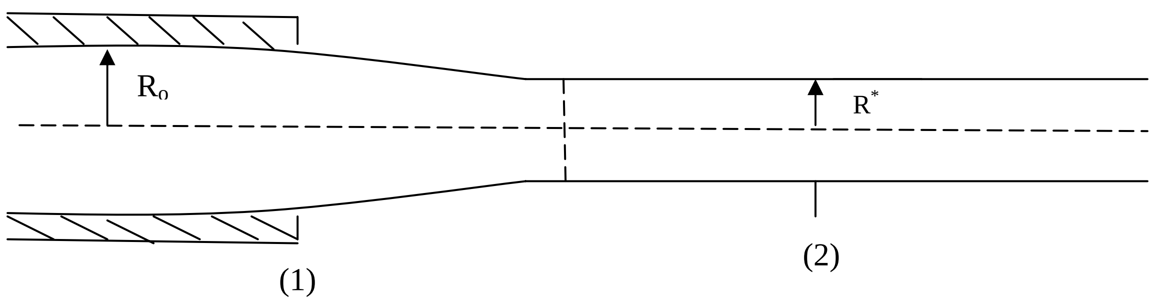
10.52
Mechanics of Fluids
Spring 2006

Problem Set 3

Problem 1

Mass transfer studies involving the transport of a solute from a gas to a liquid often involve the use of a laminar jet of liquid. The situation is a sketched below:



At section 1, the flow is laminar and the velocity profile is Poiseuillean, i.e.

$$u/u_{CL} = 1 - (r/R_o)^2$$

where u_{CL} is the center-line velocity. Upon leaving the pipe, the jet contracts and the velocity profile flattens so that, at section 2, the velocity profile is perfectly flat and no further contraction occurs, i.e.,

$$u = u^* \text{ for } r \leq R^*$$

In order to interpret the mass transfer results, a precise knowledge of R^* is essential. For this reason, you are asked to determine the <u>numerical</u> value of R^*/R_0, subject to the following assumptions:

1. Gravity may be neglected.

2. Surface tension may be neglected.

3. The pressure in the gas and in the liquid is everywhere the same.

4. The gas is of such low viscosity that any shear stresses on the surface of the liquid jet may be neglected.

<u>Problem 2</u> (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)

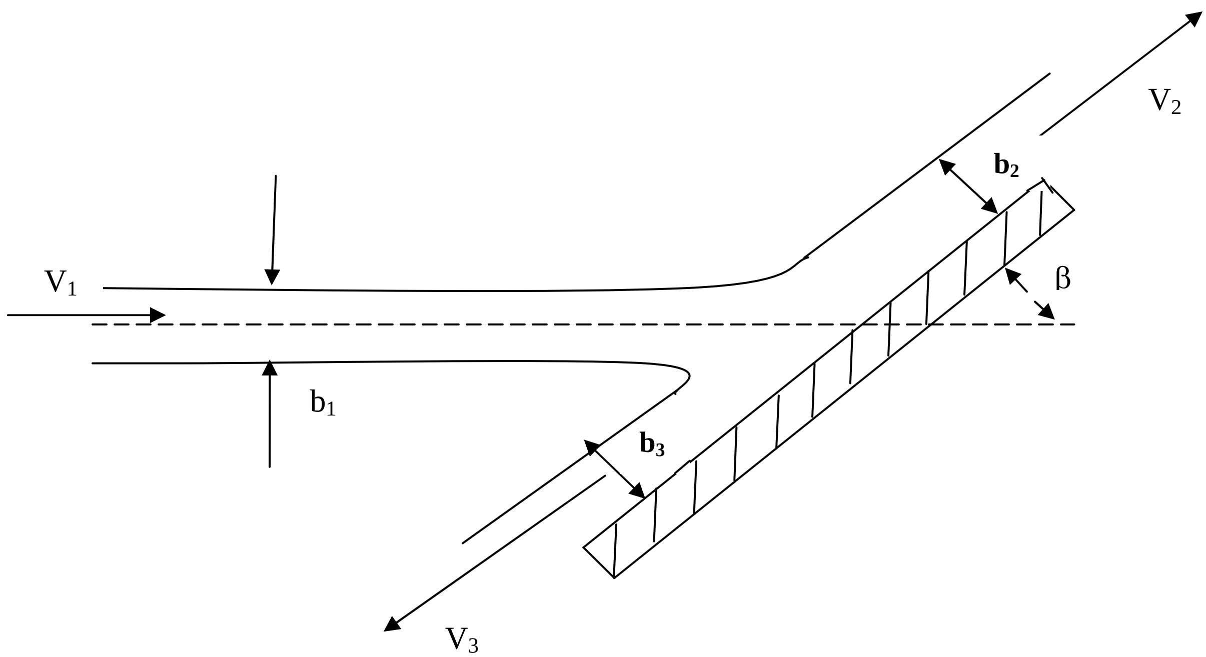
A two-dimensional jet of incompressible fluid strikes a plane surface. Find V_2, V_3, b_2, and b_3 in terms of V_1, b_1, β, and ρ.

