminoacids.gif>



Protein structure



- **Primary structure:** Sequence of amino acids
- **Secondary structure:** Protein secondary structure refers to certain common repeating structures found in proteins. There are two types of secondary structures: alpha-helix and beta-pleated sheet.
- **Tertiary structure:** Tertiary structure is the full 3-dimensional folded structure of the polypeptide chain.
- **Quaternary Structure:** Quaternary structure is only present if there is more than one polypeptide chain. With multiple polypeptide chains, quaternary structure is their interconnections and organization.

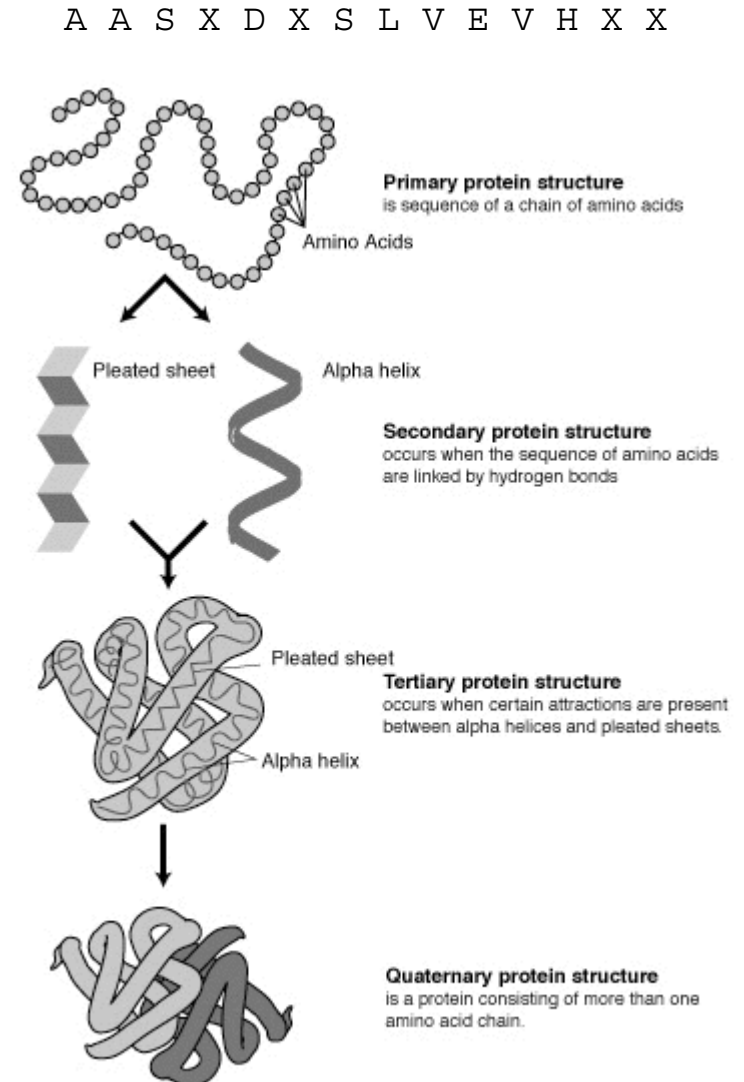


Image courtesy of NIH.



Despite the existence of a GREAT variety of protein materials, observe dominance of ‘universal building blocks’



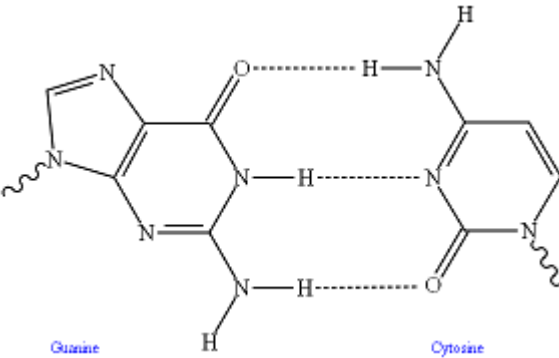
Alpha-helix (AH)



Hydrogen bonding

e.g. between O and H in H₂O
Between N and O in proteins...

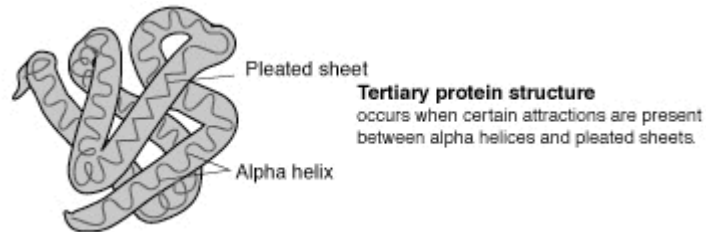
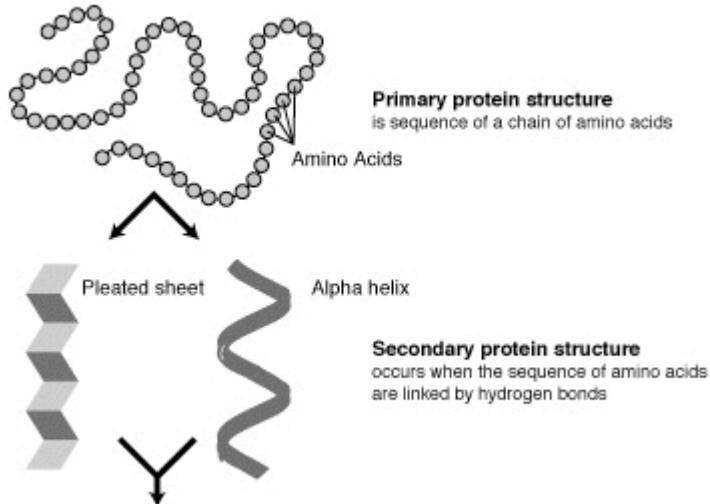
Formation of AH induced due to hydrophobic character of side chains



Images courtesy of Wikimedia Commons and NIH.

Image removed due to copyright restrictions.

For an image of the classic hydrogen bond example, hydrogen bonds in water, please see http://upload.wikimedia.org/wikipedia/commons/f/f9/3D_model_hydrogen_bonds_in_water.jpg.





Beta-sheets (BS)



Beta sheet

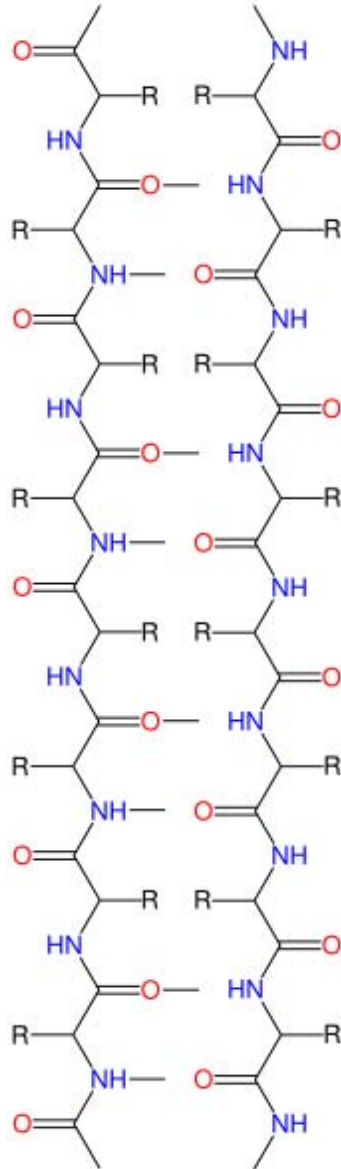


Image removed due to copyright restrictions.

For an image of the classic hydrogen bond example, hydrogen bonds in water, please see

http://upload.wikimedia.org/wikipedia/commons/f/f9/3D_model_hydrogen_bonds_in_water.jpg

Present in mechanically relevant proteins

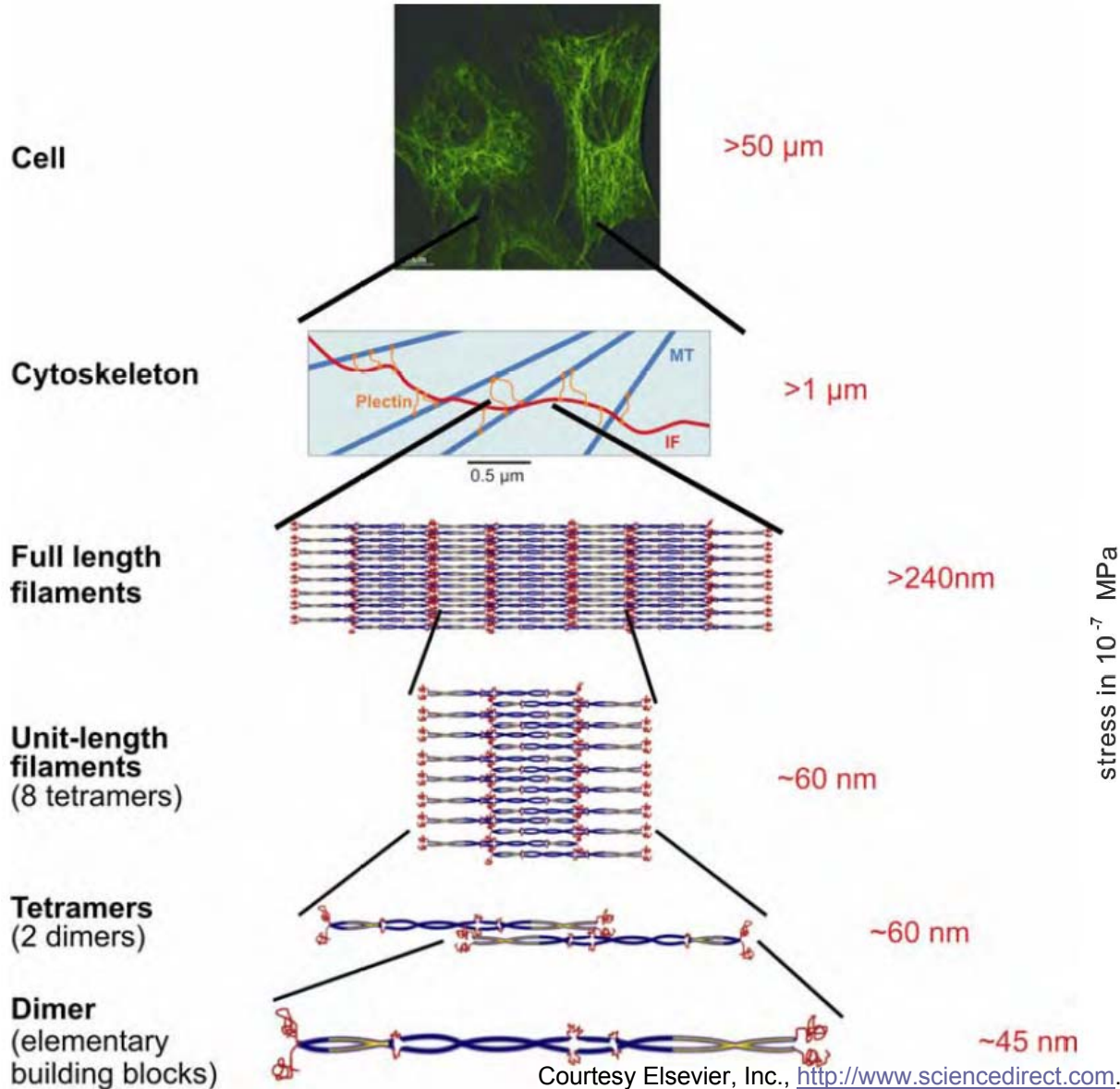
Spider silk

Fibronectin

Titin (muscle tissue)

