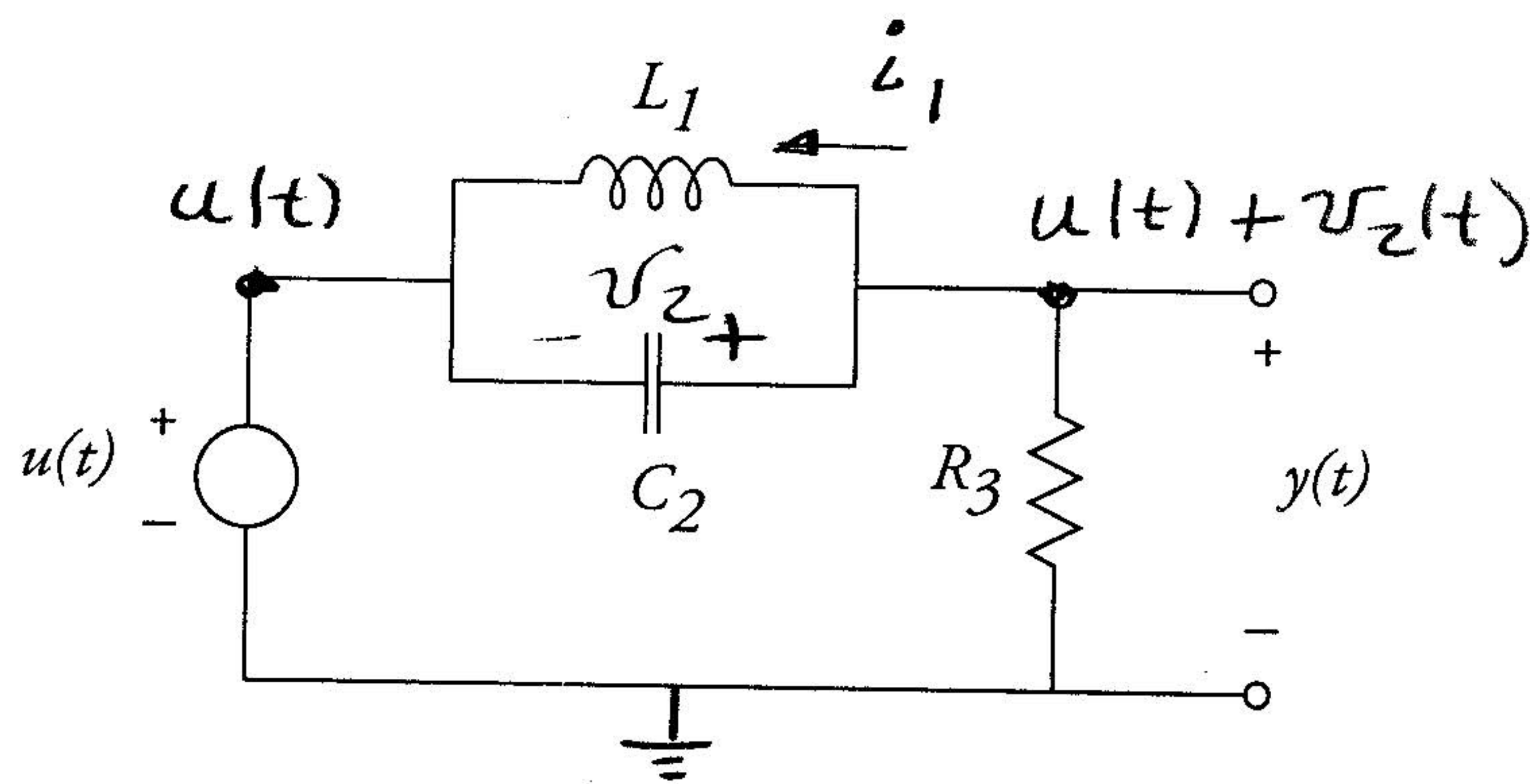


To solve the circuit, use the node method:



Note that there are no unknown nodes, which simplifies things!

The states are

$$x_1 = \dot{i}_1$$

$$x_2 = v_2$$

To find $\dot{x}_1 = di_1/dt$, need v_1 :

$$\begin{aligned} \dot{x}_1 &= \frac{di_1}{dt} = \frac{1}{L} v_1(t) \\ &= \frac{1}{L} \underbrace{[(u + v_2) - u]}_{\text{potential across } L} \\ &= \frac{1}{L} v_2 \end{aligned}$$

To find $\dot{x}_2 = dv_2/dt$, need i_2 . To find i_2 , apply KCL at $u + v_2$ node:

$$\frac{u + v_2 - 0}{R} + i_1 + i_2 = 0$$

Therefore,

$$\dot{i}_2 = -i_1 - \frac{1}{R} v_2 - \frac{1}{R} u$$

and

$$\dot{x}_2 = \frac{dv_2}{dt} = \frac{1}{C} i_2$$

$$= -\frac{1}{C} i_1 - \frac{1}{RC} v_2 - \frac{1}{RC} u$$

Therefore, the state equation is given by

$$\dot{\underline{x}} = \underbrace{\begin{bmatrix} 0 & 1/L \\ -1/C & -1/RC \end{bmatrix}}_A \underline{x} + \underbrace{\begin{bmatrix} 0 \\ -1/RC \end{bmatrix}}_B u$$

To find the measurement equation, note that

$$y(t) = v_2 + u = x_2 + u$$

Therefore,

$$y = \underbrace{[0 \quad 1]}_C \underline{x} + \underbrace{[1]}_D u$$

N.B.:

There are other possible labellings for v_2 and i_1 . If you used a different labelling, some of the signs may be different

In particular,

1) If v_2 labelled opposite mine,

$$C = [0 \quad -1]$$

$$B = \begin{bmatrix} 0 \\ +1/RC \end{bmatrix}$$

2) If v_2 or i_1 labelled opposite mine (but not both),

$$A = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix}$$

3) If both v_2 and i_1 labelled opposite mine, A remains the same.