16.901: Project #3
Impact of Variability on Turbine Blade Temperature Solution

1 Background

In Project #2, the temperature distribution in a cooled turbine blade was estimated using a finite element method. In this project, the concern is to quantify the variability in maximum temperatures and maximum temperature gradients due to variability in the environment and properties of the turbine blade. These quantities are of interest because the usable life of a blade decreases as either the temperature or the temperature gradients increase.

1.1 Design Conditions

The nominal, i.e. design-intent, conditions are the same as analyzed in Project #2. In particular,

$$k = 30\, W/(m\, K).$$

For the outer surface of the blade, the temperature will be assumed to be,

$$T_{gas} = 1500\, K.$$

The heat transfer coefficient will be larger at the leading edge, and this is modeled with the following form,

$$h_{gas}(x) = h_{TE} + (h_{LE} - h_{TE}) e^{-4\left(\frac{x/c}{0.05}\right)^2}\, W/(m^2 K), \tag{1}$$

with the design-intent values of h_{LE} and h_{TE} being 16,000 and 4,000 $W/(m^2K)$. The chord of the blade is $c = 0.04\, m$. For the internal cooling passages and trailing edge slot, we will assume that,

$$T_{cool} = 600\, K, \qquad h_{cool} = 1500\, W/(m^2 K).$$

Using the provided Matlab script, **DesignIntent.m**, the the maximum temperature and maximum temperature gradient (magnitude) in the blade at design-intent conditions are (to three significant digits),

$$T_{max}^{des} = 1430\, K, \qquad |\nabla T|_{max}^{des} = 70,000\, K/m.$$

1.2 Input Variability

We will assume the following variability in the inputs:

- The thermal conductivity of the blade will be assumed to have a triangular distribution in which $k_{min} = 28.5\, W/(m\, K)$, $k_{mpp} = 30\, W/(m\, K)$, and $k_{max} = 31.5\, W/(m\, K)$.
- The external gas temperature will be assumed to have a triangular distribution in which $T_{gas_{min}} = 1400\, K$, $T_{gas_{mpp}} = 1500\, K$, and $T_{gas_{max}} = 1600\, K$.

- The leading-edge, external gas heat transfer coefficient will be assumed to have a triangular distribution in which $h_{LEmin} = 15,500\, W/(m^2K)$, $h_{LEmpp} = 16,000\, W/(m^2K)$, and $h_{LEmax} = 18,000\, W/(m^2K)$. Note that this distribution is not centered about the most-probable value because it is assumed that the presence of additional surface roughness at the leading-edge will tend to increase the heat transfer in this region above design intent.

- The trailing-edge, external gas heat transfer coefficient will be assumed to have a triangular distribution in which $h_{TEmin} = 3,500\, W/(m^2K)$, $h_{TEmpp} = 4,000\, W/(m^2K)$, and $h_{TEmax} = 4,500\, W/(m^2K)$.

- The cooling passage temperature will be assumed to have a triangular distribution in which $T_{coolmin} = 550\, K$, $T_{coolmpp} = 600\, K$, and $T_{coolmax} = 650\, K$. Note: it is assumed that the cooling temperature in the three passages will be a single random value (not three different random values). This is because the passages are all supplied by a single common plenum and all will see the same change in temperature. This essentially assumes that the variability of the temperature from passage-to-passage is much smaller than that from blade-to-blade.

- The cooling passage heat transfer coefficient will be assumed to have a triangular distribution in which $h_{coolmin} = 1,400\, W/(m^2K)$, $h_{coolmpp} = 1,500\, W/(m^2K)$, and $h_{coolmax} = 1,600\, W/(m^2K)$). As with the cooling passage temperatures, we will assume only a single random value for all three passages.

1.3 Matlab Scripts

The following Matlab scripts and input files are available on the webpage for you to use in completing this project.

- **calcblade.m**: Function that performs the finite element analysis given the input parameter values and the mesh data. It returns the temperature at the nodes and the temperature gradient magnitude in the elements.

- **loadblade.m**: Function that loads the mesh. Note: Since the shape of the blade is not changing during the Monte Carlo simulation, this function only should be called once prior to the entering the Monte Carlo loop. Inside the loop, only **calcblade.m** needs to be called.

- **Thgas.m**: Function that calculates the external gas temperature and heat transfer coefficient dependent on the location on the surface.