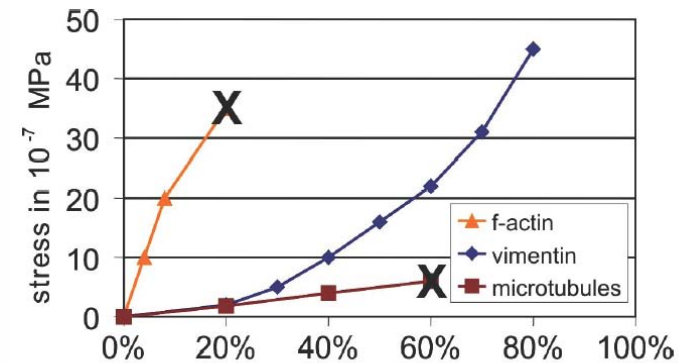


Case study: Intermediate filament proteins



Intermediate filaments (IFs)

Structure significant under large deformation
Forms nuclear envelope (lamin)

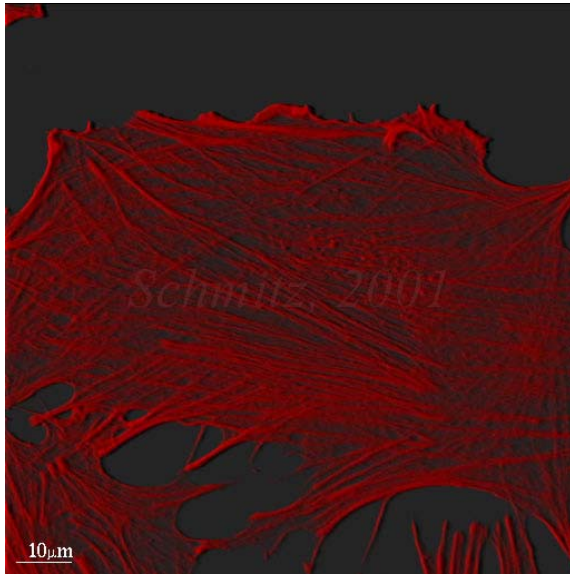


Janmey PA, Euteneuer U, Traub P, Schliwa M (1991) J Cell Biol 113:155

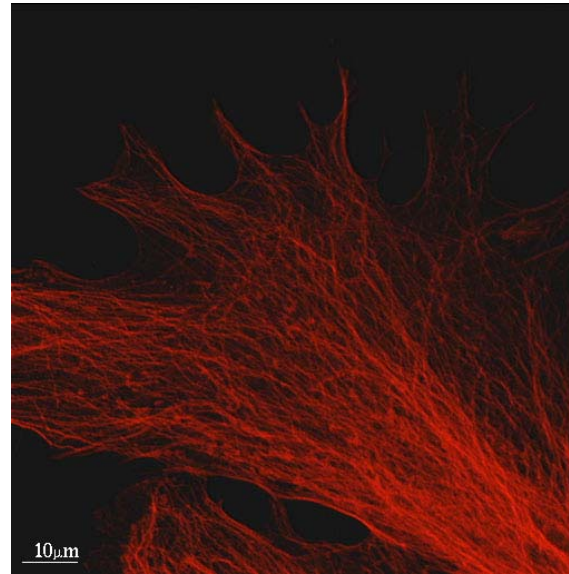
Courtesy of Rockefeller University Press via CC BY-NC-SA license.



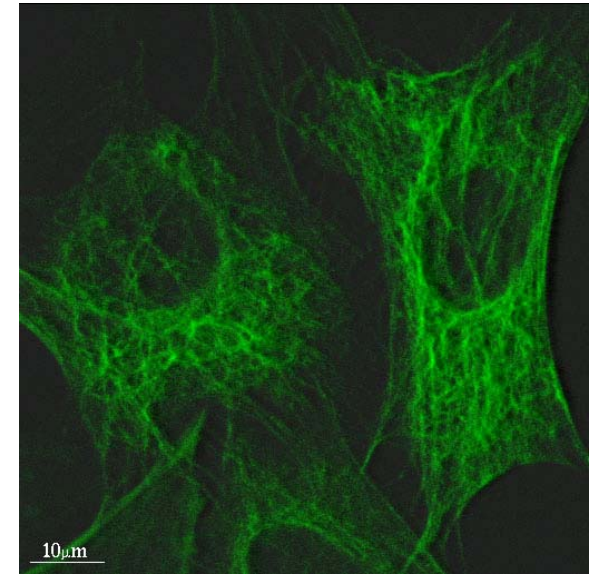
The cell's cytoskeleton



Actin

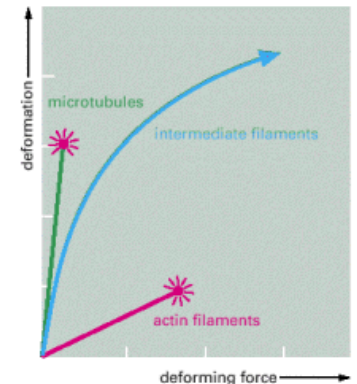


Tubulin



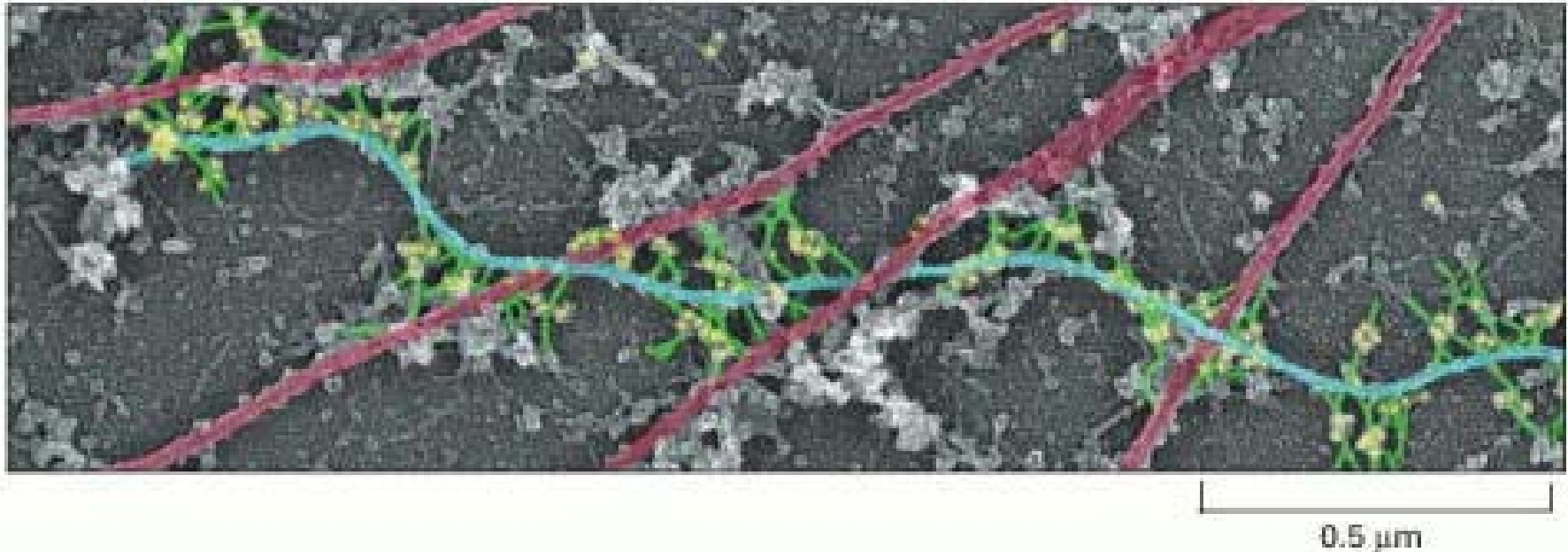
Vimentin

Fluorescent staining for different protein networks





Cross-linking of cytoskeletal elements



Courtesy of Rockefeller University Press via CC BY-NC-SA license.

Plectin cross-linking of diverse cytoskeletal elements. Plectin (*green*) makes cross-links from **intermediate filaments (*blue*)** to other intermediate filaments, to **microtubules (*red*)**, and to **myosin thick filaments**. In this electron micrograph, the dots (*yellow*) are gold particles linked to anti-plectin antibodies. The entire actin filament network was removed to reveal these proteins.

