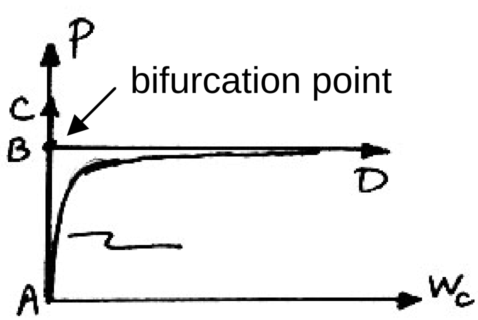
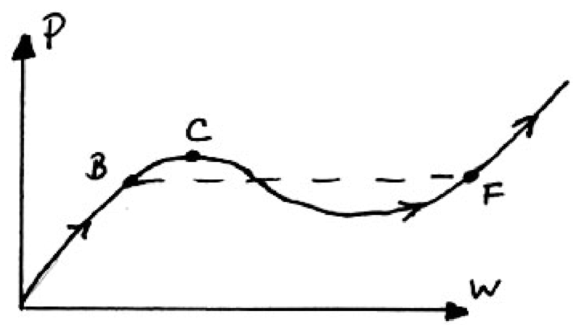
16.20 HANDOUT #5
Fall, 2002
Stability and Buckling

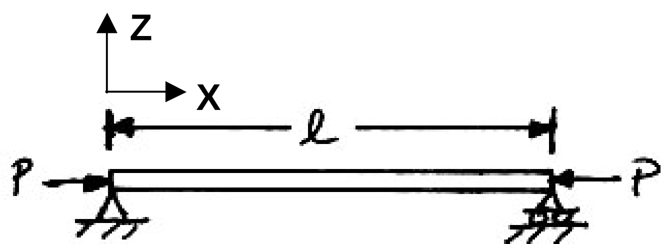
Bifurcation Buckling and *Snap-Through Buckling*





BIFURCATION BUCKLING

Perfect Column:



- Governing Equation: $$EI\frac{d^4w}{dx^4} + P\frac{d^2w}{dx^2} = 0$$

- Solution: $$w = A\sin\sqrt{\frac{P}{EI}}x + B\cos\sqrt{\frac{P}{EI}}x + C + Dx$$

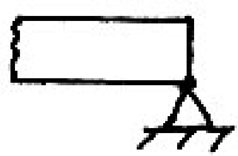
- Simply supported: $P_{cr} = \frac{n^2 \pi^2 EI}{\ell^2}$ mode shape: $w = A \sin \frac{n \pi x}{l}$

<u>*Euler buckling load:*</u> $P_{cr} = \frac{\pi^2 EI}{l^2}$

- General Case: $P_{cr} = \frac{c \pi^2 EI}{l^2}$ c = coefficient of edge fixity

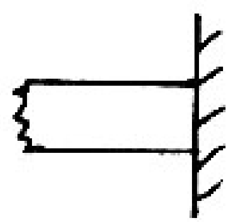
- Various Boundary Conditions

  – Simply-supported (pinned)



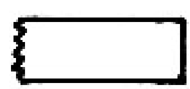
$$\begin{cases} w = 0 \\ M = EI \frac{d^2 w}{dx^2} = 0 \end{cases}$$

  – Fixed end (clamped)



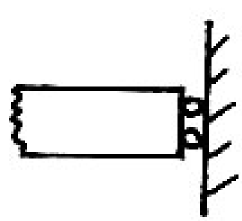
$$\begin{cases} w = 0 \\ \frac{dw}{dx} = 0 \end{cases}$$

  – Free end



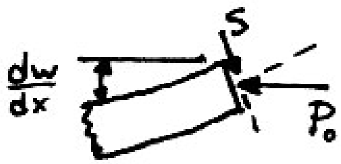
$$\begin{cases} M = EI \frac{d^2 w}{dx^2} = 0 \\ S = \frac{d}{dx}\left(EI \frac{d^2 w}{dx^2}\right) = 0 \end{cases}$$

  – Sliding



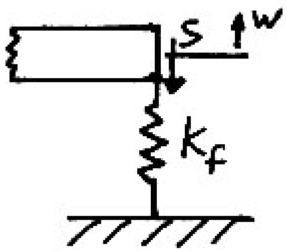
$$\begin{cases} S = \frac{d}{dx}\left(EI \frac{d^2 w}{dx^2}\right) = 0 \\ \frac{dw}{dx} = 0 \end{cases}$$

– Free end with axial load



$$\begin{cases} M = EI\dfrac{d^2w}{dx^2} = 0 \\ S = \dfrac{d}{dx}\left(EI\dfrac{d^2w}{dx^2}\right) = -P_0\dfrac{dw}{dx} \end{cases}$$

