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Spring 2005
EXAM 2
April 13, 2005

1. (50 points) In the reverse osmosis systems we considered in class and for homework (Problem Set 3), we assumed that some salt or solute leaked through the membrane so that the permeate had a low salt concentration C_P. The flow of water and salt through the membrane can be modeled as two transport processes in parallel. Pure water permeates through the membrane wall by reverse osmosis and very small amounts of salt water pass through tiny pores or imperfections in the membrane by bulk flow. The salt concentration, C_P, is a result of both the osmotic flow of pure water and the bulk flow of salt water through pores. At any location along the length of the membrane tubes these two fluxes N'' and N''_P are:

$$N''\left[\frac{m^3}{m^2 s}\right] = K_m \left(P - P_0 - \Delta\Pi_{os}\right) \tag{1}$$

for pure water through the walls by reverse osmosis and

$$N''_P\left[\frac{m^3}{m^2 s}\right] = K_P \left(P - P_0\right) \tag{2}$$

for bulk flow of salt water through the pores where $N'' >> N''_P$. In these equations, P is the pressure inside the tube, P_o is the pressure outside the tube, $\Delta\Pi_{os}$ is the osmotic pressure, and K_m and K_p are experimentally-determined coefficients. The osmotic pressure can be related to concentration by

$$\Delta\Pi_{os} = k_{os}\left(C_{BW} - C_P\right) \tag{3}$$

where C_{BW} is the solute concentration at the inner wall of the tube, C_P is the concentration of solute in the permeate and k_{os} is the osmotic coefficient.

The concentration of solute in the well-mixed bulk of the liquid within the tube is C_B and the mass transfer coefficient of solute in the tube is $k_{MT}[m/s]$.

a) Determine the concentration of salt at the inner wall of the membrane C_{BW} as a function of C_B, N'' and k_{MT}, assuming that N''_P is zero.

b) Determine the concentration of salt at the inner wall of the membrane C_{BW} as a function of C_B, N'', N''_P, and k_{MT} when N''_P is not zero.

c) Find an expression for the salt concentration at the wall C_{BW} as a function of K_m, K_P, k_{MT}, k_{os}, C_B, C_P, P, and P_0.

d) Express the well-mixed salt concentration in the permeate C_P in terms of *only* N'', N''_P and C_{BW}.

e) Using your expression from part (d) and the definitions of N'' and N''_P, show whether C_P increases, decreases, or remains constant as the pressure inside the tubular membrane increases. You may assume that $C_{BW} >> C_P$.

2. (50 points) A column packed with one inch rings is used to strip solute A from water using air. The water enters the top of the column with a mole fraction of solute A equal to 0.01 and it is desired that the water leaving the bottom of the column have a mole fraction of A of 0.001. The air entering the column is free of solute A.

The water enters the column at a rate per unit cross-sectional area of 5,000 lb/hr-ft^2. The column operates at one atmosphere and 30°C. At equilibrium the solute A follows Henry's law

$$y = 10\,x \tag{1}$$

where y is the mole fraction of solute A in air and x is the mole fraction of A in water. The rate of mass transfer of A is controlled by the liquid side resistance so that the overall liquid-phase coefficient $K_X a$ is equal to the liquid-side coefficient $k_L a$.

a) What is the minimum flow rate of air if the concentrations of A in the inlet and outlet streams are stated as above? Express your answer in lb-mole/hr-ft^2.

b) What is the mole fraction of A in the exiting air stream if the flow rate of the air stream is twice the minimum and the inlet and outlet concentrations of A are as stated above?

c) Calculate the number of overall liquid phase transfer units N_{0L} (see Note) if the flow rate of the air stream is twice the minimum and the concentrations of A in the inlet and outlet streams of water are as given above.

d) If the column is 25 feet tall, what is the value of $k_L a \left[\dfrac{\text{lb moles}}{\text{hr ft}^3 \Delta x}\right]$?

Following an electrical power disruption at the plant, the mole fraction of A in the exiting water stream is 0.0012. A check of flow rates of air and water entering the column and the concentration of A in the incoming water show no change. An engineer suspects that an old valve in a pipe half way up the column may have been partially opened by the power failure, allowing a portion of the air stream to leave the column. This valve should normally be closed.

e) What is the number of overall liquid phase transfer units N_{0L} in the bottom half of the column? What is the concentration of A in the air stream at the position of the valve when the concentration of A in the water stream exiting the column is 0.0012?

f) Sketch the lower and upper operating lines if 40% of the air stream is leaking through the valve.

<u>Note</u>: Both the equilibrium and operating lines are linear; therefore, the distance between them is linear. Similar to the way we use the logarithmic mean temperature difference in heat exchangers,

$$\int_{x_1}^{x_2} \frac{dx}{x^* - x} = \frac{x_2 - x_1}{\overline{(x^* - x)}_{LM}}$$

where the subscript LM indicates the logarithmic mean of the quantity in parentheses over the range x_1 to x_2.

The logarithmic mean of a and b is: $\dfrac{b - a}{\ln\left(\dfrac{b}{a}\right)}$.