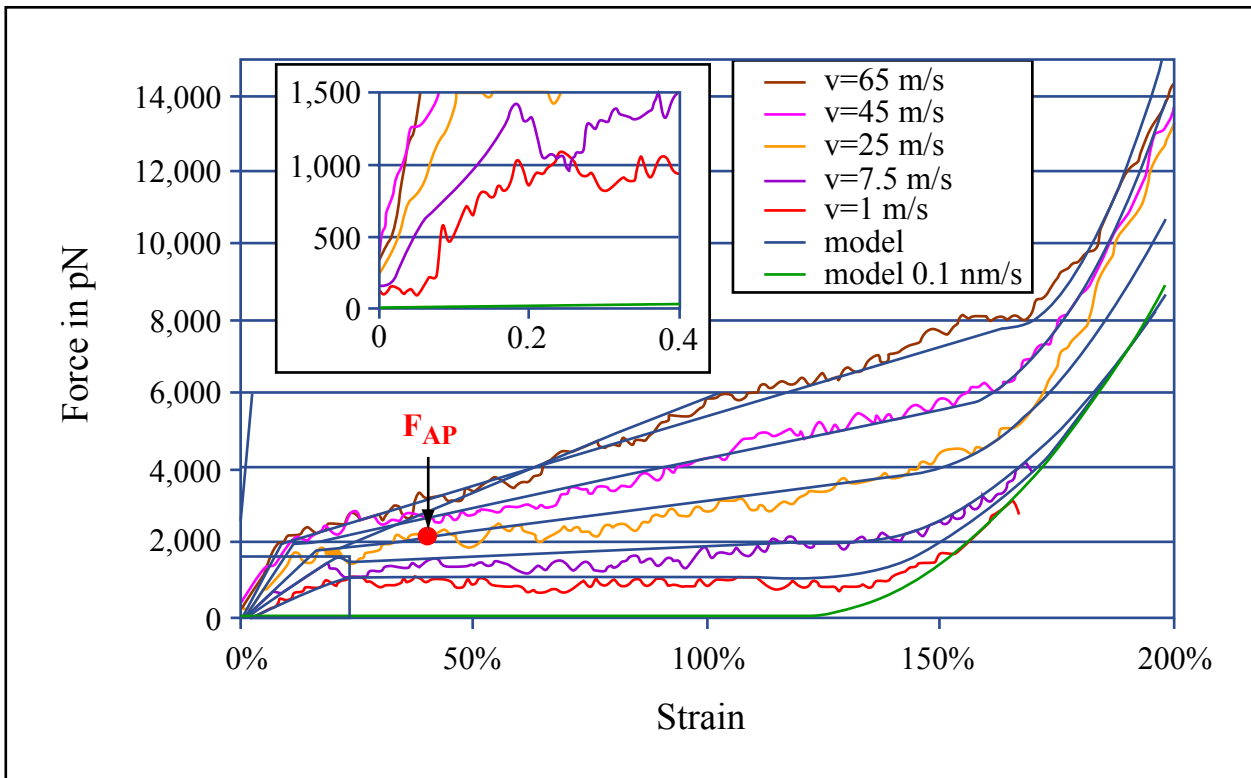
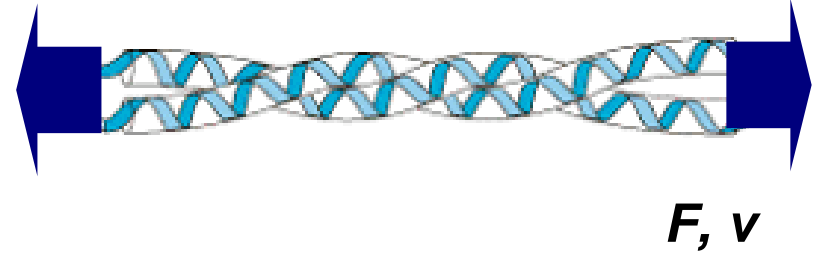
(From T.M. Svitkina and G.G. Borisy, *J. Cell Biol.* 135:991–1007, 1996)



Mechanics of vimentin dimers



- Stage I:** Uncoiling of each alpha helix
- Stage II:** Uncoiling of coiled-coil
- Stage III:** Stretching of protein backbone



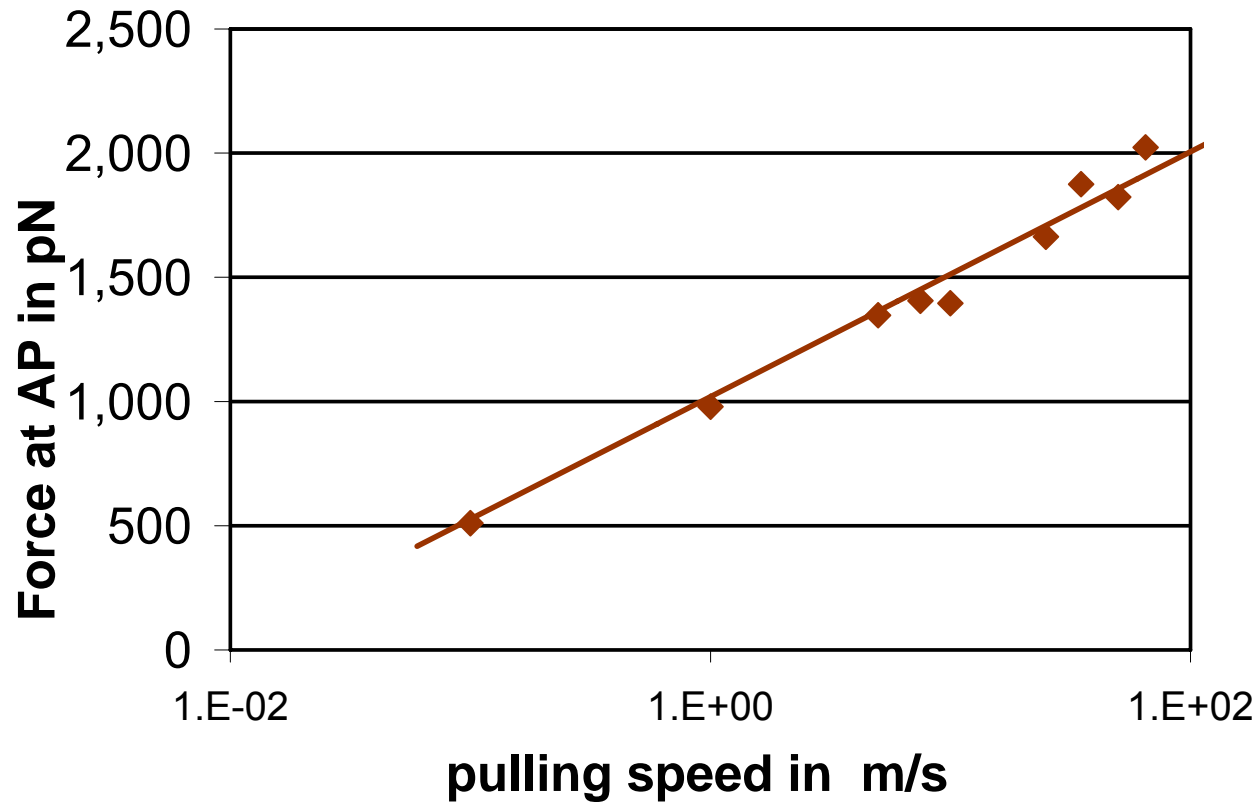
$$\sigma = F/A$$

A: Cross-sectional area of molecule

Figure by MIT OpenCourseWare. After Ackbarow and Buehler, 2006.



Unfolding force depends logarithmically on pulling speed



Larger pulling speed leads to larger resistance against unfolding



Studies on other protein structures



MD simulation



Courtesy Elsevier, Inc., <http://www.sciencedirect.com>.
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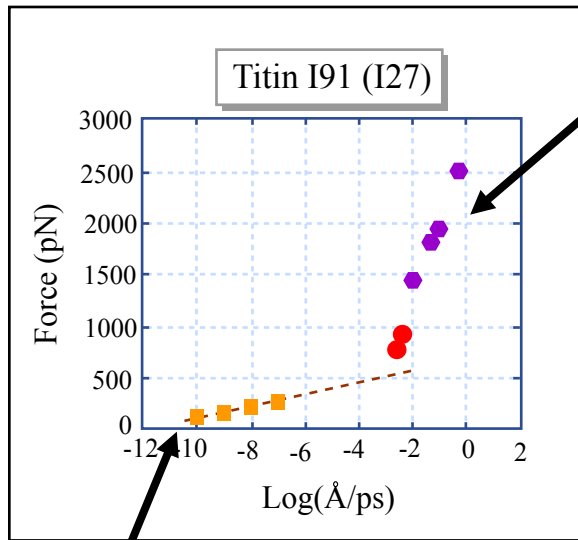
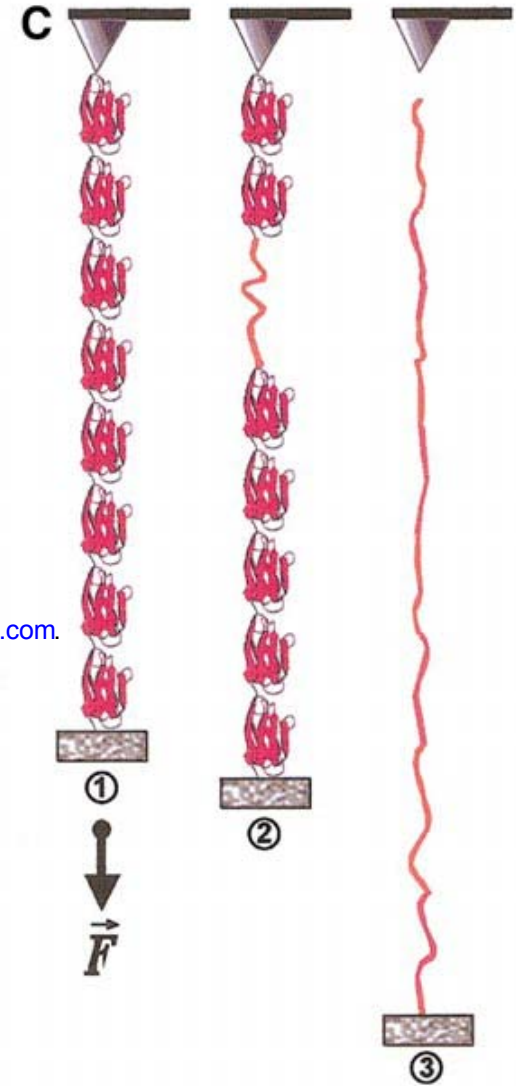


Figure by MIT OpenCourseWare. After Schulten, 2007.