Assuming the plate to be of unit length in the direction normal to the paper, find the force needed to hold the plate stationary. Base your calculations on the following assumptions:

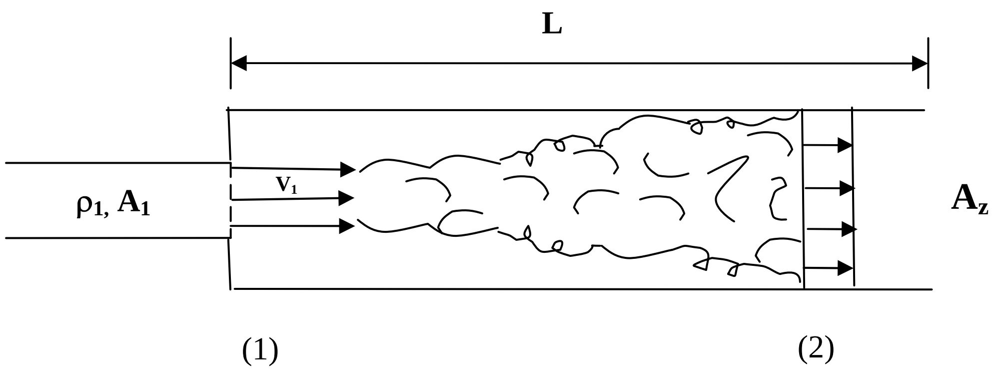
a) Gravity is not a factor.

b) The fluid is perfect.

c) The velocities V_1, V_2, V_3 are uniform at their respective cross sections.



Problem 3 (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)



An incompressible fluid passes through a sudden expansion in a pipe, from area A_1 to A_2, > A_1. The flow just downstream of the expansion looks like a jet of area A_1, which emerges into an almost stagnant, recirculating zone which surrounds it. Further downstream, the jet mixes in a turbulent, viscous manner with the fluid surrounding it, and eventually, at some station (2), the velocity is approximately uniform over the larger cross-section.

Assuming that the velocity is uniform just upstream of the expansion as well as at station (2), and that, although viscous forces acting within the flow are important and in fact cause the velocity at (2) to be uniform, the net shear force exerted by the fluid on the wall between (1) and (2) is negligible,

(a) find the rise in static pressure between (1) and (2),

(b) find the loss in stagnation pressure, or total head, between (1) and (2). Check your results by showing that in the limit $A_1/A_2 \Rightarrow 0$, they reduce to the forms which apply to a jet issuing into an infinite reservoir.

(c) Give a more concrete criterion for the shear stress to be "negligible", as assumed in the problem introduction. You may express the criterion as an upper limit on the shear stress in terms of the given quantities and the distance L from (10 to (2).

