A> 2D Interaction Models.

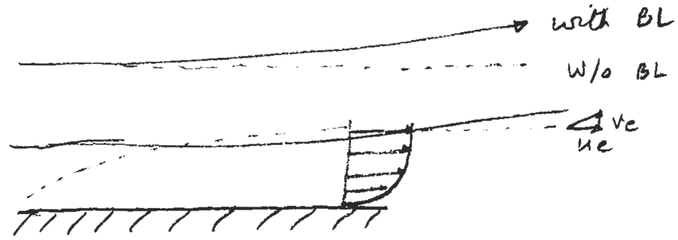
B> Implication for Lift and Drag

Reading : Handout, paper

A> Zeroth order matching $\vec{u}\cdot\hat{n} = 0$

First order matching $\vec{u}\cdot\hat{n} = V_E = V_i \quad (\sim O(1/\sqrt{Re}))$

In actual flow

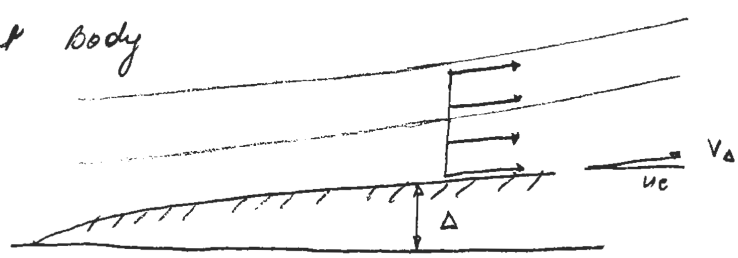


Using continuity $\nabla\cdot\vec{u} = 0$ gives

$$V_e = \frac{d}{dx}\left(u_e\delta^*\right) - \delta\frac{du_e}{dx}$$

Two models to enforce Ve :

(1) Displacement Body



$$\int_0^{y_e}(\quad) = \int_0^{\Delta}(\quad) + \int_{\Delta}^{y_e}(\quad)$$

$$\Rightarrow \quad V_e - V_\Delta = -\int_{\Delta}^{y_e}\frac{\partial u}{\partial x}dy = \frac{d(\Delta u_e)}{dx} - y_e\frac{du_e}{dx} - u\cancel{\frac{dy_e}{dx}} + u_e\cancel{\frac{dy_e}{dx}}$$

$$- u_e\frac{d\Delta}{dx}$$

$$V_e - V_\Delta = \frac{d}{dx}(\Delta u_e) - y_e \frac{du_e}{dx} - u_e \frac{d\Delta}{dx}$$

$$V_\Delta = u_e \frac{d\Delta}{dx} \qquad \text{(by observation)}$$

$$\therefore \quad V_e = \frac{d}{dx}(\Delta u_e) - y_e \frac{du_e}{dx}$$

$\Rightarrow \quad \Delta = \delta^*$ on comparing with real visc. flow.

<u>Note</u> $\quad \vec{u}_{EIF} \cdot \hat{n}_{disp} = 0$.