Experiment





Molecular defects in vimentin

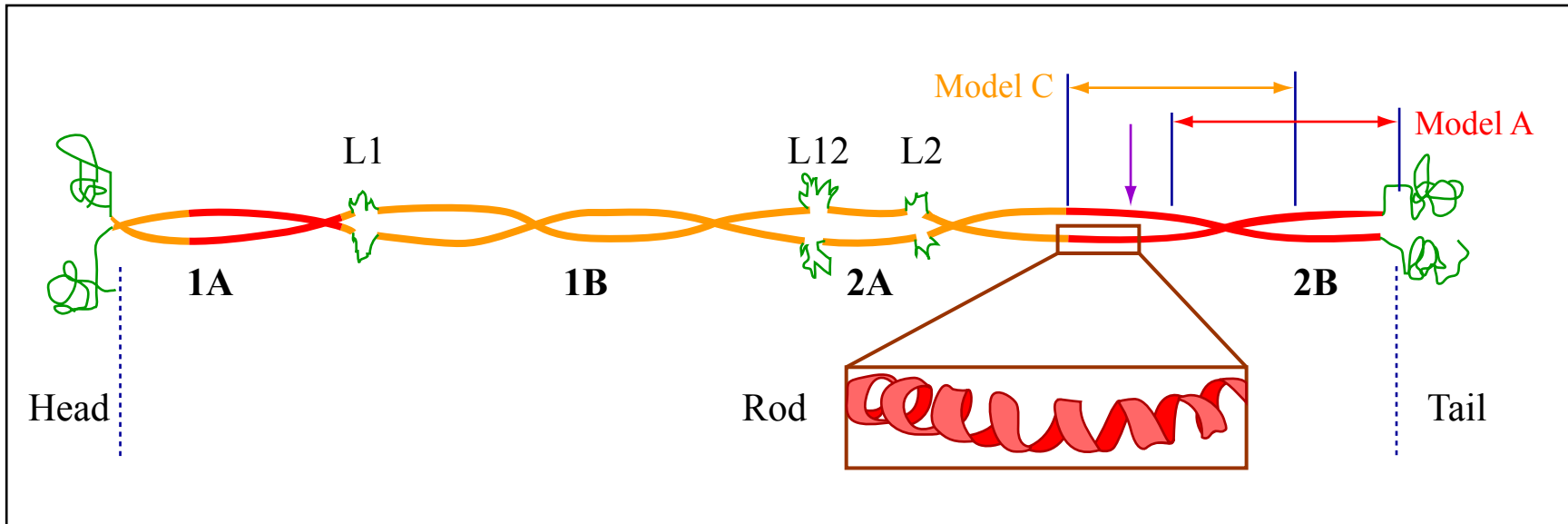


Figure by MIT OpenCourseWare. After Ackbarow and Buehler, 2008.

'Skips' are insertions of one residue into the heptad pattern

'Stammers' result through an insertion of three additional residues

'Stutters' appear if four additional residues interrupt the heptad sequence: Presence of a stutter results in an almost parallel run of both AHs without interrupting the CC geometry.



Pulling rate dependence: Bell model



Probability for bond rupture

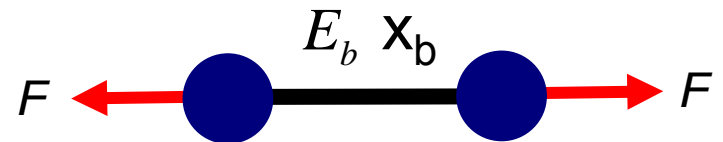
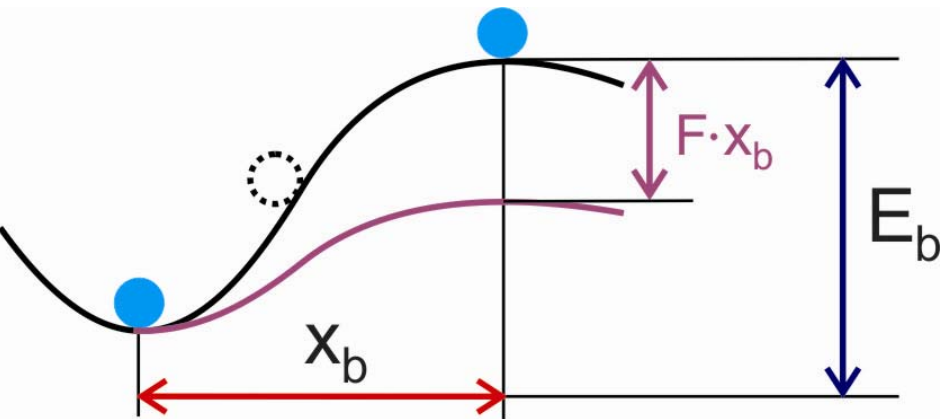
$$p = \exp \left[-\frac{E_b - Fx_B}{k_B T} \right]$$

“off rate”

$$\chi = \omega_0 \cdot \exp \left(-\frac{(E_b - F \cdot x_b)}{k_b \cdot T} \right)$$

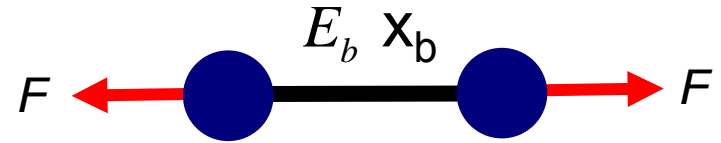
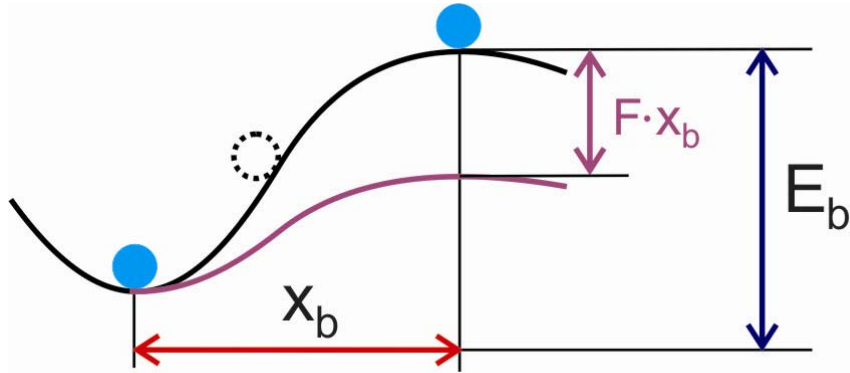
$$p = \tau^{-1} \omega^{-1}$$

$$\omega = 1 \times 10^{13} / \text{sec}$$





Pulling rate dependence: Bell model

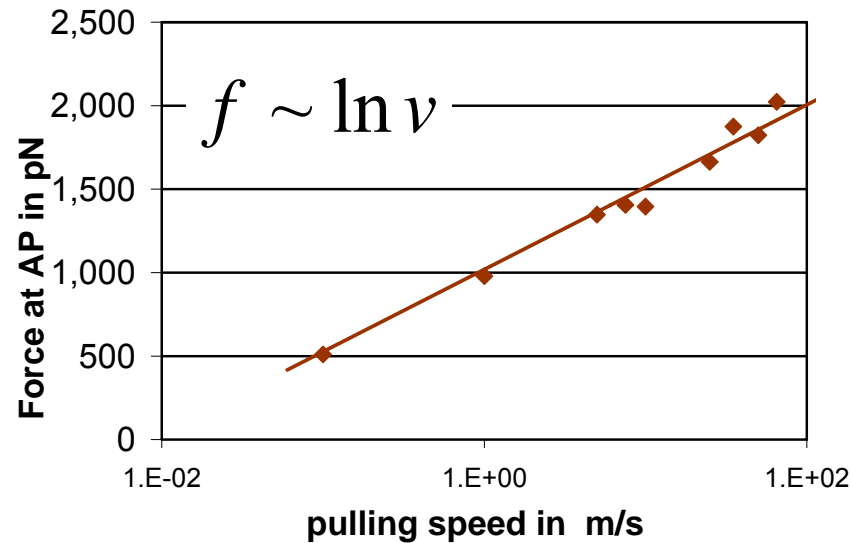


Pulling velocity

$$v = \omega_0 \cdot x_b \cdot \exp\left(-\frac{(E_b - F \cdot x_b)}{k_b \cdot T}\right)$$

Rupture force

$$F(v) = \frac{k_b \cdot T}{x_b} \cdot \ln v - \frac{k_b \cdot T}{x_b} \cdot \ln v_0 = a \cdot \ln v + b$$