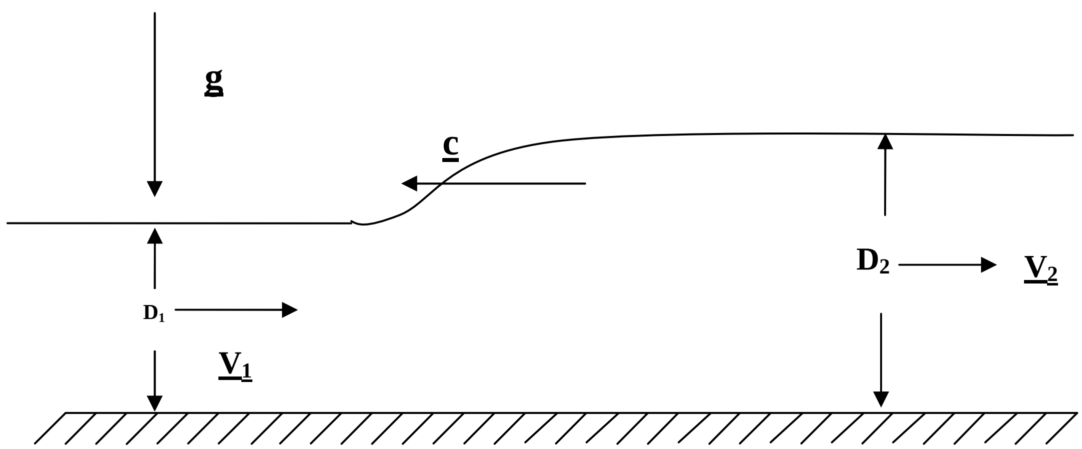
The given quantities are the upstream flow speed v_1, ρ, A_1 and A_2. Neglect gravitational effects.

Problem 4 (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)

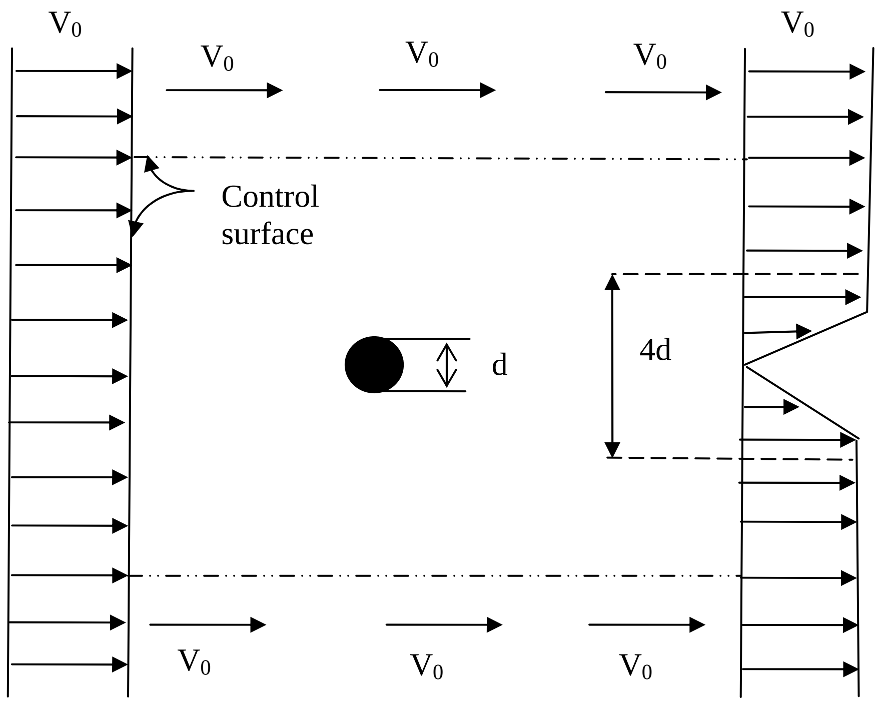
The following is a description of a process which actually occurs in practice. Consider a horizontal rectangular channel, open to the atmosphere, in which water is flowing at a (substantially) uniform velocity V_1 and depth D_1. Now a restriction is placed in the channel (perhaps by partially closing a gate downstream). As a result of the restriction, it is observed that a "surge wave" moves upstream. The front of the surge wave moves with the constant velocity "c" relative to the stationary co-ordinate axes. A short distance downstream of the wave front, the flow is again essentially uniform with velocity V_2 and depth D_2. The shear stresses applied by the channel walls are small and hence may be neglected.

a) Using a control volume which moves with the wave, derive the relation which gives the wave speed in terms of V_1, D_1, and D_2. (V_2 must not appear).

b) Why is a moving control volume more convenient than a stationary one?



