10.37 Chemical and Biological Reaction Engineering, Spring 2007
Prof. K. Dane Wittrup

Lecture 19: Oxygen transfer in fermentors

This lecture covers: Applications of gas-liquid transport with reaction

Gas-liquid mass transfer in bioreactors

Microbial cells often grown aerobically in stirred tank reactors
-oxygen supply is often limiting

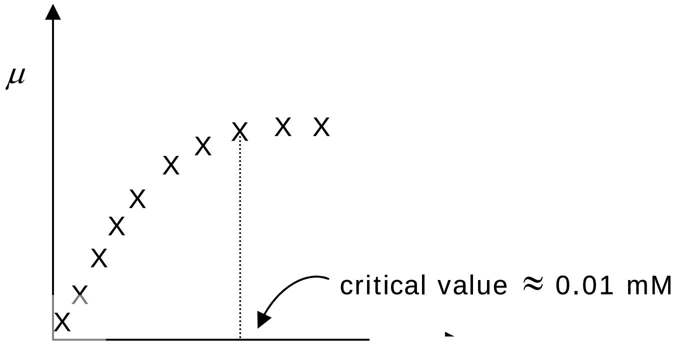

Figure 1. μ vs dissolved oxygen.

D.O. = dissolved oxygen

Equilibrium solubility of O_2 ≈ 1 mM

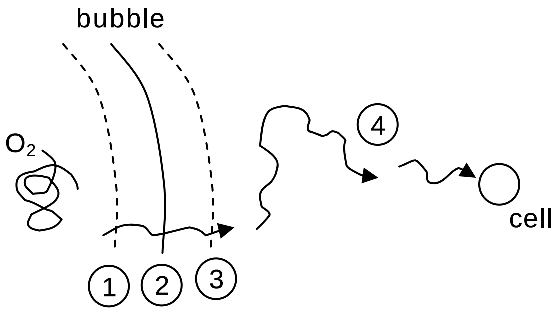

Figure 2. Oxygen pathway.

1) Diffusion across stagnate gas film
2) Absorption
3) Stagnate liquid layer (rate-limiting step)
4) Diffusion and convection

Cite as: K. Dane Wittrup, course materials for 10.37 Chemical and Biological Reaction Engineering, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].

at equilibrium

3) O_2 flux $= k_l(C^*_{O_2} - C_{O_2})$ [=] $\frac{\text{mol}}{\text{area time}}$

mass transfer coefficient

bulk liquid concentration

What is the value for the interfacial area?

Important system parameters:

- liquid physical properties (surface tension, viscosity)
- power input/volume (stirring, propeller size)
- superficial gas velocity

empirical correlations (TIB 1:113 '83)

$$k_l a = constant\, U_s^{\alpha}\left(\frac{P}{V}\right)^{\beta}$$ where U_s is the superficial gas velocity

$$k_l a [=] \left(\frac{\text{length}}{\text{time}}\right)\left(\frac{\text{area}}{\text{volume}}\right) = \text{time}^{-1} \qquad (\text{s}^{-1})$$

$$U_S [=] \frac{\text{length}}{\text{time}} \qquad (\text{m/s})$$

$$\frac{P}{V} = \frac{\text{power}}{\text{volume}} \qquad (\text{W/m}^3)$$

const. = 0.002

α = 0.2

β = 0.7

@ SS, O_2 transport = O_2 uptake by biomass

biomass growth rate or $\frac{dX}{dt}$

$$k_l a(C^*_{O_2} - C_{O_2}) = \frac{\boxed{\mu X}}{Y_{X/O_2}}$$

yield coefficient ≈ .4-.9 $\frac{\text{g cell dry wt.}}{\text{g } O_2}$

Crude limit: $\frac{dX}{dt} < k_l a C^*_{O_2} Y_{X/O_2}$

O_2 transport in tissues

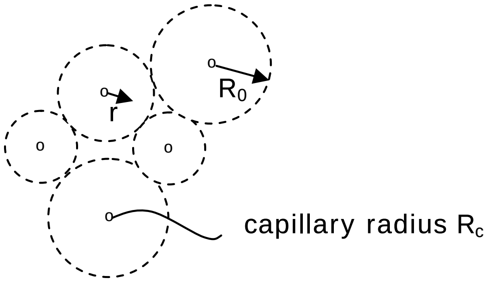

Figure 3. Krogh cylinder model.

One-dimensional steady-state diffusion:

$$\underbrace{\frac{D_{O_2}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_{O_2}}{\partial r}\right)}_{\text{Fick's Law}} = V_{O_2} \longleftarrow \text{metabolic consumption rate of oxygen, zero-order}$$

(cylindrical coordinates)

Boundary conditions:

symmetry
no-flux

flux=0 @ r=R_0

$$D_{O_2}\frac{\partial C_{O_2}}{\partial r} = 0 \quad @\ r=R_0$$

$$C_{O_2} = C_{O_2,\,plasma} \quad @\ r=R_c$$

Integrate twice:

$$\frac{C_{O_2}}{C_{O_2,\,plasma}} = 1 + \Phi\left(r^{*2} - R^{*2} - 2\ln\frac{r^*}{R^*}\right)$$

where $r^* = r/R_0$, $R^* = R_c/R_0$, $\Phi = \frac{1}{4}\frac{V_{O_2}}{C_{O_2,\,plasma}}\frac{R^2}{D_{O_2}} = \frac{\text{char. rxn rate}}{\text{char. transport rate}}$

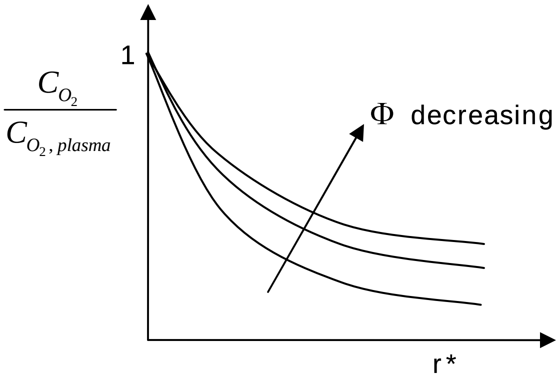

Figure 4. Dissolved oxygen vs. radius for various values of Φ.

O_2 diffuses further before consumption as Φ decreases.

When $R^* \approx 0.05$, $C_{O_2} = 0$ @ $r^* = 1$ when $\Phi \geq 0.2$

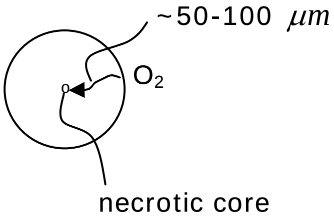

Figure 5. Tumor micrometastases.

Cite as: K. Dane Wittrup, course materials for 10.37 Chemical and Biological Reaction Engineering, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].