Energy barrier: $E_b \approx 5.6$ kcal/mol

$x_b \approx 0.17$ Å: Suggests rupture of single H-bond



Change in deformation mechanism

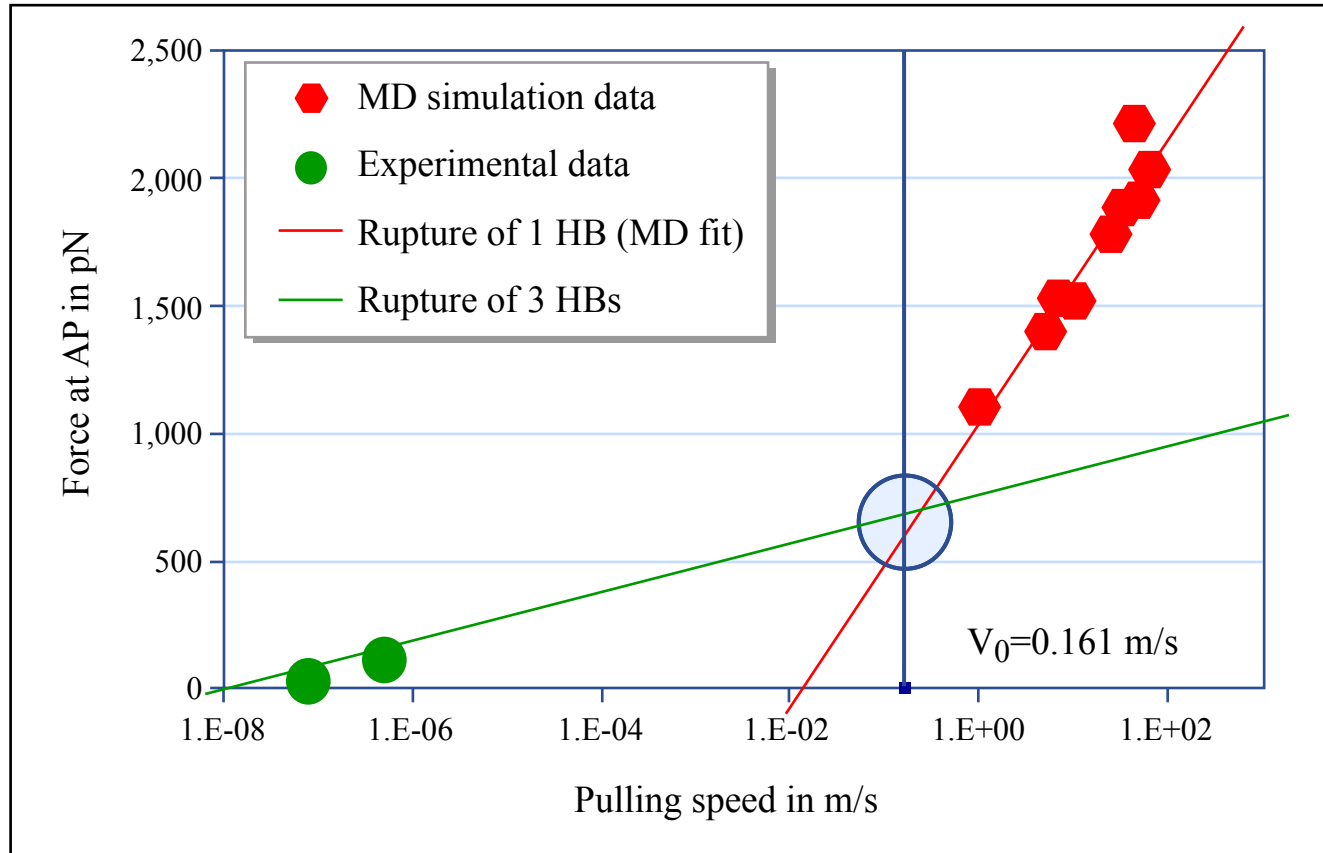


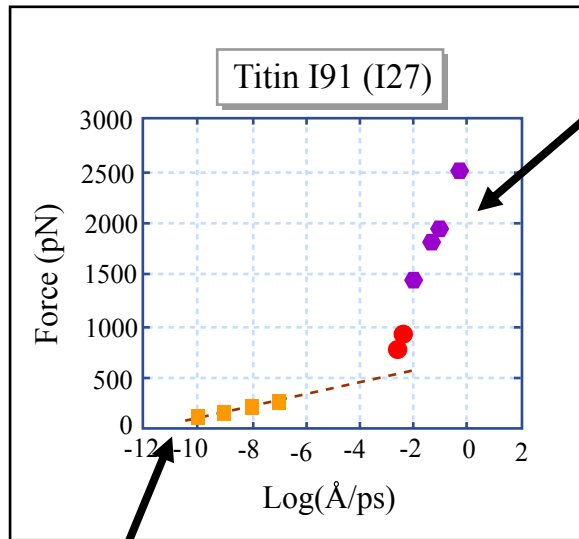
Figure by MIT OpenCourseWare. After Ackbarow and Buehler, 2006.



Studies on other protein structures



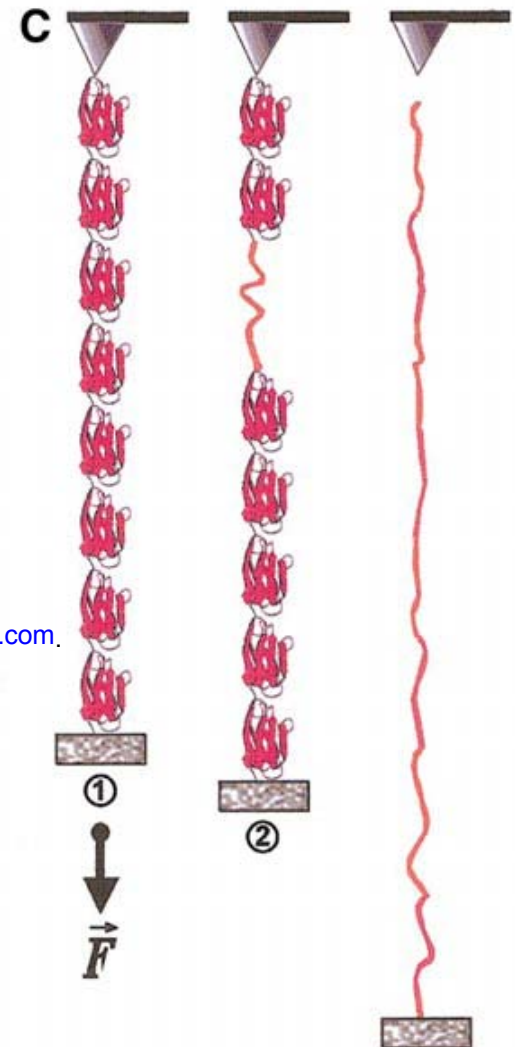
MD simulation



Courtesy Elsevier, Inc., <http://www.sciencedirect.com>.
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Figure by MIT OpenCourseWare. After Schulten, 2007.

Experiment





Change in deformation mechanism

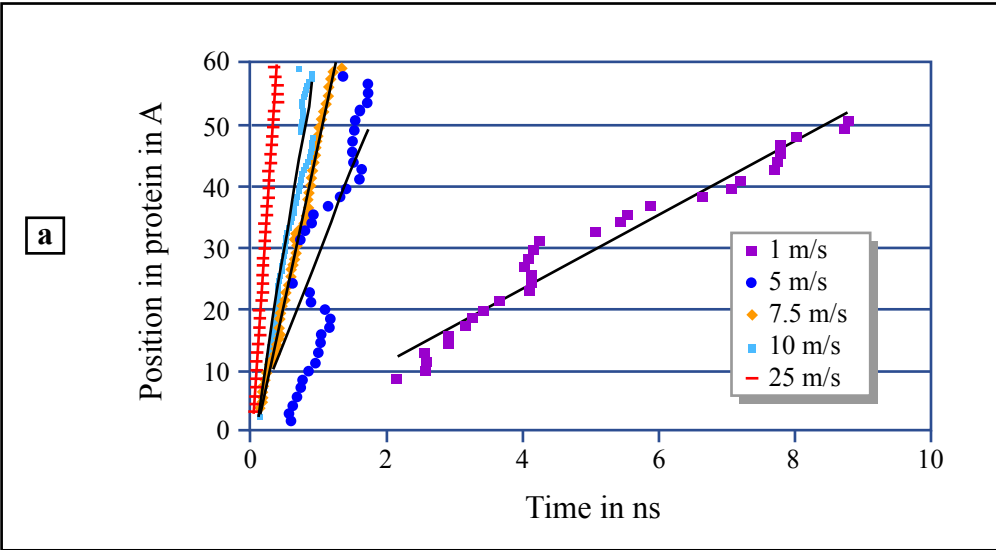


Figure by MIT OpenCourseWare. After Ackbarow and Buehler, 2006.

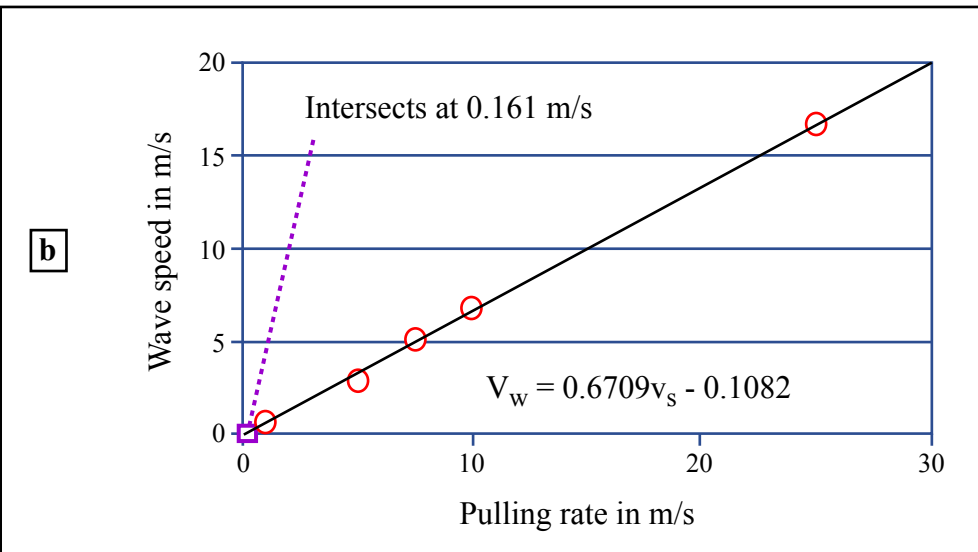
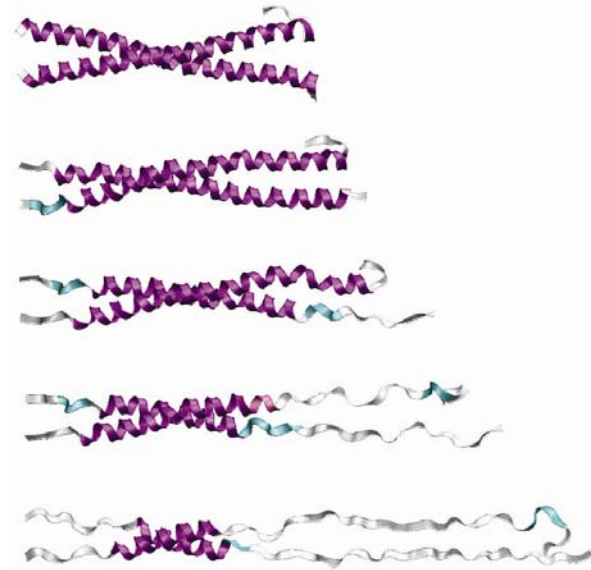


Figure by MIT OpenCourseWare.