Problem 5 (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)



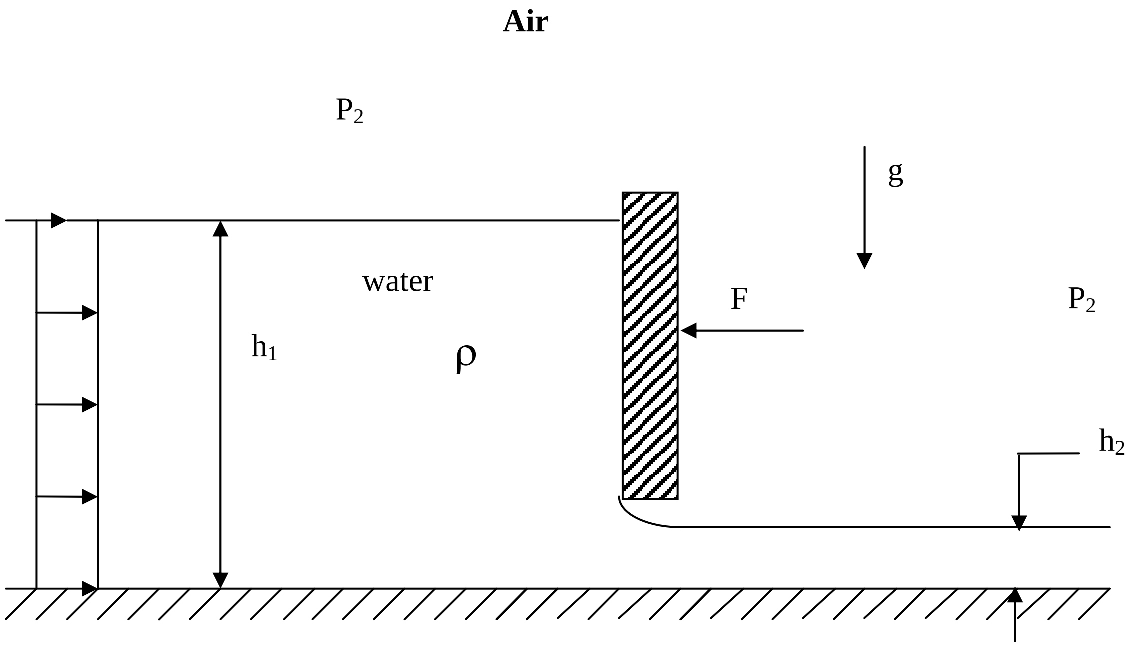
In an experiment to determine drag, a circular cylinder of diameter d was immersed in a steady, two-dimensional incompressible flow. Measurements of velocity and pressure were made at the boundaries of the control surface shown. The pressure was found to be uniform over the entire control surface. The x-component of velocity at the control surface boundary was approximately as indicated by the sketch.

From the measured data, calculate the drag coefficient of the cylinder, based on the projected area and on the free stream dynamic head.

$$C_D \equiv \frac{\text{Drag force per unit length}}{(1/2)\,\rho V_0^2 d}$$

answer: $C_D = 4/3$

Problem 6 (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)



A sluice gate is installed in a steady water stream of depth h_1 and speed V_1 (as measured far upstream of the gate). Downstream of the gate the stream has a depth h_2 which is less than h_1.

The flow is incompressible and inviscid.

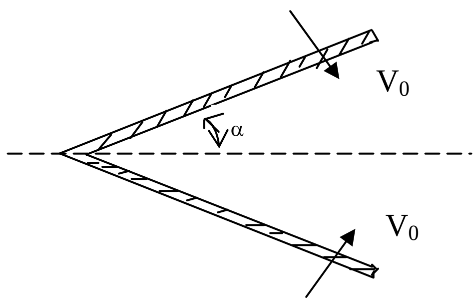
(a) Assuming uniform velocities at (1) and (2), derive an expression for the horizontal force F, per unit width, required to hold the gate in place given ρ, V_1, h_1 and h_2. Check your result by showing that it is zero when $h_2 = h_1$ and equal to the hydrostatic result when $h_2 = 0$.

