16.346 Astrodynamics
Fall 2008

Exercises 18

1. Derive the Lagrange Interpolation Formulas for a sequence of four measurements.
2. Derive the differential equation used in Gibbs Method to determine the parameter p:

$$\frac{d^2r}{dt^2} = \epsilon(p - r)$$

3. Derive the relation

$$\left(\frac{dr}{dt}\right)^2 = \mu\left(\frac{2}{r} - \frac{p}{r^2} - \frac{1}{a}\right)$$

4. Derive the differential equation used in Gibbs Method to determine the semimajor axis a:

$$\frac{d^2}{dt^2}(r^2) = 2\mu\left(\frac{1}{r} - \frac{1}{a}\right)$$

5. Given a pair of range and range-rate measurements for a spacecraft in interplanetary space:

$$r_1 = 0.600762027 \text{ a.u.} \qquad r_2 = 0.603053915 \text{ a.u.}$$

$$\frac{dr_1}{dt} = 0.288618834 \text{ a.u./year} \qquad \frac{dr_2}{dt} = 0.575041077 \text{ a.u./year}$$

Determine the elements a, e, and p of the orbit. **Problem 3–35**

Answer: $a = 1.19999993$ a.u. and $p = 0.89999998$ a.u.

6. Three observations of a satellite are made from the earth

$$t_1 = 0.005274926 \text{ year} \qquad t_2 = 0.010576712 \text{ year} \qquad t_3 = 0.021370777 \text{ year}$$

$$\mathbf{i}_{\rho_1} = \begin{bmatrix} -0.830593168 \\ 0.554953271 \\ 0.046280201 \end{bmatrix} \qquad \mathbf{i}_{\rho_2} = \begin{bmatrix} -0.851865512 \\ 0.521052142 \\ 0.053196007 \end{bmatrix} \qquad \mathbf{i}_{\rho_3} = \begin{bmatrix} -0.890539844 \\ 0.450207593 \\ 0.065206661 \end{bmatrix}$$

when the earth is positioned at

$$\mathbf{d}_1 = \begin{bmatrix} 0.999450810 \\ 0.033137270 \\ 0 \end{bmatrix} \text{ a.u.} \qquad \mathbf{d}_2 = \begin{bmatrix} 0.997792649 \\ 0.066406537 \\ 0 \end{bmatrix} \text{ a.u.} \qquad \mathbf{d}_3 = \begin{bmatrix} 0.990998441 \\ 0.133873410 \\ 0 \end{bmatrix} \text{ a.u.}$$

Find the position vector of the satellite at time t_2.

Note: The earth's orbit is assumed to be a circle of radius one astronomical unit and the earth crossed the reference x-axis at time zero.

Answer:

$$\mathbf{r}_2 \approx \begin{bmatrix} 0.1489\ldots \\ 0.5856\ldots \\ 0.0530\ldots \end{bmatrix} \qquad \text{Exact answer} \quad \mathbf{r}_2 = \begin{bmatrix} 0.159321004 \\ 0.579266185 \\ 0.052359607 \end{bmatrix}$$

$$\rho_1 = 0.952633214 \text{ a.u.} \qquad \rho_2 = 0.984277017 \text{ a.u.} \qquad \rho_3 = 1.048132894 \text{ a.u.}$$

$$\left.\frac{d\mathbf{i}_\rho}{dt}\right|_{t_2} = \begin{bmatrix} -3.870867561 \\ -6.449952154 \\ 1.241279013 \end{bmatrix} \qquad \left.\frac{d^2\mathbf{i}_\rho}{dt^2}\right|_{t_2} = \begin{bmatrix} 53.35190114 \\ -20.99951742 \\ -23.82234043 \end{bmatrix}$$

$$D_1 = -144.0082104 \quad D_2 = 1.044242145 \quad D_3 = -12.42980519$$

$$r(t_2) = 0.606571899 \qquad \rho(t_2) = 0.996437289 \qquad \left.\frac{d\rho}{dt}\right|_{t_2} = 5.930383272$$