Muddy Card Responses Lecture M18 3/04/2004

Synopsis. Examined Tresca and Von Mises multiaxial yield criteria, which utilize the empirical observation that yield is controlled by the maximum shear stresses. Noted that use of a 3-D Mohr's circle (3 circles on one diagram) is a useful way to visualize an entire stress-state. Moved on to discuss fracture. Applied the energy approach due originally to Griffith to obtain the result that fracture occurs when the strain energy release rate exceeds the toughness (energy per area required to create new crack surface). From this obtained the result that fracture occurs when the stress intensity factor, K, ($=K = Y\sigma\sqrt{\pi a}$) equals or exceeds the fracture toughness K_c. Thus fracture depends on the applied stress level and the crack size.

What size crack can be visually detected? It depends how hard you look!. If you have very highly polished surfaces and perhaps use a stereo microscope cracks of a few 10's of micron's (50-100 μm) might be detected. In routine inspections on aerospace structures a trained inspector may be able to reliably identify cracks in the range 1-10 mm.

Does the equation $\frac{\pi a\sigma^2}{E} \geq G_c$ assume that the applied stress is perpendicular to the lengthwise dimension a ? Good point. Yes it effectively does. Generally cracks will propagate perpendicular to the maximum tensile stress. If the crack is not originally orientated in this direction it will grow and change direction so that it is. Cracks to not generally propagate if there is a compressive load.

How do more round cracks affect the toughness etc.? Are there conditions on the shape of the crack for the equations we learned to day. For instances a sharp crack vs. a rounded (elliptical) notch. This is a very important point and we will discuss more on Monday. Strictly speaking for the "fracture mechanics" we sped through today to apply there has to be a very sharp crack. In reality the mechanics works well even if the crack is slightly blunted, but one has to be careful. A round ended notch in a ductile metal has a finite stress concentration factor and is likely to yield at the notch tip rather than to fracture.

I didn't understand why you said yield happens when G = G_c. If $G_c = \frac{\pi a\sigma^2}{E}$ what is G?
There are a couple of issues here. Firstly fracture (or fast fracture) is fundamentally a different failure mechanism than yield. Yield involves permanent deformation, but the material remains in one piece. Fast fracture involves the catastrophic and sudden propagation of a crack which will result in at least two pieces of material at the end of it – or more. Second, the two equations you wrote are consistent. In general the strain energy release rate G has the form: $G = C\frac{\pi a\sigma^2}{E}$ where C is a constant that depends on the

particular geometry. In our case C=1. At the point where fracture occurs G=G_c, thus $G_c = \frac{\pi a \sigma^2}{E}$.

Cracks would seem to be very bad in an application like turbine or fan blades. Is this true? And what would you do about it? You are absolutely correct.. If we look at the equations we have derived: $Y\sigma\sqrt{\pi a} \geq K_c$ and $\frac{\pi a \sigma^2}{E} \geq G_c$, then we can see that if the stresses are high, which they are in the blades of turbomachinery, then the maximum crack we can tolerate will be relatively small. Critical crack sizes in fan blades and turbine disks may be ~ 1-2 mm. This is very small to be able to detect reliably. In order to avoid catastrophic fractures and particularly bursting disks, the manufacturing process is very carefully controlled. In addition very careful, high resolution Non Destructive Inspection (NDI) is performed after manufacturing to ensure that the parts are defect free. This consists of ultrasound and –X-ray inspections. Finally, these highly stressed parts are given very conservative maximum lifetimes, to eliminate the possibility of fatigue failure (which we will talk about in M19).

Tough vs. Strong vs. Stiff. Now you know what toughness is (resistance to crack propagation – an energy/area), strength (maximum stress a material can carry) and stiffness (stress to achieve a given strain). Note you could also define these quantities slightly differently at the structural level, so one has to be a little careful distinguishing between material and structural properties.

Mud: Could you write on the board instead of pointing to slides when you lecture? It is easier to take notes when things are written on the board. My apologies for the speed of the second half of today's lecture. I went too fast. Normally I try to write the important points/equations or draw the key diagrams on the board as well as showing them on slides. I was intent on making sure that we covered enough of fracture to allow you to start on problem M18. We will have some time for questions and answers in lecture M19 before we move on to fatigue. Also, please note that the notes are on the web, so you should not need to copy down any of the text and figures that I put on the overhead projector – although you may want to annotate it.

Mud Culture: "Mother indulgent. Said I read too much. Not true. Read too little and understood less" James Joyce. I am impressed and can only add a counter example.

Adverse Mud Culture: …..*One fine morning in the middle of the Precession of the Equinoxes this 'satiable Elephant's Child asked a new fine question that he had never asked before. He asked, 'What does the Crocodile have for dinner?' Then everybody said, 'Hush!' in a loud and dreadful tone, and they spanked him immediately and directly, without stopping, for a long time.*

By and by, when that was finished, he came upon Kolokolo Bird sitting in the middle of a wait-a-bit thorn-bush, and he said, 'My father has spanked me, and my mother has spanked me; all my aunts and uncles have spanked me for my 'satiable curiosity; and still I want to know what the Crocodile has for dinner!'

Then Kolokolo Bird said, with a mournful cry, 'Go to the banks of the great grey-green, greasy Limpopo River, all set about with fever-trees, and find out.'…… *Rudyard Kipling (from the Elephant's Child).* – which says something about asking questions in a non-supportive environment, looking for answers for oneself (and finding them in the mud of rivers), and, if you read the whole story it also says something about suffering for the sake of obtaining a unique talent…. Truly a metaphor for Unified perhaps!

There were 2muddy cards with no mud, or positive responses. Thank you