Lecture 31 — Nov 26, 03.

Compressible TSL

A) Aerodynamic effect of ht / cooling

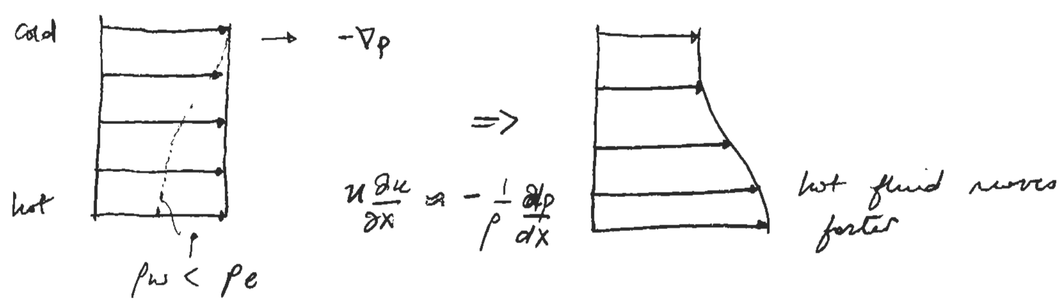
B) Compressible Scaling Transformation

Recap: Last lecture: approximate temperature profile ($Pr \neq 1$) for adiabatic flows. Using temp profile

$$\rho/\rho_e = \frac{T_e}{T}$$

we can get density profile. Correct integral thicknesses and closure relations.

A) Effect of Heating / Cooling

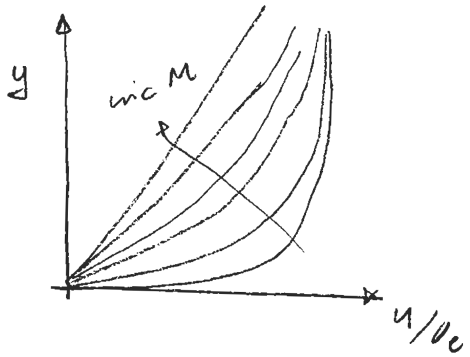


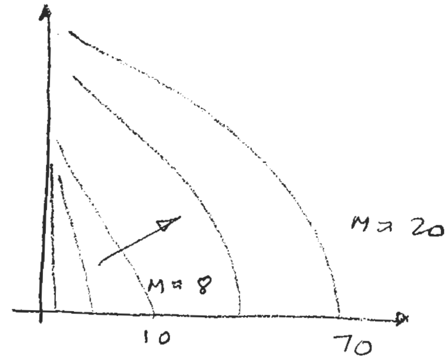
$$u\frac{\partial u}{\partial x} \approx -\frac{1}{\rho}\frac{dp}{dx}$$

$\therefore$ heated wall (due to ht transfer, viscous dissipation adiabatic) increases influence of $\frac{dp}{dx}$ on shape parameter.

Opposite effect for cold wall $\rho_w > \rho_e$.

For an insulated wall (adiabatic), high M flows heat the wall due to viscous dissipation in BL





Effect on profile drag.

$$\cancel{\frac{d\theta}{dx} = \frac{c_f}{2} - (2+H-M_e^2)\frac{\theta}{u_e}\frac{du_e}{dx}}$$

$$\frac{d}{dx}(\rho_e u_e^2 \theta) = \tau_w - \rho_e u_e \delta^* \frac{du_e}{dx}$$

$H_k = H - (\gamma-1)M_e^2$

$H+2-M_e^2$

$H_k + (\gamma-1)M_e^2 + 2 - M_e^2$

$H_k + 2$

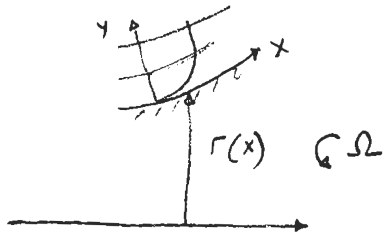
or $\frac{d}{dx}\left[\ln \rho_e u_e^2 \theta\right] = \frac{1}{\theta}\frac{c_f}{2} - ($

$H_k = H - (\gamma-1)M_e^2$

$M_e^2 \approx 0.9$, $H_k \approx 1.8$ $\Rightarrow$ % form drag inc.

$$\left.\frac{(\gamma-1)M_e^2}{H_k}\right|_{avg} \approx 20\%$$

B) BL on rotating Blade.



x, y – streamwise, normal

x-mom $\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} + \rho\Omega^2 r\frac{dr}{dx}$

integral form → $\frac{1}{\theta}\frac{d\theta}{dx} = \frac{1}{\theta}\frac{c_f}{2} - (H+2-M_e^2)\frac{1}{u_e}\frac{du_e}{dx} + (H_\rho - M_e^2)\left(\frac{\Omega r}{u_e}\right)^2 \frac{1}{r}\frac{dr}{dx}$

where $H_\rho = \frac{\delta_\rho}{\theta}$, $\delta_\rho = \int_0^\infty (1-\rho/\rho_e)\, r\, dy$.