(2) Wall blowing

$$V_e - V_{wall} = -\int_0^{y_e} \frac{\partial u}{\partial x}\, dy$$

$$= -\frac{d}{dx}\int_0^{y_e} u\, dy + u_e \frac{dy_e}{dx}$$

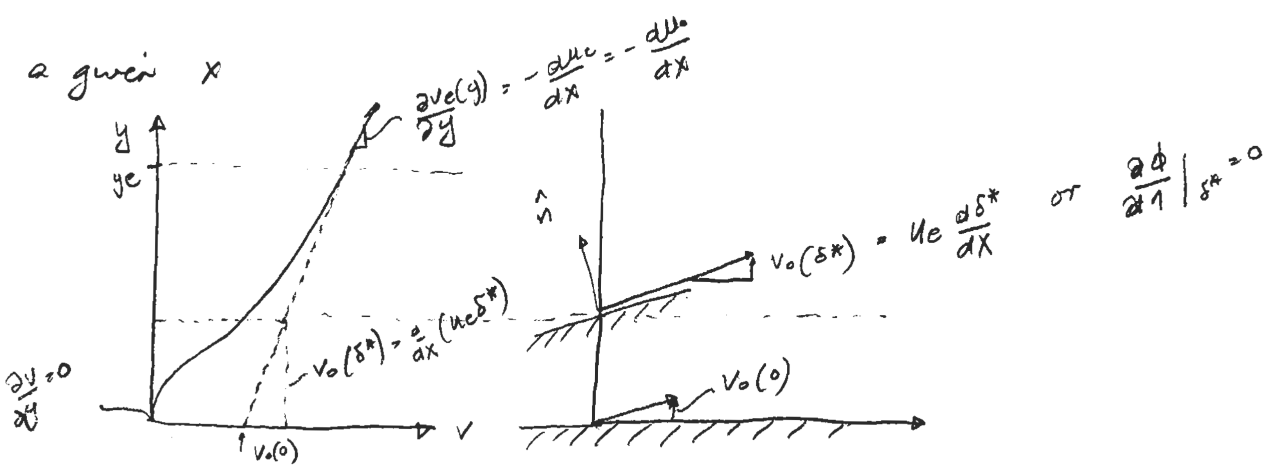
$$= -\frac{d}{dx} u_e y_e + u_e \frac{dy_e}{dx}$$

$$= -y_e \frac{du_e}{dx}$$

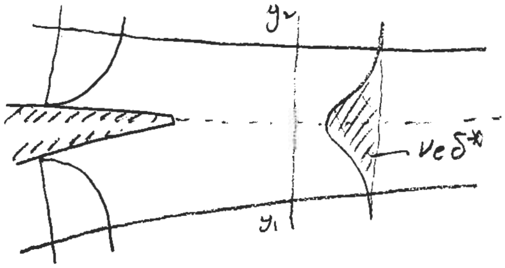
$$\therefore \quad V_e = V_{wall} - y_e \frac{du_e}{dx}$$

$$\Rightarrow \quad V_{wall} = \frac{d}{dx}(u_e \delta^*)$$

At a given x



Wake



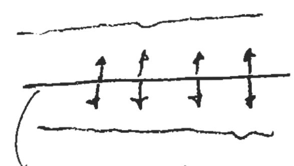
Real Flow: $$v_2 - v_1 = -\int_{y_1}^{y_2} \frac{\partial u}{\partial x} = \frac{d}{dx}(u_e\delta^*) - (y_2 - y_1)\frac{du_e}{dx}$$

Disp. Body $$V_2 - V_1 = \frac{d}{dx}(u_e\Delta) - (y_2 - y_1)\frac{du_e}{dx} \implies \Delta = \delta^*$$

Wall Blowing

$$V_2 - V_1 = V_{w_2} - V_{w_1} - (y_2 - y_1)\frac{du_e}{dx}$$

$$\implies V_{w_2} - V_{w_1} = \frac{d}{dx}(u_e\delta^*)$$



source sheet
$\sigma = \Delta V_W$

- No info on where to put body.
- No info on individual v_{w_1}, v_{w_2} $\left(\Delta\left(\frac{\partial\phi}{\partial\eta}\right) = \Delta v_W\right)$
- wake position set $\Delta p = 0$ $\left(\Delta \frac{\partial\phi}{\partial s} = 0\right)$

potential flow controls difference in u_e
b.l " difference in v_W

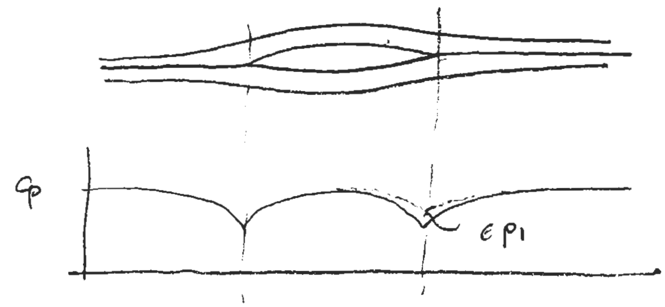
2D Drag (implication of IBLT)

$$D = \oint -p\,dy + \oint \tau\,dx$$

D_{form} or D_{press} — $D_{friction}$

For inviscid, shock free, flow $D_{form} \equiv 0$ (d'Alembert's Paradox)