– Vertical spring

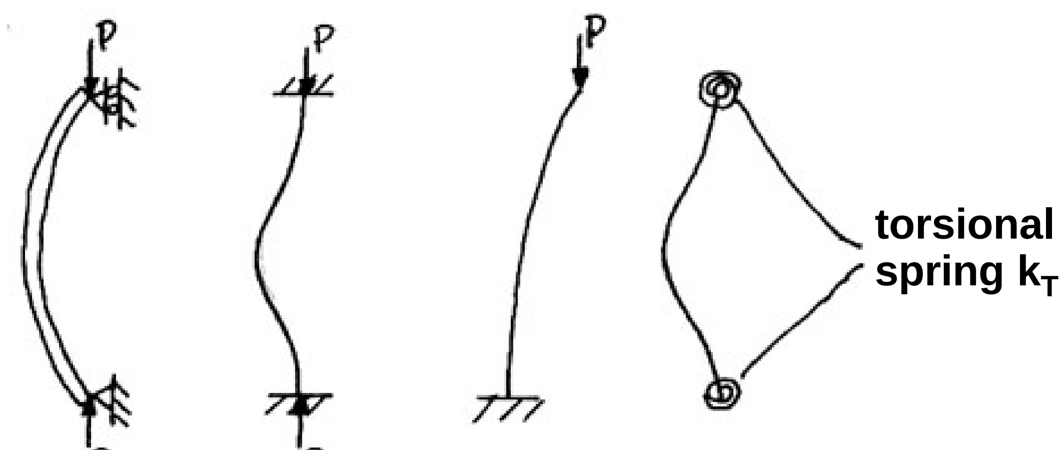


$$\begin{cases} M = EI\dfrac{d^2w}{dx^2} = 0 \\ S = \dfrac{d}{dx}\left(EI\dfrac{d^2w}{dx^2}\right) = k_f\, w \end{cases}$$

– Torsional spring

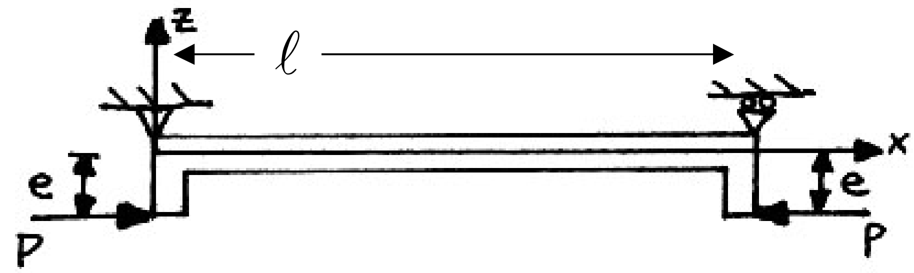
$$\begin{cases} w = 0 \\ M = EI\dfrac{d^2w}{dx^2} = -k_T\dfrac{dw}{dx} \end{cases}$$

- Various Configurations



c = 1 c= 4 c = 0.25 1 < c < 4

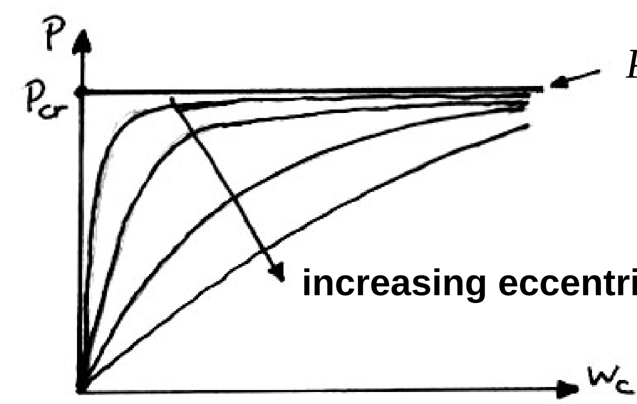
- Important Definitions
  - radius of gyration = $\rho = \left(I / A\right)^{1/2}$
  - slenderness ratio = L / ρ
  - effective length = $L' = \dfrac{L}{\sqrt{c}}$
- Effects of Initial Imperfections