- **DesignIntent.m**: Script that performs the heat transfer analysis at the design intent conditions. This script calls the above functions.

- **hpblade_coarse.mat**: The coarse mesh data file which is to be used for this project.

2 Tasks

2.1 Estimation of Probability of Failure

A failure will occur when the maximum temperature or temperature gradients become too large. For simplicity, a limiting value of the temperature and temperature gradient will be assumed. Specifically, we will assume that,

$$T_{limit} = 1500\,K, \qquad |\nabla T|_{limit} = 80,000\,K/m.$$

As a simple method to combine damage due to high temperature and damage due to high temperature gradients, a blade will be assumed to fail when $D > 1$ where,

$$D \equiv \frac{T_{max} - T_{max}^{des}}{T_{limit} - T_{max}^{des}} + \frac{|\nabla T|_{max} - |\nabla T|_{max}^{des}}{|\nabla T|_{limit} - |\nabla T|_{max}^{des}}.$$

Implement a Monte Carlo method to determine the probability of failure to within ± 0.01 at 99% confidence. Specifically:

1. Develop well-commented Matlab scripts to implement the Monte Carlo analysis for this problem. Upload your Matlab scripts on the webpage. Note: no additional documentation is required beyond well-documented, clearly-written Matlab scripts.

2. What is the estimated probability of failure? What sample size was required to achieve the required accuracy?

   Solution: The estimated probability of failure is 0.166 ± 0.01 at 99% confidence. This required a sample size of 12,489 to achieve.

3. Include histograms and CDF plots of the distribution of T_{max}, $|\nabla T|_{max}$, and D. Note: use the Matlab command **cdfplot** for plotting CDF's.

   Solution: See Figures 1-3.

2.2 Screening for Important Factors

In this section, you are to determine which of the input variabilities have the strongest effect on the probability of failure. In principle, it would be possible to run additional Monte Carlo simulations in which the input variability were individually decreased (or increased) and the change in the probability of failure could be calculated. However, this could involve significant time especially when the number of inputs is large. Another possibility is to analyze the previous Monte Carlo results (from Section 2.1) to estimate the input variabilities that are strong drivers of the probability of failure and/or the damage. In this task, you will determine which inputs have the strongest effects on the damage, and assume that these inputs also have a strong effect on the failure probability.

One standard method for quantifying the strength of coupling between variables in a sample is through the Spearman rank correlation coefficient. The Spearman rank correlation coefficient is constructed by sorting the two variables and then constructing the correlation

	k	T_{gas}	h_{LE}	h_{TE}	T_{cool}	h_{cool}
ρ	-0.08	0.95	0.10	-0.18	-0.13	0.13

Table 1: Spearman correlation coefficient for each input with respect to damage, D

coefficient based on the difference in ranks between them. In this project, the correlation will be quantified between the damage, D, and each of the 6 input parameters (k, T_{gas}, h_{LE}, h_{TE}, T_{cool} and h_{cool}). For example, to find the rank correlation coefficient between D and k, D is sorted (from low to high) and k is sorted similarly. Then, the rank distance for the pair (D_i, k_i) is defined as,

$$d_i = \text{rank}(D_i) - \text{rank}(k_i),$$

where the function $\text{rank}(x_i)$ returns the integer index in the sorted set of x_i's. Using this definition, the correlation coefficient, ρ, between the ranks of the two variables is,

$$\rho = 1 - 6\frac{\sum_{i=1}^{N} d_i^2}{N(N^2-1)}.$$

To calculate this correlation coefficient, you can use Matlab's function **corr** which is available in Matlab's Statistical Toolbox. When using this function, make sure to set the type to Spearman. Note that ρ is guaranteed to range between -1 and 1. Strong correlations are indicated when ρ approaches either ± 1. No correlation is indicated for $|\rho| \to 0$. The sign of ρ indicates whether the two variables increase together, or in opposition.

For this task, determine the input variables which are most strongly rank-correlated to the damage using Spearman's rank correlation coefficient applied to the sample from Section 2.1. Specifically, give the value of ρ for each input.

Solution: The correlation coefficients are shown in Table 1. The results shows that T_{gas} is by far the most strongly correlated to the damage. Following far behind as the second most strongly correlated is h_{TE}.