perfect cancellation



For classical theory, wall BC: $V_0 = 0$ → flow tangency

$$D_{form} = 0$$

For IBLT, wall BC: $V_0 = V_e \sim O(1/\sqrt{Re})$ attached

$\sim O(1)$ separated

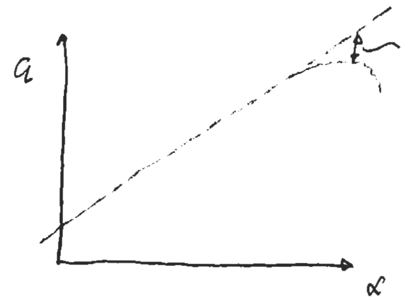
$$V_0 = \underbrace{u_{inv}}_{O(1)} + \underbrace{u_{vis}}_{O(1/\sqrt{Re})}$$

Ily $p = p_{inv} + p_{vis}$

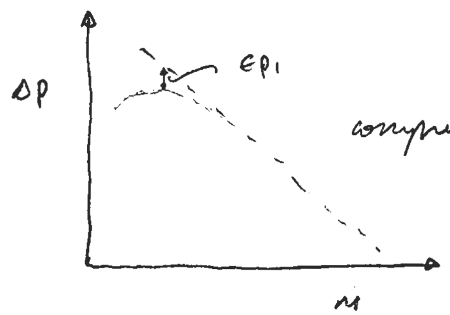
$$D_{form} = 0 + O(1/\sqrt{Re}) \leftarrow \text{about } 1/2 \text{ the total drag.}$$

$O(1)$

Similarly for lift (Stall prediction)



airfoil



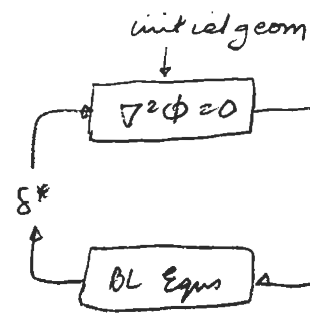
compressor charact

5.3 IBLT

A) Iteration Stability

B) Fully Coupled Solution

A) Iteration Stability



Classical Iteration

0) assume some δ^* ($=0$)

1) add δ^* to airfoil contour ($\frac{\partial\phi}{\partial n}$ or $= V_{wall}$)

2) solve $\nabla^2\phi = 0 \rightarrow u_e = \frac{\partial\phi}{\partial x}$

3) solve BL eqn $\rightarrow$ calc δ^*

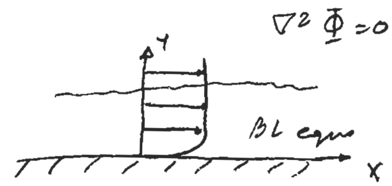
4) iterate

Problem: Almost never works due to numerical instability

Stability Analysis

- Assume a converged solution
- Perturb solution
- Examine if perturbation decays with iteration $n \to \infty$

Consider BL flow over a wall



Let $\Phi = u_\infty (x + \varphi)$

$\uparrow$ perturbation potential

$$u = \Phi_x = u_\infty (1 + \varphi_x) \qquad \frac{du}{dx} = u_\infty \varphi_{xx} \qquad \left(\nabla^2 \Phi = \nabla^2 \varphi = 0 \right)$$

$$\frac{\partial \varphi}{\partial y} = \alpha \quad \text{boundary condition}$$

Linearize BL eqn.

$$\begin{aligned} &H \, [\ \text{mom}\ ] \\ &\theta \, [\ \text{K.E shape par}\ ] \end{aligned} \quad \Rightarrow \quad \frac{d\delta^*}{dx} = \underset{\substack{\uparrow \\ C_f,\, C_D \\ \text{const}}}{A} + \underset{\substack{\uparrow \\ H \text{ constant}}}{B} \frac{du_e}{dx}$$

$$\frac{d\delta^*}{dx} = \theta \frac{dH}{dx} + H \frac{d\theta}{dx}$$

Assume A, B fixed and depend on base solution

Note that

$$\alpha = \frac{d\delta^*}{dx} \quad \text{— slope of displacement surface (no flow through surface)}$$