(b) Also obtain an expression for V_2. Show that as h_2 approaches zero, V_2 approaches $\sqrt{2gh_1}$ and F approaches $\rho g h_1^2 / 2$. Explain.

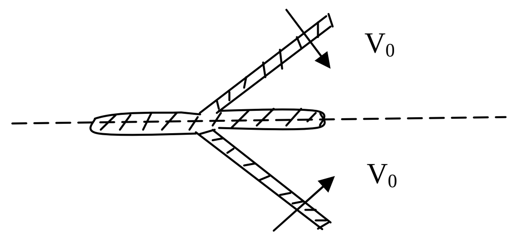
Problem 7

Consider two thin planar sheets (of an ideal, incompressible liquid) which are propelled perpendicular to their respective surfaces with a constant velocity V_0 as shown in Figure A. As the two sheets proceed to interact with each other, jets will be produced along the axis of symmetry, as shown in Figure B. What is the velocity to be expected in the two jets?

Hint: It is very helpful to use a reference frame which moves with the point of intersection. Why is this so?



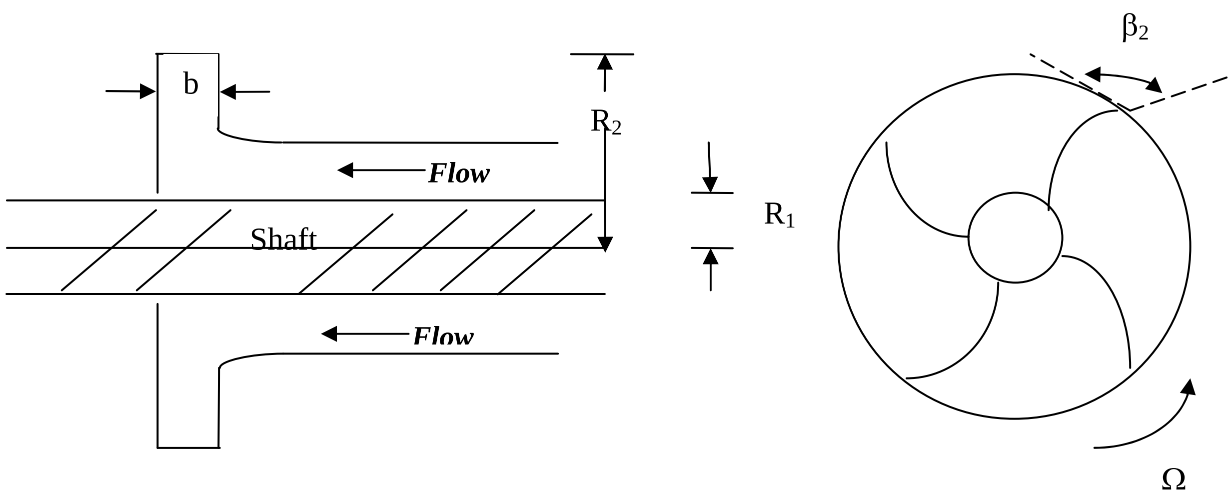
(A)



(B)

<u>Problem 8</u> (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)

Water flows through the impeller of a centrifugal pump as shown. Find the torque exerted on the impeller and the power required to drive it. The absolute velocity of the water at the entrance has no tangential component. The total flow rate is 60 liters/sec.



Ω	=	100 radiams/sec
R_1	=	5 cm
R_2	=	15 cm
b	=	1 cm
β_2	=	120°

Problem 9

It has been proposed that a flow meter be built which utilizes a whirling arm as shown in Figure 1. It is the claim of the inventors that, due to the rotation, the fluid will exert a force on the arm and in the direction shown in Figure 2. It is further claimed that this force will be directly proportional to the mass flow rate. If true, such a device could be used as a mass flow rate meter.