2.3 Impact of Important Factors on Probability of Failure

The purpose of this task is to quantify the impact that decreasing the variability of the inputs would have on the probability of failure. This type of information would then be used to decide if resources should be spent on controlling these sources of variability. To quantify this impact, the following procedure is suggested. For an input at a time, the input variability will be decreased by 50% by decreasing the separation of the minimum value and the maximum value to the most probable value by 50%. For example, for h_{LE}, the new triangular distribution would be defined by, $h_{LEmin} = 15,750\,W/(m^2K)$, $h_{LEmpp} = 16,000\,W/(m^2K)$, and $h_{LEmax} = 17,000\,W/(m^2K)$. Starting from the input variable that is most strongly correlated with the damage (as determined in Section 2.2), perform a Monte Carlo simulation in which only that input variability is decreased (while the other 5 input distributions remain the same) and calculate the probability of failure to within ± 0.01 at 99% confidence. When the probability of failure from reducing an input's variability cannot be discerned from statistical uncertainty, then the impact of the remaining more weakly correlated inputs does not need to be quantified. As a simple test of statistically significant differences, we will require that the confidence intervals for the probability of failure do not overlap.

In this task, specifically perform the following tasks:

1. In order of strongest correlation, determine the probability of failure for a 50% reduction in an input's variability as described above until the first input that does not have a statistically significant impact. Including up to the first non-significant result, determine the probability of failure within ± 0.01 at 99% confidence and the sample size required to achieve this accuracy for each input.

2. Include histogram and CDF plots for the reduced input variability simulations that were performed.

Solution:

The following probability of failures were found:

- For a 50% reduction in T_{gas} variability, the probability of failure was estimated as $P = 0.0375 \pm 0.01$ at 99% confidence using 3251 trials. The histogram and CDF plots are shown in Figures 4-6.

- For a 50% reduction in h_{TE} variability, the probability of failure was estimated as $P = 0.162 \pm 0.01$ at 99% confidence using 12209 trials. The histogram and CDF plots are shown in Figures 7-9. Since this range of probability of failure overlaps with the full variability result (which was $P = 0.166 \pm 0.01$), there is no longer a statistical significance to the differences observed.

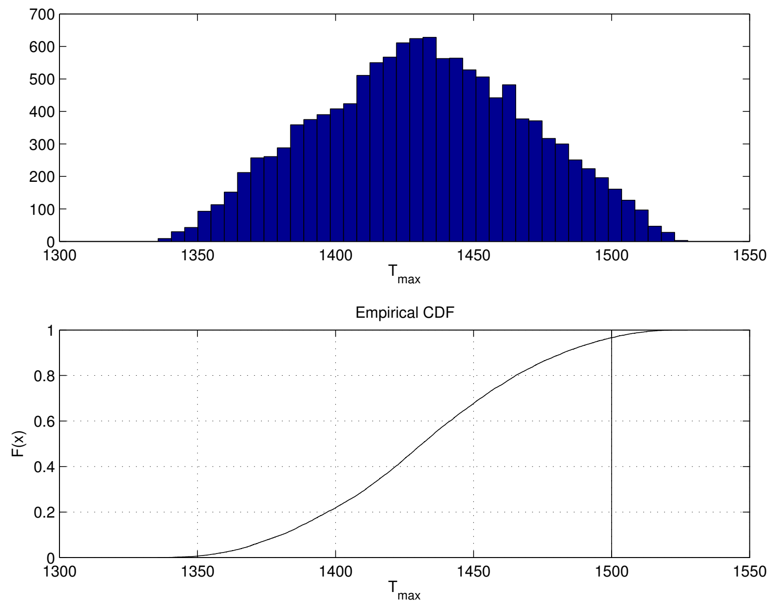

Figure 1: Histogram and CDF of T_{max} distribution.

