1a) $\frac{u}{u_e} \equiv U = \frac{y}{\delta}$ $\delta^* = \int (1-U)\,dy = \frac{\delta}{2}$ $\theta = \int (U - U^2)\,dy = \int \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy = \frac{\delta}{6}$, $H = 3$

log form of mom. eq'n: $\frac{d}{dx} \ln(\rho u_e^2 \theta) = \frac{1}{\theta}\frac{C_f}{2} - H \frac{d}{dx} \ln u_e$

$$\Delta\left(\ln \rho u_e^2 \theta\right) \simeq \int \frac{C_f}{2}\frac{dx}{\theta} - H\,\Delta\left(\ln u_e\right)$$

$$\ln \frac{(\rho u_e^2 \theta)_{exit}}{(\rho u_e^2 \theta)_i} \simeq -H \ln \frac{u_{e\,exit}}{u_{e_i}} = -H \ln \frac{V}{V/2}$$

Initial $\rho u_e^2 \theta = \rho V^2 \frac{\delta}{24} = \rho V^2 \delta \cdot 0.0417$

Exit $\rho u_e^2 \theta = \rho u_e^2 \theta_i \cdot 2^{-H} = (\rho u_e^2 \theta)_i \cdot \frac{1}{8} = \rho V^2 \delta \cdot 0.0052$

1b) $\frac{\rho}{\rho_e} \equiv R = a + b\frac{y}{\delta}$ $a = 0.9$, $b = 0.1$

$$\delta^* = \int_0^\delta (1 - RU)\,dy = \delta \int_0^1 \left[1 - (a\eta + b\eta^2)\right] d\eta = \delta\left[1 - \frac{a}{2} - \frac{b}{3}\right] = 0.517\,\delta$$

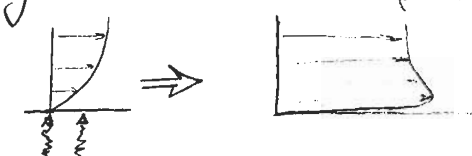
$$\theta = \int (1-U) RU\,dy = \delta \int_0^1 \left[(a\eta + b\eta^2) - (a\eta^2 + b\eta^3)\right] d\eta = \delta\left[\frac{a}{6} + \frac{b}{12}\right] = 0.158\,\delta$$

$H = 3.263$

Initial $\rho u_e^2 \theta = \rho V^2 \cdot \frac{1}{4}\delta\left[\frac{a}{6} + \frac{b}{12}\right] = \rho V^2 \delta \cdot 0.0395$

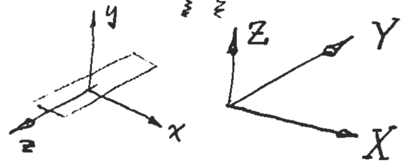
Exit $\rho u_e^2 \theta = (\rho u_e^2 \theta_i) \cdot 2^{-H} = \rho V^2 \delta \cdot 0.0041$

1c) These estimates assume that the C_f term is negligible and that H is constant. In reality θ can get driven below 0 if the wall is hot enough and there is a very fast acceleration. This will result in a velocity overshoot since $\frac{\partial u}{\partial x} \simeq \frac{\rho_e u_e}{\rho u}\frac{du_e}{dx}$ can be quite large near the wall if $\frac{\rho}{\rho_e}$ is small:



The device is a ramjet in this case

2a) Aligning the BL coordinates as shown:



$x \parallel X$
$y \parallel Z$

$$D = \iint (V - u)\rho u\,dZ\,dY = \int \left[\int (V-u)\rho u\,dy\right] dz$$

$$D = \int \left(\rho_e q_e^2 \theta_{xx}\right)_{wake} dz$$ the drag is the spanwise integral of $\left(\rho q_e^2 \theta_{xx}\right)$

Integrate x-mom eq'n: $\int dx \int dz \left\{ \frac{\partial}{\partial x}\left(\rho q_e^2 \theta_{xx}\right) + \frac{\partial}{\partial z}\left(\rho q_e^2 \theta_{xz}\right) = \tau_x - \rho_e q_e \delta_x^* \frac{\partial u_e}{\partial x} - \rho q_e \delta_z^* \frac{\partial w_e}{\partial x} \right\}$

$$\int \left(\rho_e q_e^2 \theta_{xx}\right)_{wake} dz = D = \underbrace{\iint \tau_x\,dx\,dz}_{\text{friction drag}} - \underbrace{\iint \rho q_e \delta_x^* \frac{\partial u_e}{\partial x}\,dx\,dz}_{\text{pressure drag}} - \iint \rho q_e \delta_z^* \frac{\partial w_e}{\partial x}\,dx\,dz - \int dx \left\{ \rho q_e^2 \theta_{xz} \right|_{z_{min}}^{z_{max}}$$

small for high aspect ratio wing

0 at tips