Governing equation still: $$EI\frac{d^4 w}{dx^4} + P\frac{d^2 w}{dx^2} = 0$$

--> Boundary Conditions Change: Primary Moment = -eP

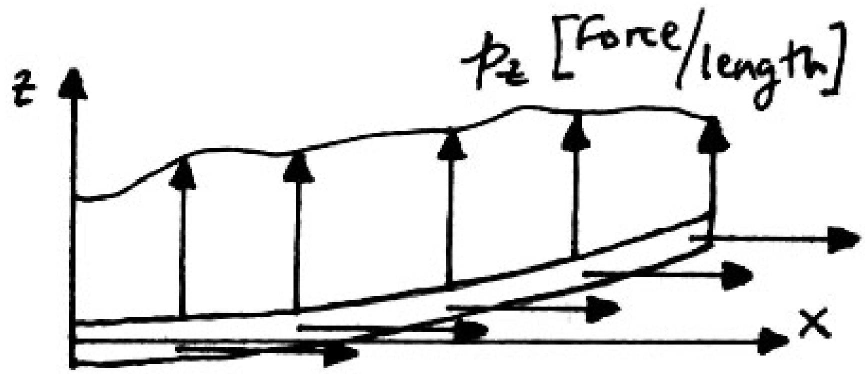
$$w = e\left\{\frac{\left(1 - \cos\sqrt{\frac{P}{EI}}\,l\right)}{\sin\sqrt{\frac{P}{EI}}\,l}\sin\sqrt{\frac{P}{EI}}\,x + \cos\sqrt{\frac{P}{EI}}\,x - 1\right\}$$



$$P_{cr} = \frac{\pi^2 EI}{l^2}$$

$$M = EI\frac{d^2 w}{dx^2} = -eP\left\{\frac{\left(1 - \cos\sqrt{\frac{P}{EI}}\,l\right)}{\sin\sqrt{\frac{P}{EI}}\,l}\sin\sqrt{\frac{P}{EI}}\,x + \cos\sqrt{\frac{P}{EI}}\,x\right\}$$

BEAM-COLUMN



- Resultant Relations

$$\frac{dF}{dx} = -p_x - \frac{d}{dx}\left(S\frac{dw}{dx}\right) \approx -p_x$$

$$\frac{dS}{dx} = p_z + \frac{d}{dx}\left(F\frac{dw}{dx}\right)$$

$$\frac{dM}{dx} = S$$

- Governing Equation:

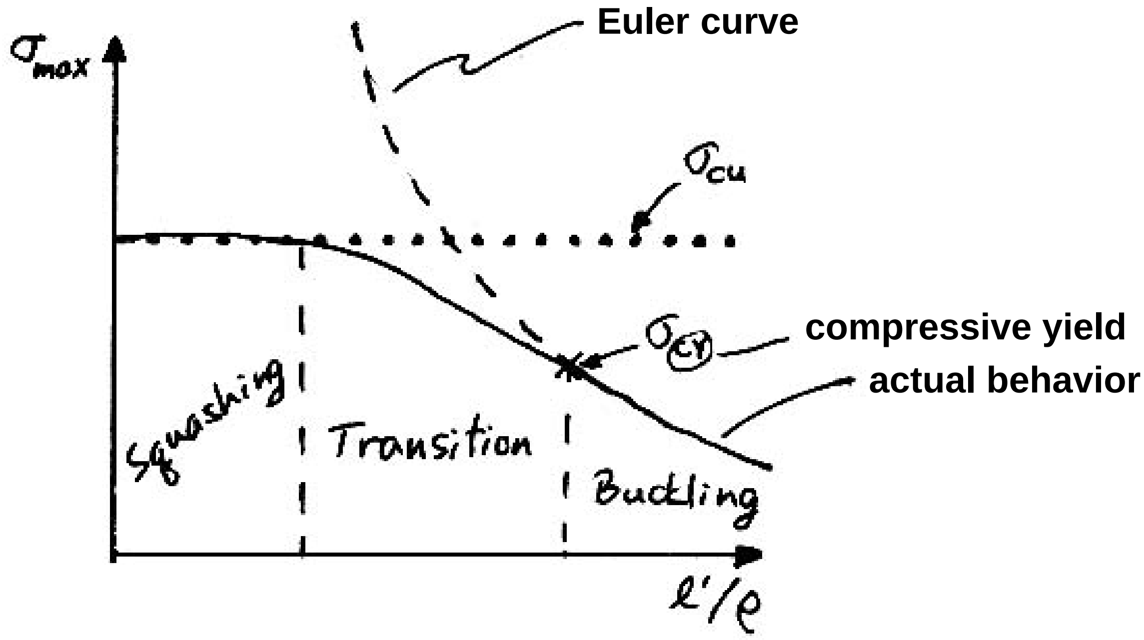
$$\frac{d^2}{dx^2}\left(EI\,\frac{d^2w}{dx^2}\right) - \frac{d}{dx}\left(F\,\frac{dw}{dx}\right) = p_z$$