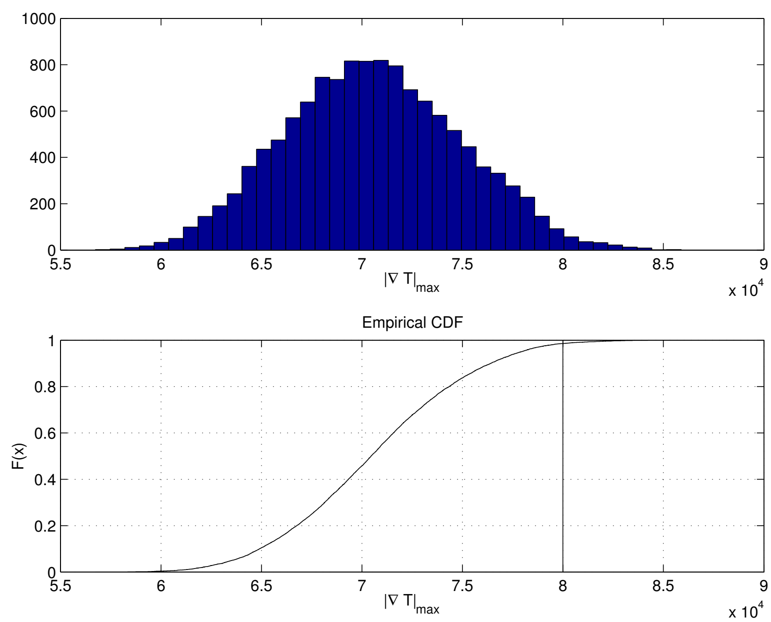

Figure 2: Histogram and CDF of $|\nabla T|_{max}$ distribution.

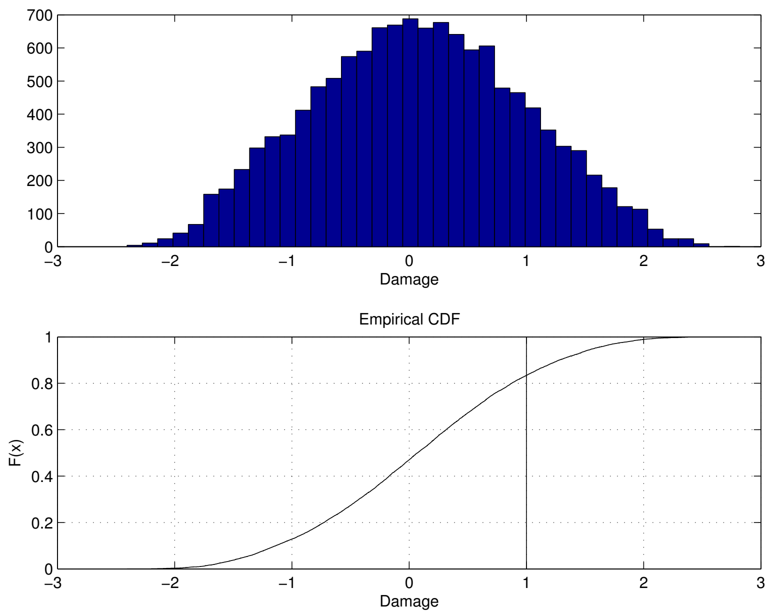

Figure 3: Histogram and CDF of D distribution.

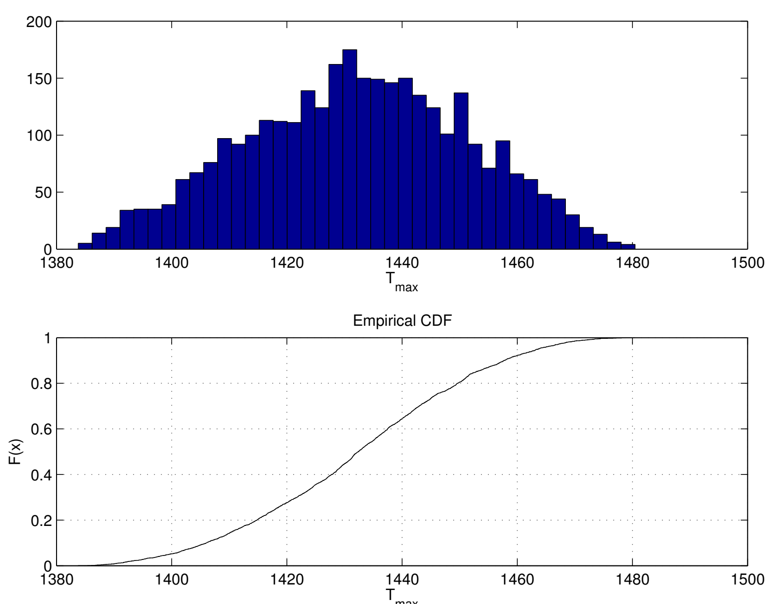

Figure 4: Histogram and CDF of T_{max} distribution for 50% reduction in T_{gas} variability.

