Recitation Section 21
April 27-28, 2005

## Life Tables

## A. Cohort life table

You are studying the life of the common tribble. In a large tribble colony, you mark 1000 newborn tribbles and observe them for the next 5 years. You find the following:

| Year <br> (age) | \# tribbles <br> alive at <br> start of <br> year <br> $\mathbf{n}_{\mathbf{x}}$ | Survivor- <br> ship | Mortality <br> rate | Mean \# <br> tribbles <br> alive in <br> year <br> $\mathbf{L}_{\mathbf{x}}$ | Average <br> Remaining <br> Life <br> Expectancy <br> $\mathbf{e}_{\mathbf{x}}$ | Average life <br> expectancy <br> for <br> individuals <br> of age $\mathbf{x}$ |
| :---: | :---: | :---: | :---: | :---: | :--- | :--- |
| 0 | 1000 |  | $\mathbf{m}_{\mathbf{x}}$ |  |  |  |
| 1 | 900 |  |  |  |  |  |
| 2 | 700 |  |  |  |  |  |
| 3 | 200 |  |  |  |  |  |
| 4 | 50 |  |  |  |  |  |
| 5 | 0 |  |  |  |  |  |

1. Derive formulas for and calculate the remaining values in the table, based on the following definitions.
$l_{\mathrm{x}} \quad$ survivorship in year $\mathrm{x}=$ survival of individuals to age x
$\mathrm{m}_{\mathrm{x}} \quad$ mortality rate in year $\mathrm{x}=$ proportion of individuals of age x dying by age $\mathrm{x}+1$
$L_{x} \quad$ age units lived in year $x=$ mean $\#$ of individuals alive between year $x$ and $x+1$
$e_{x} \quad$ remaining life expectancy at age $x=$ expectation of further life for individuals of age $x$

$$
\mathrm{e}_{\mathrm{x}}=\frac{\sum_{\mathrm{i}=\mathrm{x}}^{5} \mathrm{~L}_{\mathrm{i}}}{\mathrm{n}_{\mathrm{x}}}
$$

2. Sketch the survivorship curve for tribbles:
3. Describe this curve in words. Why does the shape of the curve make sense qualitatively?
4. What other types of curves are there? Describe qualitative conditions that produce these curves.

## B. Replacement rates

You also collected data on the tribbles born to the cohort you are studying. This is summarized below:

| YEAR <br> (age) <br> $\mathbf{x}$ | \# tribbles alive <br> at start of year <br> $\mathbf{n}_{\mathbf{x}}$ | \# individuals born to <br> members of cohort <br> during year $\mathbf{x}$ | Fecundity <br> $\mathbf{b}_{\mathbf{x}}$ | $\mathbf{l}_{\mathbf{x}} \mathbf{b}_{\mathbf{x}}$ | for part 3 | for part 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1000 | 0 |  |  |  |  |
| 1 | 900 | 1200 |  |  |  |  |
| 2 | 700 | 50 |  |  |  |  |
| 3 | 200 | 5 |  |  |  |  |
| 4 | 50 | 0 |  |  |  |  |
| 5 | 0 |  |  |  |  |  |

1. Calculate the fecundity and realized fecundity $\left(l_{x} b_{x}\right)$ for each age group.
2. Calculate the net reproductive rate, $\mathrm{R}_{0}=\sum_{\mathrm{i}=0}^{\mathrm{i}=5} \mathrm{l}_{\mathrm{i}} \mathrm{b}_{\mathrm{i}}$. Is this population stable $\left(\mathrm{R}_{0}=1\right)$, growing ( R ${ }_{0}>1$ ), or shrinking ( $\mathrm{R}_{0}<1$ )?
3. Suppose you find tribbles with the same life expectancies except that they all give birth to 2 new tribbles only once in their lifetime, at an age of 2 years. Will the resulting population be stable?
4. Suppose you find tribbles with the same life expectancies except that they all give birth to 4 new tribbles only once in their lifetime, at an age of 3 years. Will the resulting population be stable?
