Finite-Difference Solution of TSL Equations

Local Scaling Transformation: $\xi = x \qquad \eta = \dfrac{y}{\Delta(x)}$

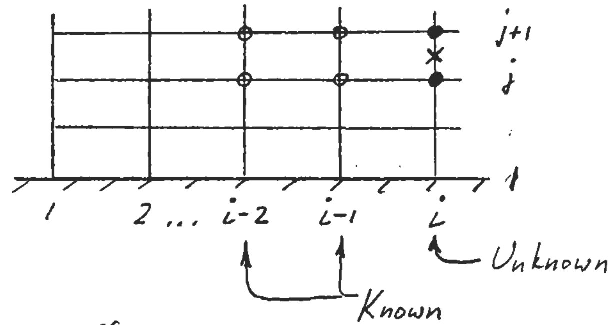
$$U = \frac{\partial F}{\partial \eta} \qquad S = \frac{\nu \xi u_e}{n^2}\left(1 + \frac{\nu_t}{\nu}\right)\frac{\partial U}{\partial \eta}$$

$$\frac{\partial S}{\partial \eta} + \beta_n F \frac{\partial U}{\partial \eta} + \beta_u\left[1 - U\frac{\partial F}{\partial \eta}\right] = \xi\left[\frac{\partial F}{\partial \eta}\frac{\partial U}{\partial \xi} - \frac{\partial F}{\partial \xi}\frac{\partial U}{\partial \eta}\right]$$

$$n \equiv u_e \Delta$$

$$\beta_{()} \equiv \frac{\xi}{()}\frac{d()}{d\xi} = \frac{d[\ln()]}{d[\ln \xi]}$$

Discretization



η-derivative at "x" – trapezoidal rule

$$\left.\frac{\partial F}{\partial \eta}\right|_{j+1/2} = \frac{1}{\Delta\eta}\left(F_{j+1} - F_j\right)_i \qquad \left.\frac{\partial U}{\partial \eta}\right|_{j+1/2} = \text{etc.}$$

ξ-derivative at "x" – 3-point backward difference.

$$\left.\frac{\partial F}{\partial \xi}\right|_{j+1/2} = \frac{1}{2}\left\{\frac{1}{\Delta\xi}\left(\frac{3}{4}F_i - F_{i-1} + \frac{1}{4}F_{i-2}\right)_{j+1} + \frac{1}{\Delta\xi}\left(\frac{3}{4}F_i - F_{i-1} + \frac{1}{4}F_{i-2}\right)_j\right\}$$

$$\beta_u = \frac{3}{4}\,\frac{\ln(u_{e_i}/u_{e_{i-1}})}{\ln(\xi_i/\xi_{i-1})} + \frac{1}{4}\,\frac{\ln(u_{e_{i-1}}/u_{e_{i-2}})}{\ln(\xi_{i-1}/\xi_{i-2})}\,, \qquad \beta_n = \text{etc.}\ldots$$

- At $i=2$, use Backward Euler for ξ-derivatives ($i-2$ not available)
- At $i=1$, assume similarity $\rightarrow \dfrac{\partial}{\partial \xi} = 0$, β_u β_n prescribed.

At each i station: $3N+2$ unknowns: $F_j\ U_j\ S_j\ (1 \le j \le N)$, u_{e_i}, n_i

Equations: $3N-3$ discrete equations, 3 BC's, 2 constraints for u_e, n

Constraints for u_e:
- Specified u_e: $u_{e_i} = u_{e\,spec}$ "direct problem"
- Specified δ^*: $\delta^*_i = \delta^*_{spec}$ "inverse problem"

Constraints for n:
- Define $\Delta = \sqrt{\dfrac{\nu\xi}{u_e}}$ (works OK)
- Define $\Delta = \theta \Rightarrow 1 = \int U(1-U)\,d\eta$ (works better)

Basic Procedure: At each i station, solve Falkner-Skan-like problem for F, U, S, u_e, n. Extra terms come from $\frac{\partial}{\partial \xi} \neq 0$.
Requires turbulence model $\nu_t(F, U, S, u_e, n, \nu)$