- Buckling of Beam-Column:

$$EI\frac{d^2w}{dx^2} + Pw = M_{primary}$$

OTHER ISSUES

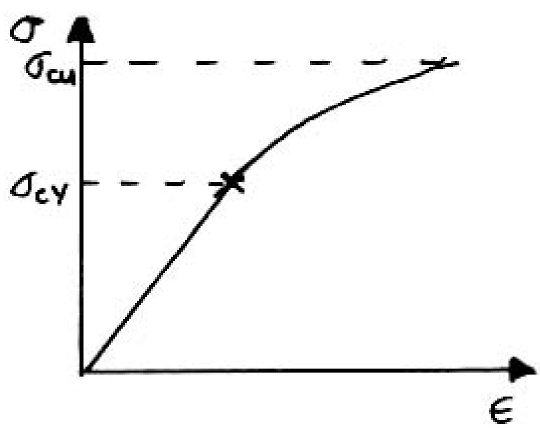
- Fracture/Failure via "squashing"



$$\sigma = \frac{P}{A} = \sigma_{cu} \text{ for "squashing"}$$

compressive ultimate

- Progressive Yielding

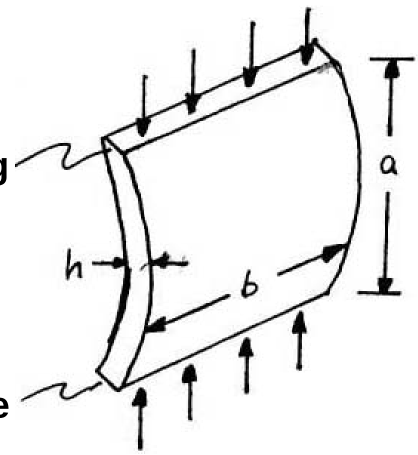


use tangent modulus

• Nonuniform Beams

$$\frac{d^2}{dx^2}\left(EI\,\frac{d^2w}{dx^2}\right) + P\,\frac{d^2w}{dx^2} = 0$$

• Plates

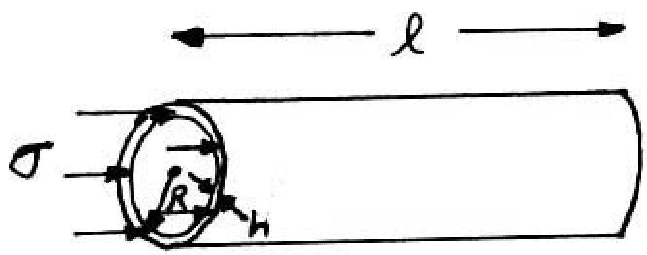


$$P_{cr} = \frac{\pi^2 EI}{l^2\left(1-\nu^2\right)}$$

simply-supported
isotropic plate

$$w = w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

• Cylinders



local "crippling"

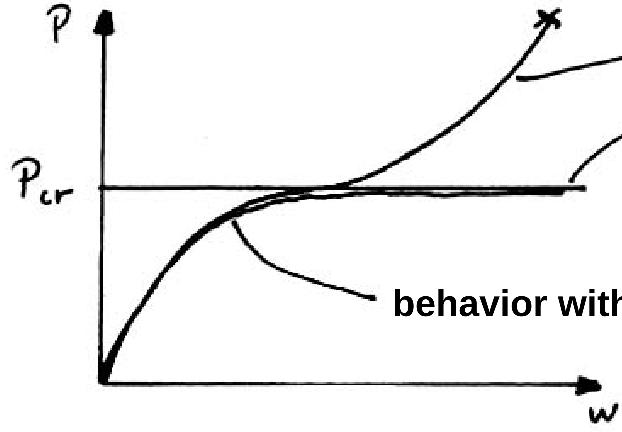
$$\sigma_{cr(linear)} = 0.606\,E\,\frac{h}{R} \quad \textit{(isotropic)}$$

due to imperfections: $\sigma_{cr_{(actual)}} \approx (0.15 \text{ to } 0.9)\,\sigma_{cr_{(linear)}}$

• Reinforced Plates

Consider buckling/crippling of elements of stiffness as well as of panels

- Postbuckling



large deformations ⟶ $\text{curvature} = \frac{d\theta}{ds} = \frac{1}{\sqrt{1 - \left(\frac{dw}{ds}\right)^2}} \frac{d^2w}{ds^2}$

Basic Equation:

$$\left[1 + \frac{1}{2}\left(\frac{dw}{ds}\right)^2 + \text{H.O.T.}\right] \frac{d^2w}{ds^2} + \frac{P}{EI} w = 0$$

Use Galerkin Method (minimize residuals)