

16.901: Homework # 10

Due Date: April 4, 2pm

In this homework, you will complete the Matlab script, `fem_dif1d.m`, to solve the one-dimensional heat transfer problem,

$$T_{xx} + q(x) = 0, \quad q(x) = 50e^x,$$

with boundary conditions $T(\pm 1) = 100$. This is the same problem we have been looking at in the lecture notes. The script is available on the webpage under the Homework #10 link. You need to complete the two lines in the script that calculate the forcing term integrals,

$$\int_{-L/2}^{L/2} \phi_j q dx.$$

The nodal basis for linear finite elements has been used in the script. The lines you need to add are in a loop over the elements. For each element, you calculate two forcing term contributions: the forcing term contribution to the weighted residual associated with node 1 of the element, and the forcing term contribution to the weighted residual associated with node 2 of the element. To do these contributions, the following integral will be required,

$$\int_{x_1}^{x_2} x e^x dx = [x e^x - e^x]_{x_1}^{x_2}.$$

1. Complete the Matlab script and run the simulation for 5 elements and 10 elements. For your homework, include the plots of the solutions as well as a hard copy of your completed script.
2. What do you think the order of accuracy is for this finite element method? Justify your answer using the plots from the 5 and 10 element solutions. Note: the exact solution has been included in these plots as the solid line without symbols.