Similarly K.E

$$\frac{1}{H^*}\frac{dH^*}{dx} = \frac{1}{\theta}\left(\frac{2c_D}{H^*} - \frac{c_f}{2}\right) - \left(\frac{2H^{**}}{H^*} \;\cdots\right)\frac{1}{u_e}\frac{du_e}{dx}$$

$$- \left(H_p - \frac{2H^{**}}{H^*}\right)\left(\frac{\Omega r}{u_e}\right)^2 \frac{1}{r}\frac{dr}{dx}$$

$$H_p \simeq (0.185\, H_k + 0.15)\, M_e^2$$

C/ Compressible Scaling Transformation

$$\rho u = \frac{\partial \psi}{\partial y} \quad , \quad \rho v = -\frac{\partial \psi}{\partial x}$$

$$\tau = (\mu + \mu_t)\frac{\partial u}{\partial y} \quad , \quad q_0 = \left(\frac{\mu}{P_r} + \frac{\mu_t}{P_{r_t}}\right)\frac{\partial h_0}{\partial y} + \left[\mu\left(1 - \frac{1}{P_r}\right) u \frac{\partial u}{\partial y}\right]$$

Governing eqns :

$$\frac{\partial \psi}{\partial y}\cdot\frac{\partial u}{\partial x} - \frac{\partial \psi}{\partial x}\cdot\frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{\partial \tau}{\partial y}$$

$$\frac{\partial \psi}{\partial y}\cdot\frac{\partial h_0}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial h_0}{\partial y} = \frac{\partial q_0}{\partial y}$$

$$\rho = \rho_e \cdot \left(\frac{T_e}{T}\right) = \rho_e \frac{h_{oe} - \frac{1}{2}u_e^2}{h_o - \frac{1}{2}u^2}$$

Addition variables – h_o, q, ρ

$$\xi = x \quad , \quad \eta = y/\Delta(x) \quad - \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} - \frac{\eta}{\Delta} \cdot \frac{d\Delta}{d\xi} \frac{\partial}{\partial \eta}$$

$$\frac{\partial}{\partial y} = \frac{1}{\Delta} \frac{\partial}{\partial \eta}$$

$\psi = mF$ | $h_o = h_{oe} H$

$u = u_e V$ | $\rho = \rho_e R$ | $R = \dfrac{1 - (u_e^2/2h_{oe})}{H - V^2 (u_e^2/2h_{oe})}$

$\tau = \dfrac{m u_e}{\xi} \cdot S$ | $q_o = \dfrac{m h_{oe}}{\xi} Q$

$\dot{m} \equiv \rho_e u_e \Delta$

$$RV = \frac{\partial F}{\partial \eta} \quad , \quad S = \frac{\xi \mu_e}{\rho_e u_e \Delta^2} \left(\frac{\mu}{\mu_e} + \frac{\mu_t}{\mu_e} \right) \frac{\partial V}{\partial \eta} \quad , \quad Q = \frac{\xi \mu_e}{\rho_e u_e \Delta^2} \left[\left(\frac{\mu/\mu_e}{Pr} + \frac{\mu_t/\mu_e}{Pr_t} \right) \frac{\partial H}{\partial \eta} \right.$$

$$+ \mu/\mu_e \left(1 - 1/Pr\right) \frac{u_e^2}{h_{oe}} \cdot V \frac{\partial V}{\partial \eta}$$

$$\left. + \frac{\mu_t}{\mu_e} \left(1 - 1/Pr_t\right) \cdots \right]$$

$$\Rightarrow \quad \frac{\partial S}{\partial \eta} + \beta_m F \frac{\partial V}{\partial \eta} + \beta_u \left(1 - V \frac{\partial F}{\partial \eta}\right) = \xi \left[\qquad \right]$$

$$\frac{\partial Q}{\partial \eta} + \beta_m F \frac{\partial H}{\partial \eta} - \beta_{H_o} \cdot H \frac{\partial F}{\partial \eta} = \xi \left[\frac{\partial F}{\partial \eta} \cdot \frac{\partial H}{\partial \xi} - \frac{\partial F}{\partial \xi} \cdot \frac{\partial H}{\partial \eta} \right]$$

B.C : $\eta = 0$, $F = 0$, $V = 0$, $H = H_{wall}$ or $Q = Q_{wall}$

$\eta = \eta_e$, $V = 1$, $H = 1$

~~Incomp. limit~~ $\cancel{\left(\frac{u_e^2}{h_{oe}} = 0\right)}$ ~~$R = 1$, $H = 1$, $Q =$~~

Similarity Requirements

① $\beta_{()}$ const , $\beta_{h_o} = 0$ in practice → h_{oe} is const $= u_e$ – const , or locally incompressible $u_e^2/h_{oe} \ll 1$

② $\dfrac{\xi \mu_e}{\rho_e u_e \Delta^2}$, μ/μ_e const , ③ $\mu_t/\mu_e = 0$ (laminar) ④ BCs const

$$H_k = \frac{H - 0.29 M_e^2}{1 + 0.113 M_e^2}$$

$$H_k (1 + 0.113 M_e^2) + 0.29 M_e^2 = H$$

$$H_k (1 + 0.113 M_e^2) + 0.29 M_e^2 + 2 - M_e^2$$

$$(H_k + 2) + M_e^2 (0.113 H_k + 0.29 - 1)$$

$$\approx (H_k + 2) - M_e^2 \, 0.55$$

$$(1.5 + 2$$

$$\left(\frac{\theta_2}{\theta_1}\right) \approx \left(\frac{U_{e2}}{U_e}\right)^{-(H+2 - M_e^2 \, 0.55)}$$

$$\left(\frac{U_{e2}}{U_{e1}}\right)^{-(H+2)} \cdot \left(\frac{U_{e2}}{U_e}\right)^{0.55 M_e^2}$$

3

2.5

$$\frac{U_{e2}}{U_{e1}} \approx 0.8 \quad \Rightarrow \quad 0.8^{-3.5} \cdot 0.8$$

$$2.18$$

Solution Procedure

$$\left.\begin{array}{l} \psi,\ \mu,\ \tau \longrightarrow F,\ V,\ S \\ h_o,\ q \longrightarrow H,\ Q \end{array}\right\}$$ 5×5 Block, 5 BCs solve using Newton Method

- coupling is via density

$$\rho/\rho_e = R\left(U, H, u_e, h_{oe}\right)$$