Slower rates –
deformation ‘wave’
vanishes



Change in deformation mechanism

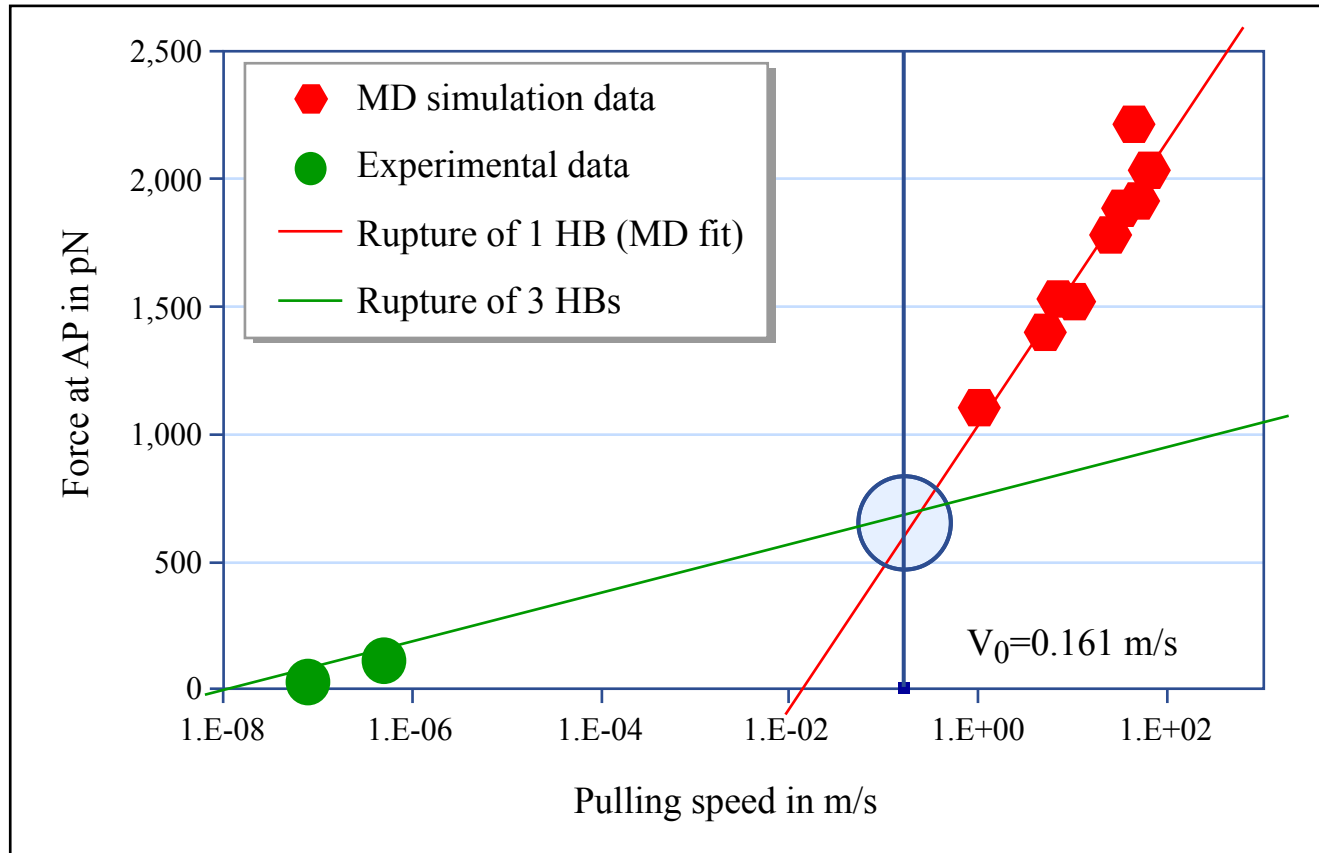


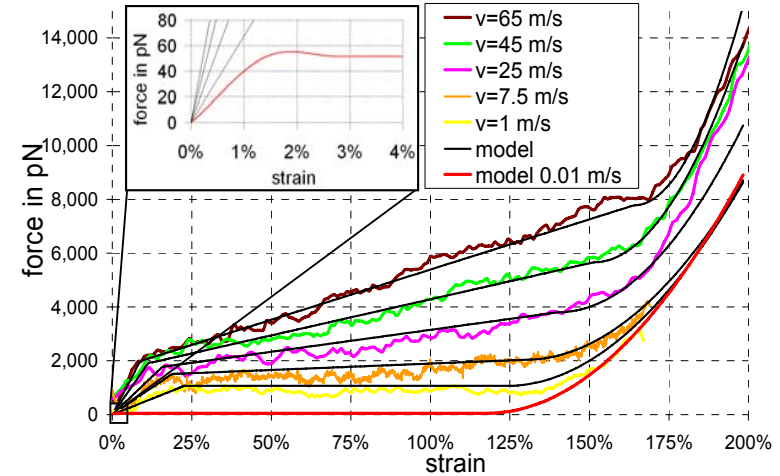
Figure by MIT OpenCourseWare. After Ackbarow and Buehler, 2006.

Predict: Strain distribution homogeneous below v_0 , no localization



Three regimes

$$F(x, v) = \begin{cases} F_1 = (F_a \cdot x) / x_a & \text{if } x < x_a \\ F_2 = s_2 \cdot (x - x_a) + F_a & \text{if } x_a \leq x < x_s \\ F_3 = s_3 \cdot (x - x_s)^2 + F_s & \text{if } x \geq x_s \end{cases}$$



$$s_1 = c_5 \cdot v + c_6, \quad s_2 = \max(c_7 \cdot v + c_8; 0), \quad \underline{F_a = c_1 \cdot \ln(v) + c_2}, \quad \text{and} \quad x_a = F_a / s_1$$

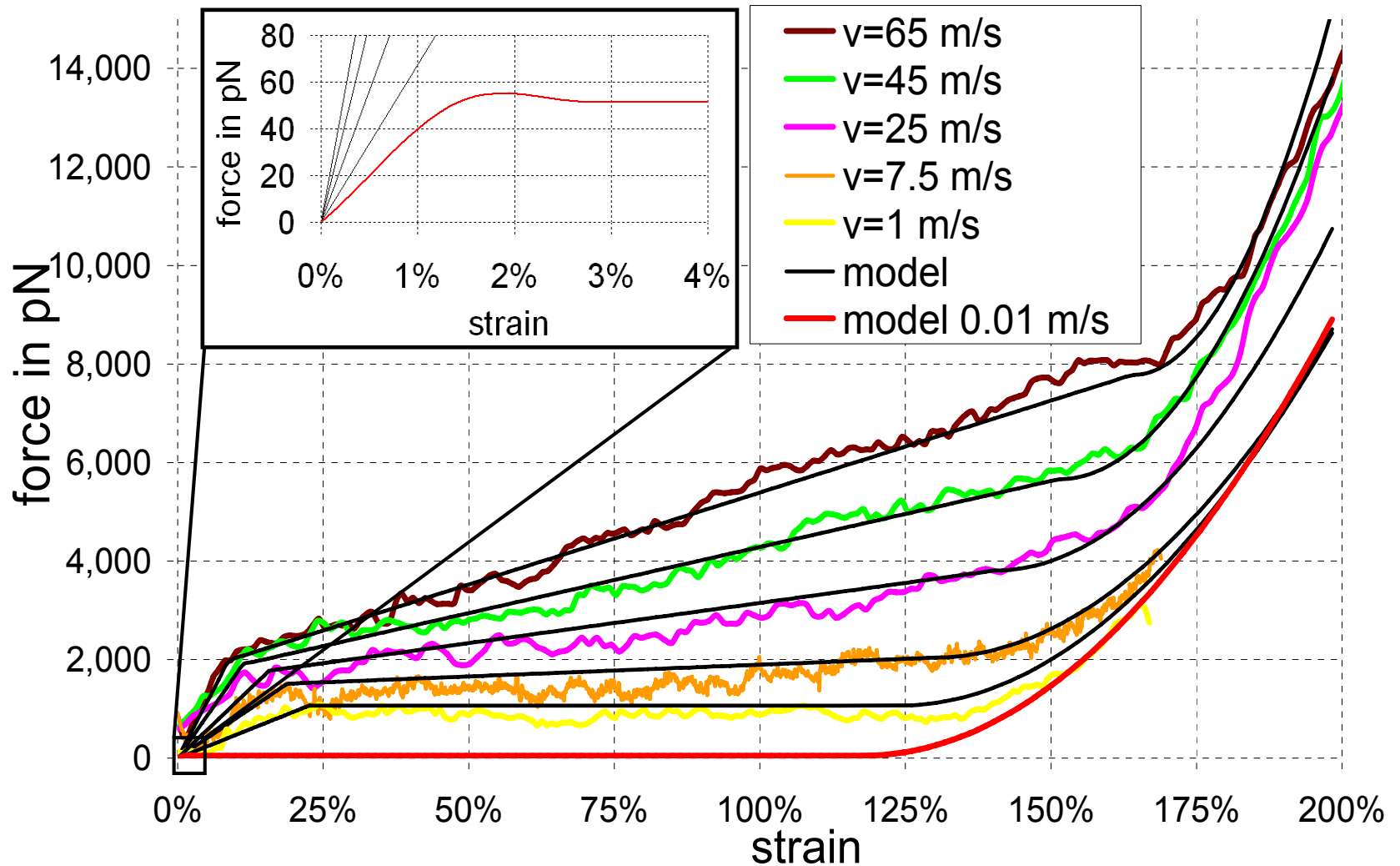
$$F_s = c_3 \cdot x_s + c_4,$$

$$s_3 = c_9 \cdot v^2 + c_{10} \cdot v + c_{11} \quad \text{and} \quad x_s = \left(F_a \cdot \left(1 - \frac{s_2}{s_1} \right) - c_4 \right) / (c_3 - s_2)$$

Numerical parameters and its units										
c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}
220	3,604	237	-19,65	723,894	59.6	173,519	-6.3	2.4E7	-4.55	2.7
pN	pN	pN/Å	pN	pN·fs/Å ²	pN/Å	pN·fs/Å ²	pN/Å	pN·fs ² /Å ³	pN·fs/Å ²	pN/Å



