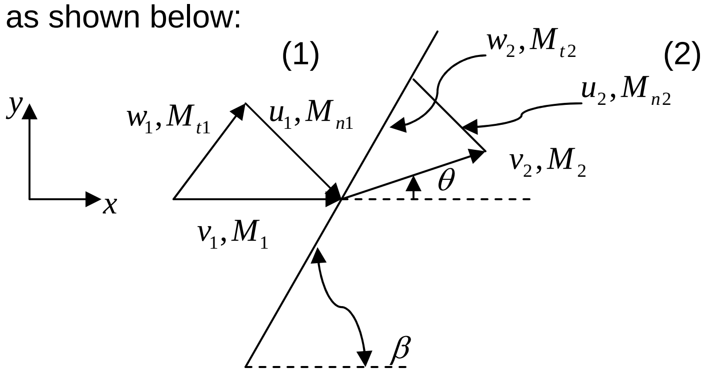
Oblique Shock Waves

Here's a quick refresher on oblique shock waves. We start with the oblique shock as shown below:



$(\ )_1$: upstream flow condition

$(\ )_2$: downstream flow condition

β : angle of shock wave $w.r.t.$ upstream flow

θ : deflection angle of flow

Also, the specific flow quantities above are:

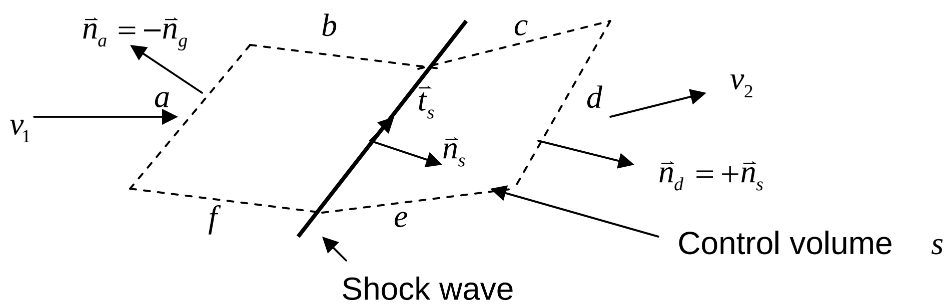
v : flowspeed — M : Mach number = $\dfrac{v}{a}$

u : normal velocity to shock — M_n : Normal Mach $\# = \dfrac{u}{a}$

w : tangential velocity to shock — M_t : Tangential Mach $\# = \dfrac{w}{a}$

The next step is to apply the 2-D Euler equations to derive jump conditions.

Let's consider the following (well-chosen) control volume across the shock:



Where: $a \,\&\, d$ are parallel to shock

b, f, c, e are parallel to local flow

Apply conservation of mass:

$$\int_s \rho \vec{V} \bullet \vec{n}\, ds = 0$$

But $\vec{V} \bullet \vec{n} = 0$ on $b, f, c \,\&\, e,$ thus:

$$\int_a \rho \vec{V} \bullet \vec{n}\, ds + \int_d \rho \vec{V} \bullet \vec{n}\, ds = 0$$

$$-\int_a \rho_1 \underbrace{\vec{V}_1 \bullet \vec{n}_s}_{u_1} ds + \int_d \rho_2 \underbrace{\vec{V}_2 \bullet \vec{n}_s}_{u_2} ds = 0$$

$$-\rho_1 u_1 \underbrace{\int_a ds}_{A_1} + \rho_2 u_2 d \underbrace{\int_d ds}_{A_2} = 0$$

But, $A_1 = A_2$ since all lines of control volume edges b, c, e, f are parallel.

$$\Rightarrow \boxed{\rho_1 u_1 = \rho_2 u_2}$$

The next equation we'll look at is tangential momentum.

$$\int_s \rho w \vec{V} \bullet \vec{n} ds = -\int_s p\vec{n} \bullet \vec{t}_s ds$$

$$-\rho_1 w_1 u_1 A_1 + \rho_2 w_2 u_2 A_2 = -\int_b p\vec{n} \bullet \vec{t}_s ds - \int_f p\vec{n} \bullet \vec{t}_s ds - \int_c p\vec{n} \bullet \vec{t}_s ds - \int_e p\vec{n} \bullet \vec{t}_s ds$$

Plugging into the pressure terms:

$$-\rho_1 w_1 u_1 A_1 + \rho_2 w_2 u_2 A_2 = -p_1 \vec{n}_b \bullet \vec{t}_s \int_b ds - p_1 \vec{n}_f \bullet \vec{t}_s \int_f ds - p_2 \vec{n}_c \bullet \vec{t}_s \int_c ds - p_2 \vec{n}_e \bullet \vec{t}_s \int_e ds$$

But $\vec{n}_b = -\vec{n}_f$ & $\vec{n}_c = -\vec{n}_e$

$\Rightarrow \rho_1 u_1 w_1 A_1 = \rho_2 u_2 w_2 A_2$

Using $\rho u A = const$.

$$\Rightarrow \boxed{w_1 = w_2}$$

Tangential velocity is unchanged across a shock wave!
The last two equations (see Anderson for more) give:

Normal momentum : $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$

Energy: $h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2$

These equations can be solved and results are displayed in graph and tables in Anderson.

Example: Find β & M_2 ?