16.55 Ionized Gases
Problem Set #4

Problem 1

Combine the total energy equation for a species s in terms of $(\vec{u}_s, T'_s, \vec{P}'_s, \vec{q}'_s)$, as derived in the notes, with the momentum equation in terms of $(\vec{u}_s, \vec{P}'_s)$ to obtain the internal energy equation. You should obtain,

$$\frac{\partial}{\partial t}\left(\frac{3}{2}n_s k T'_s\right) + \nabla \cdot \left(n_s \vec{u}_s \frac{3}{2}kT'_s + \vec{q}'_s\right) + \vec{P}'_s : \nabla \vec{u}_s \;=\; \sum_r E'_{rs} \tag{1}$$

with,

$$E'_{rs} \;=\; E_{rs} - \vec{u}_s \cdot \vec{M}_{rs}$$

Notice the absence of any electric or magnetic terms on the LHS.

Problem 2

Consider a plasma that is homogeneous, steady and stationary, subject to uniform $\vec{E}$ and $\vec{B}$ fields, which produce an electron current density $\vec{j}_e$. The gas may be undergoing steady ionization, at a rate $\dot{n}_e = R_i(T'_e)n_n n_e$ per unit volume, where $R_i = \langle c_e Q_{ioniz}\rangle_e$. Each ionization event absorbs from the electron gas an energy $\alpha_i E_i$, where $\alpha_i \approx 2$ to 3 accounts for excitation losses that happen simultaneously. Only one species of ions and of their parent neutrals are present, and it can be assumed that electron collisions are much more frequent with the neutrals than with the ions.

(a) For this situation, formulate Eq. (1) of Problem 1, as the equation that will supply the electron temperature. The dissipative term in E'_{rs} should reduce to j_e^2/σ; eliminate $\vec{j}_e$ using the Ohm's law, and show that the equation reduces to,

$$\frac{1}{m_e}\left(\frac{eE}{\sqrt{1+\beta_e^2}\,\nu_{en}}\right)^2 \;=\; 3\frac{m_e}{m_n}k\,(T'_e - T'_n) + \frac{\langle c_e Q_{ioniz}\rangle_e}{\bar{c}_e Q^*_{en}}\alpha_i E_i \tag{2}$$

(b) A model for the ionization rate is given by,

$$R_i \;\approx\; \sigma_0 \bar{c}_e \left(1 + 2\frac{kT'_e}{E_i}\right)e^{-E_i/kT'_e} \tag{3}$$

Derive an approximate criterion, in the form of a critical value of kT'_e/E_i, to test whether the ionization loss term or the elastic energy transfer term in Eq. (2) are dominant. Note that the exponential term in (3) is the controlling factor.

(c) For conditions of low magnetization, $\beta_e \ll 1$, show that T'_e is a function of T'_n and of the "reduced field" E/n_n. Conversely, for high magnetization, show that T'_e is a function of T'_n and of the drift velocity E/B. As a numerical example, consider a Hall thruster plasma, with $\beta_e \gg 1$, where (in the ionization region) E is about $100V/cm$ and B is 200Gauss. The gas is Xenon, with $V_i = 12.1$ eV, $\sigma_0 = 3.6 \times 10^{-20}m^2$ and $Q^* \approx 3 \times 10^{-19}m^2$. Take $\alpha_i = 2.5$.

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