Atomistically informed continuum theory





"cell's safety belt"

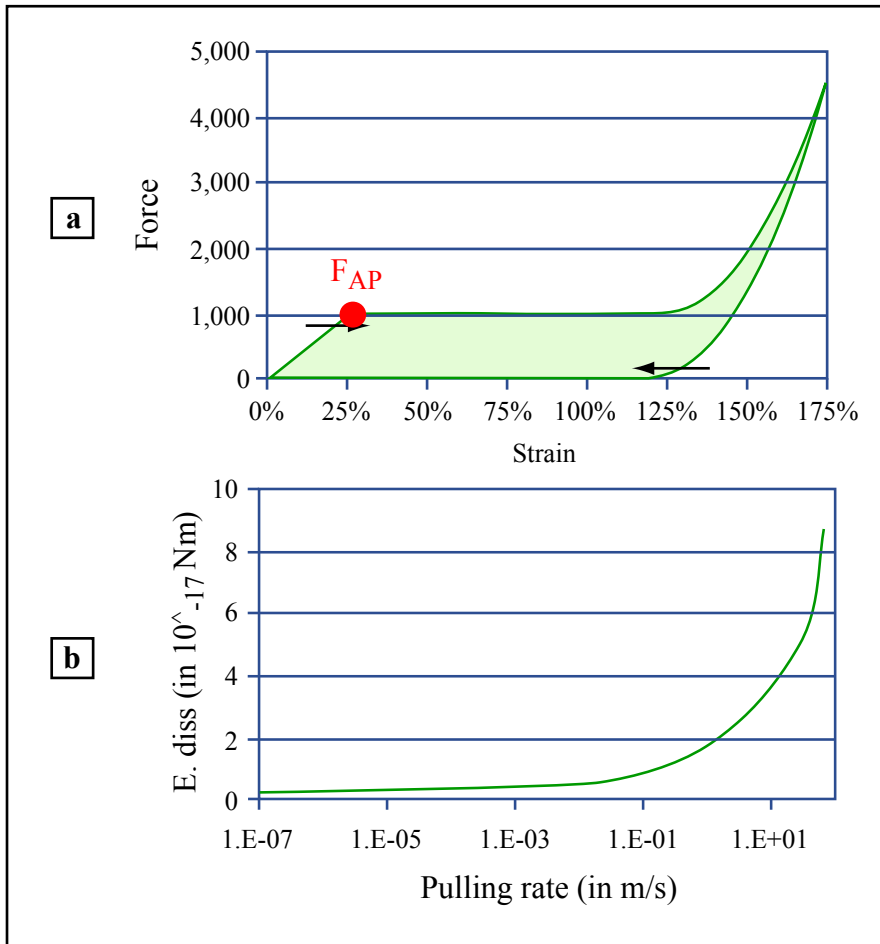
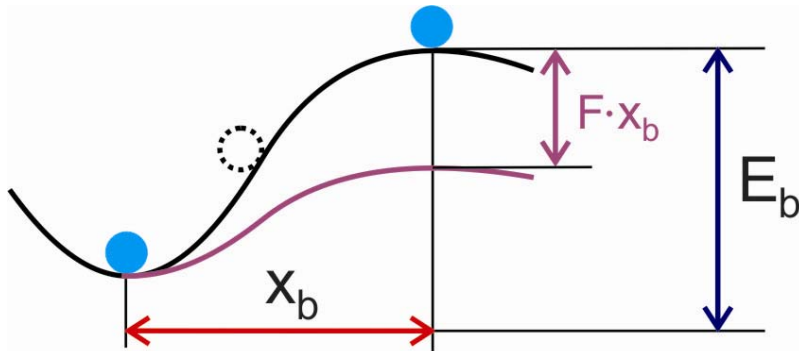


Figure by MIT OpenCourseWare. After Buehler and Ackbarow.

Superelastic material

High impact resistance

- **Coiled-coil:** Enables large superelastic strains (low forces, since H-bonds break easily) – scale $\sim 10..100$ nm
- **Different mechanisms:** Breaking of single H-bond leads to large slope of AP w.r.t. strain rate (significant strengthening at large deformation rates)
- **At small deformation rates:** strengthening not as strong, as three H-bonds break simultaneously
- **Refolding possible,** since sequence has not been destroyed (if forces below rupture of covalent bonds)



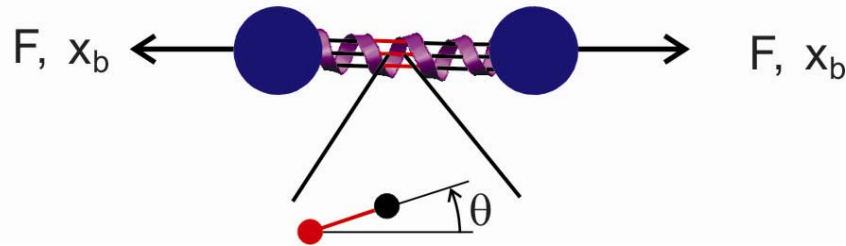
“off rate”

$$\chi = \omega_0 \cdot \exp\left(-\frac{(E_b - F \cdot x_b \cdot \cos(\theta))}{k_b \cdot T}\right)$$

Pulling velocity

$$v = \chi \cdot x_b$$

$$v = \omega_0 \cdot x_b \cdot \exp\left(-\frac{(E_b - F \cdot x_b \cdot \cos(\theta))}{k_b \cdot T}\right)$$



Rupture force

$$F(v) = \frac{k_b \cdot T}{x_b \cdot \cos(\theta)} \cdot \ln v - \frac{k_b \cdot T}{x_b \cdot \cos(\theta)} \cdot \ln v_0 = a \cdot \ln v + b$$



Molecular defects in vimentin

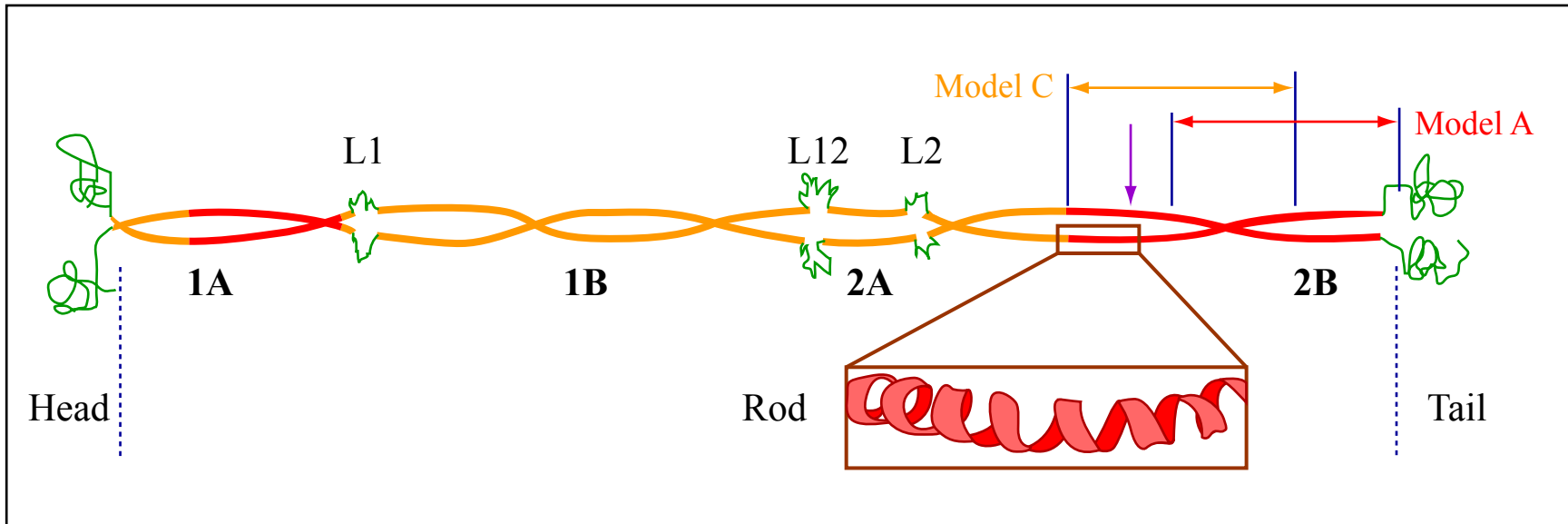


Figure by MIT OpenCourseWare. After Ackbarow and Buehler, 2008.

‘Skips’ are insertions of one residue into the heptad pattern

‘Stammers’ result through an insertion of three additional residues

‘Stutters’ appear if four additional residues interrupt the heptad sequence: Presence of a stutter results in an almost parallel run of both AHs without interrupting the CC geometry.