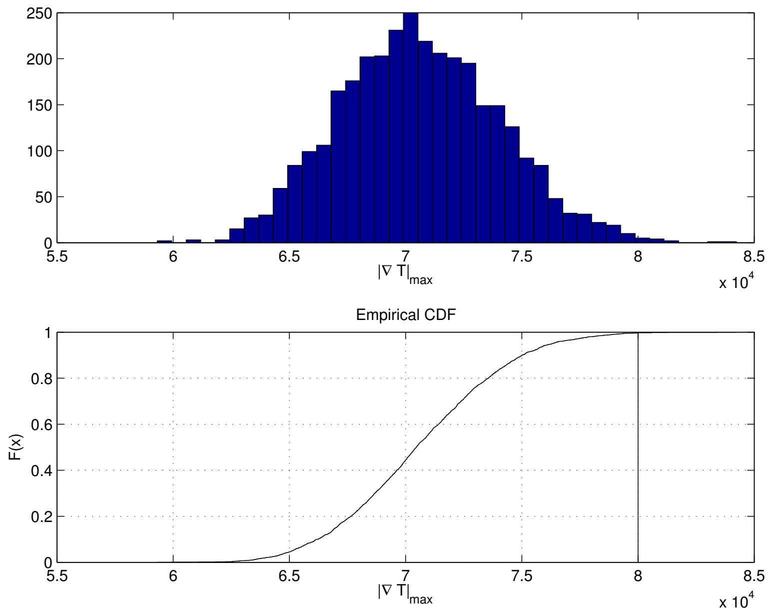

Figure 5: Histogram and CDF of $|\nabla T|_{max}$ distribution for 50% reduction in T_{gas} variability.

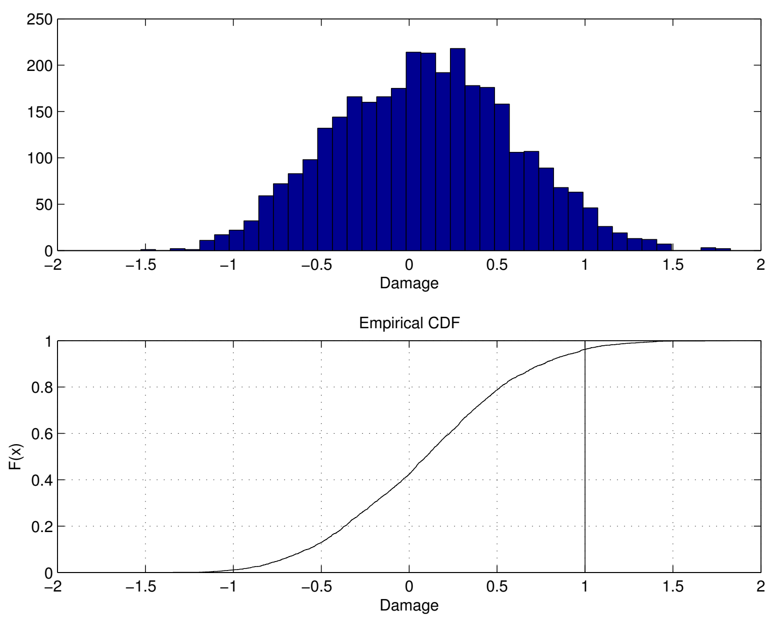

Figure 6: Histogram and CDF of D distribution for 50% reduction in T_{gas} variability.

