16.06 Principles of Automatic Control
Recitation 12

Design (by emulation) a discrete time controller for:

$$G(s) = \frac{100}{(s+2)^2} \qquad \text{satisfying:}$$

$$\begin{aligned} &\omega_c = 50\,\mathrm{r/s} \\ &M_p \leqslant 25\% \\ &K_p \geqslant 5000 \\ &T = 0.01\,\mathrm{sec} \end{aligned}$$

$$G(s) = \frac{25}{(\frac{s}{2}+1)^2}$$

$$M_p = 0.25 = e^{-\pi \tan\theta} \rightarrow \zeta = \sin\,\theta \;\rightarrow\; \text{PM} \sim 40.4^\circ$$

The strategy is to first design a lead compensator followed by a lag compensator (to satisfy the high K_p value), while accounting for the ZOH delay of $\frac{\omega_c T}{2} \cdot \frac{180^\circ}{\pi}$.

Step 1. Lead compensator:

$$\frac{k\left(\frac{s}{a}+1\right)}{\left(\frac{s}{b}+1\right)} \rightarrow \sqrt{ab} = \omega_c = 50 \qquad (1)$$

$$\begin{aligned} 2\tan^{-1}\left(\sqrt{\frac{b}{a}}\right) - 90^\circ &= -\,180^\circ - sG(\omega_c) + \text{PM} + \frac{\omega_c T}{2} \cdot \frac{180^\circ}{\pi} + 6^\circ(\text{lag}) \\ &= -\,180^\circ - (-175.4^\circ) + 40.4^\circ + 14.3^\circ + 6^\circ \\ &\approx 56.1^\circ \end{aligned}$$

$$\Rightarrow \sqrt{\frac{b}{a}} \approx 3.28 \qquad (2)$$

Equations (1) and (2) yield:

$$\begin{aligned} b =& 164 \\ a =& 15.2 \\ K_{\text{lead}}(s) =& 7.5\, \frac{(\frac{s}{15}+1)}{(\frac{s}{164}+1)} \end{aligned}$$

To find k :

$$\begin{aligned} 1 =& 25\, k\, \frac{\sqrt{\left(\frac{50}{15}\right)^2 + 1^2}}{\sqrt{(\frac{50}{164}) + 1^2}} \\ k =& 7.5 \end{aligned}$$

Step 2. Lag compensator:

Desired low frequency gain is 5000 :

$$\therefore \text{ for } K_{lag}(s) = \frac{s+a}{s+b} \;\rightarrow\; \frac{a}{b} \cdot 25 \cdot 7.5 = 5000$$

choose a to be 1 decade below ω_c : $a = 5$.

$$\therefore\; b = \frac{5 \cdot 25 \cdot 7.5}{5000} \approx 0.19$$

$$\boxed{\begin{aligned} &\therefore \text{overall compensator:} \\ K(s) =& \frac{7.5\left(\frac{s}{15}+1\right)}{\left(\frac{s}{164}+1\right)}\, \frac{(s+5)}{(s+0.19)} \end{aligned}}$$

Using MATLAB, we see that there is excessive overshoot:

38% $\rightarrow$ 13%, too much.

Therefore, we will need to change ζ by ~ 0.1, PM by $\sim 10^\circ$.

Need to redesign lead compensator: $2\tan^{-1}\left(\sqrt{\frac{b}{a}}\right) - 90^\circ = 56.1^\circ$.

Need to re-design lag compensator:

$$K(s) = \frac{5.3\left(\frac{s}{10.6}+1\right)}{\left(\frac{s}{236.2}+1\right)} \cdot \frac{(s+5)}{(s+0.13)}$$

which yields:

$K_p = 5000$, and 23.3% overshoot.