Governing irregularities and defects in proteins with HPT



Stutter: insert of residues into the heptad repeat periodicity

→ local uncoiling of the coiled-coil

→ different angles within one structure

alpha helix and stutter: $\theta_1 = 16^\circ$

coiled-coil : $\theta_2 = 23^\circ$

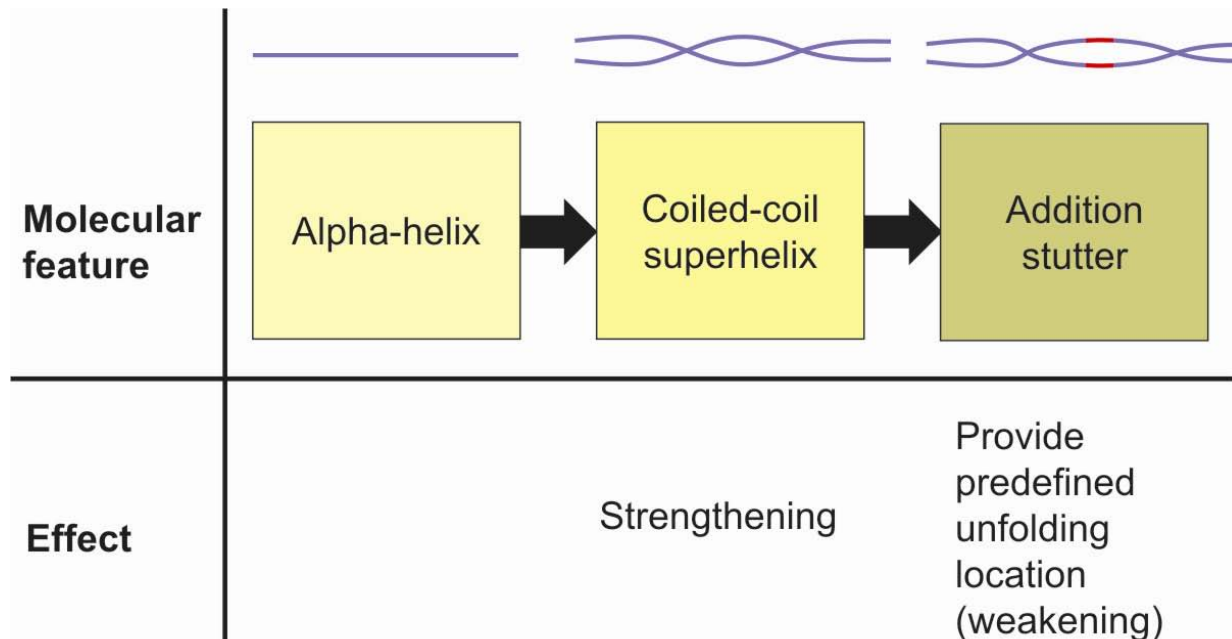
$$\theta_2 > \theta_1$$

$$\cos \theta_2 < \cos \theta_1$$

$$f_2(v) = \frac{\cos \theta_1}{\cos \theta_2} \cdot f_1(v)$$

$$f(v) = \frac{k_b \cdot T}{x_b \cdot \cos(\theta)} \cdot \ln v - \frac{k_b \cdot T}{x_b \cdot \cos(\theta)} \cdot \ln v_0$$

Difference in unfolding force
5% weaker at stutter

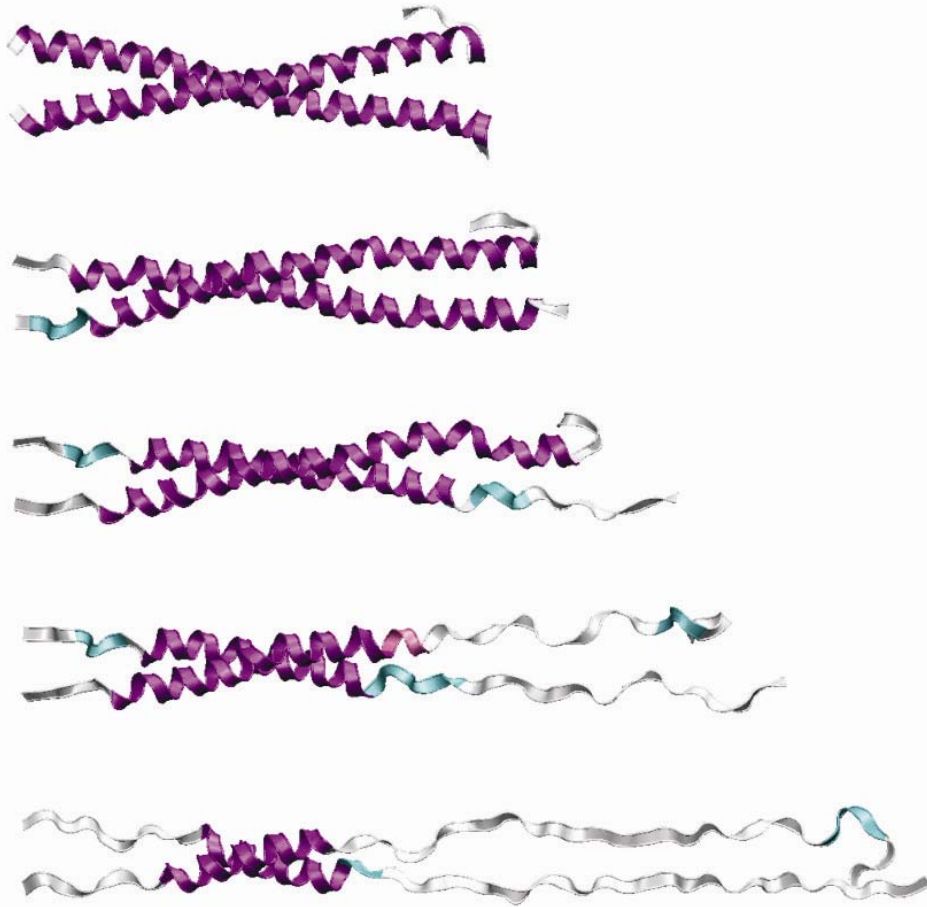




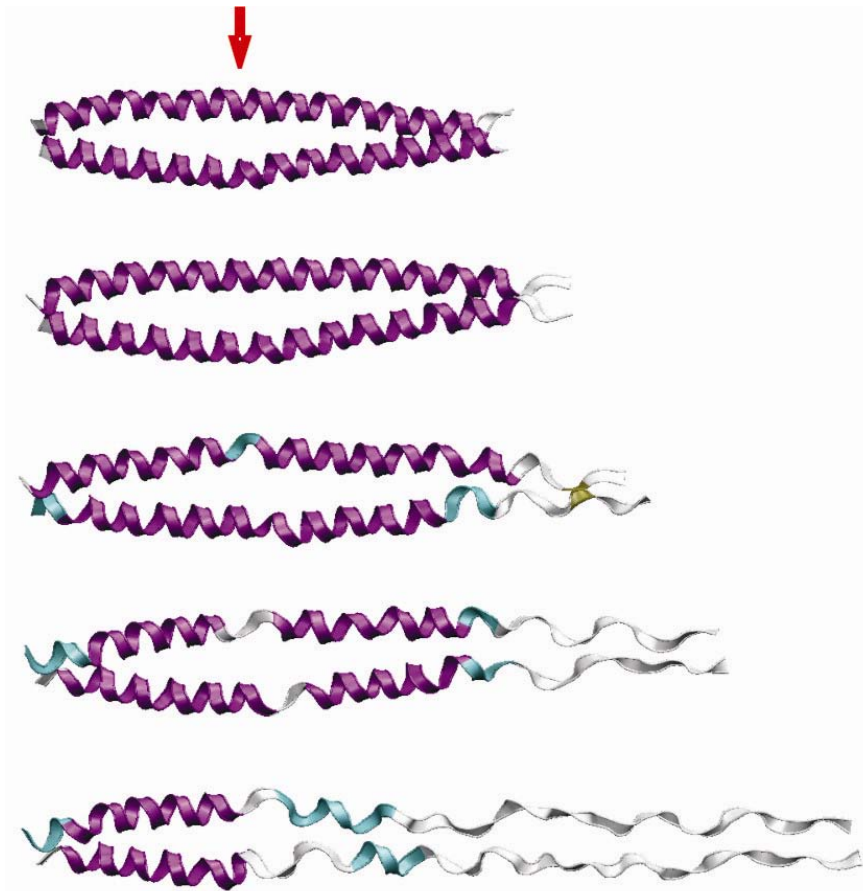
Stutter: location of predetermined unfolding



Without stutter



With stutter

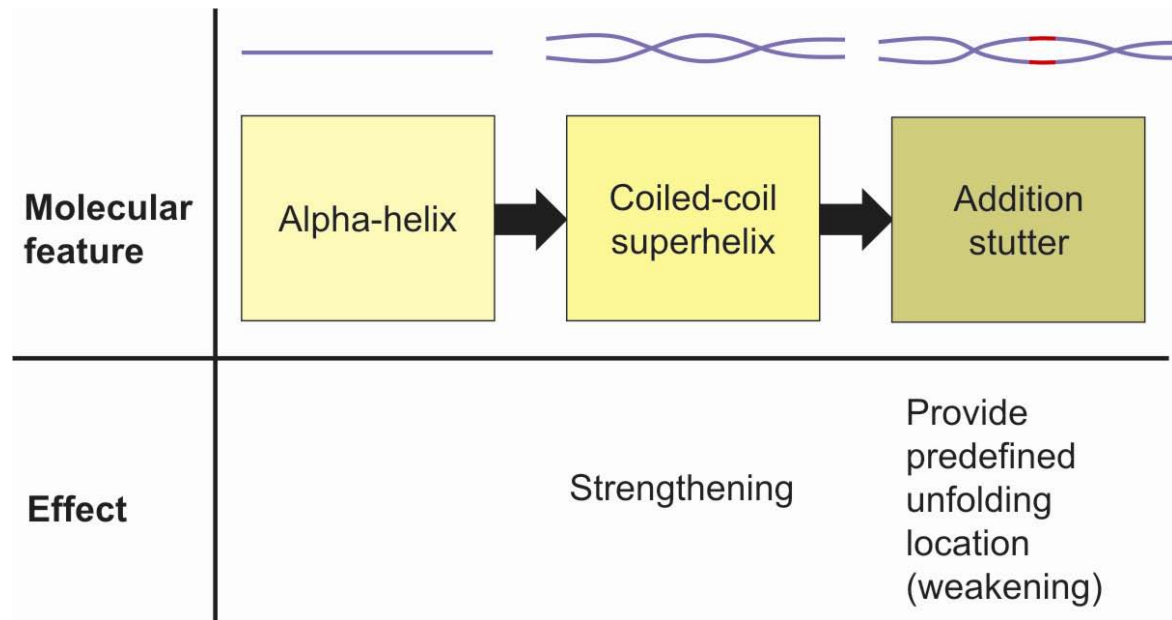




Summary: Effects of the stutter



- Renders the molecular structures softer, that is, unfolding occurs at lower tensile forces,
- Introduces predefined locations of unfolding, and
- Thus leads to a more homogeneous distribution of plastic strains throughout the molecular geometry.





Summary: 'Take home message'



- Significance of protein mechanics for biological processes (signaling, cancer, mechanical integrity..)
- Chemical bonds within protein & theoretical description (force field)
- Free energy source of elasticity: Link atomistic/molecular concepts with continuum theory (rubber elasticity, WLC model)
- Case study: Vimentin dimer – response to mechanical cue (stretching, unfolding, rate dependence)

Bell model – statistical concept to predict unfolding force 'onset of plasticity' – permanent deformation

Structure-function relationship to describe role of molecular defects in protein mechanics (example: stutter defect)