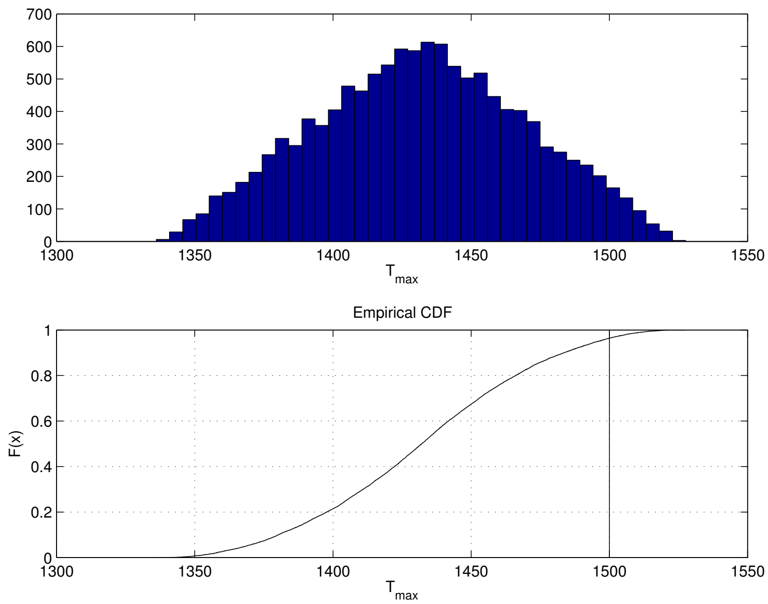

Figure 7: Histogram and CDF of T_{max} distribution for 50% reduction in h_{TE} variability.

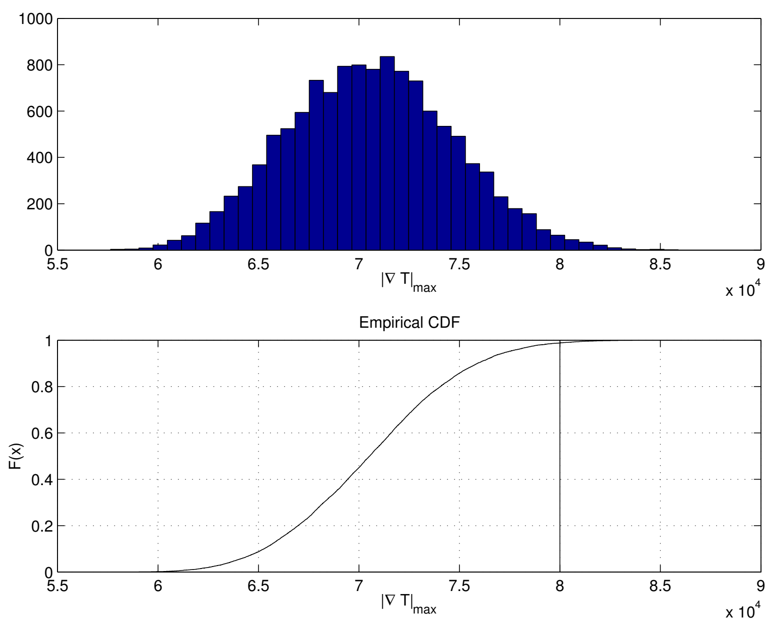

Figure 8: Histogram and CDF of $|\nabla T|_{max}$ distribution for 50% reduction in h_{TE} variability.

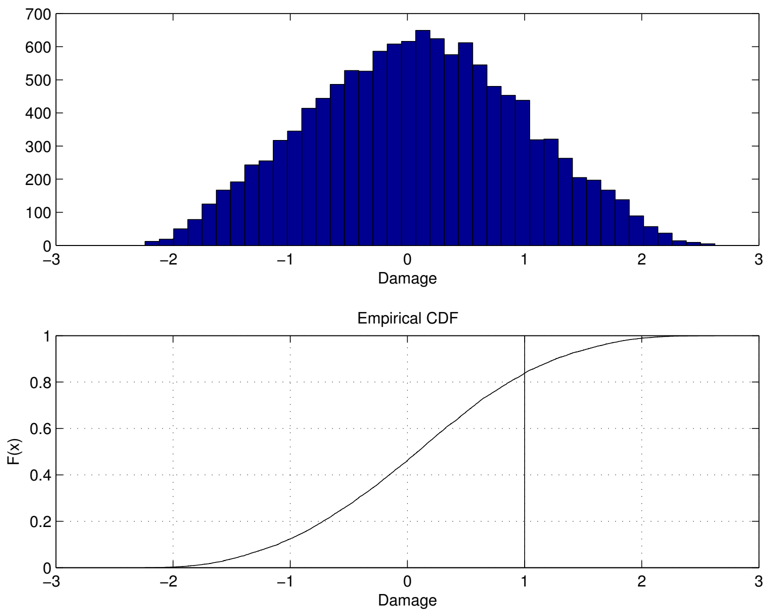

Figure 9: Histogram and CDF of D distribution for 50% reduction in h_{TE} variability.