Derive a relation for this force in terms of the mass flow rate, Ω, r_2, r_1, etc. Also, present a relation for the torque which this force exerts about the axis of rotation. Or, alternatively show that this force does not exist.

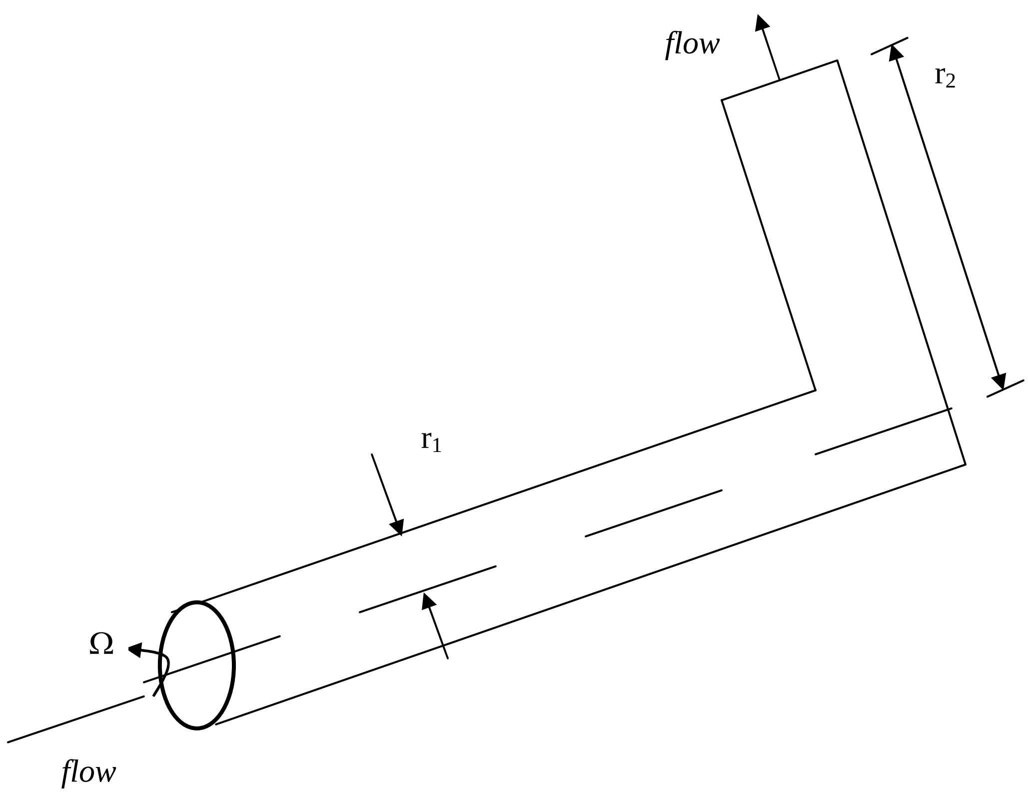


Figure 1

End View
(Along Axis of Rotation)

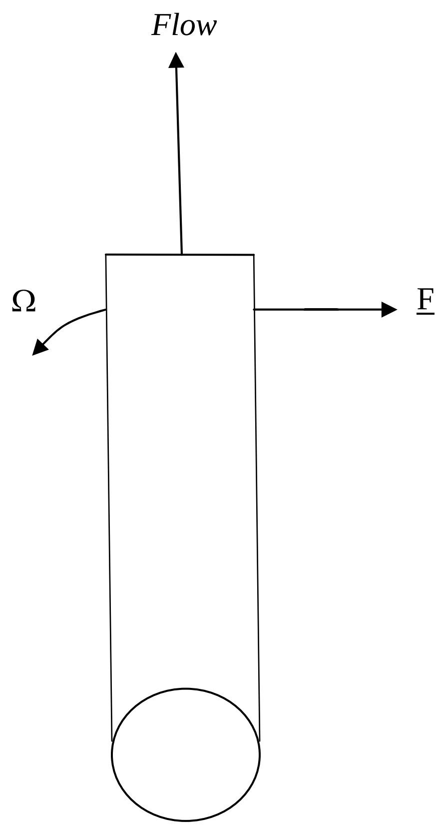
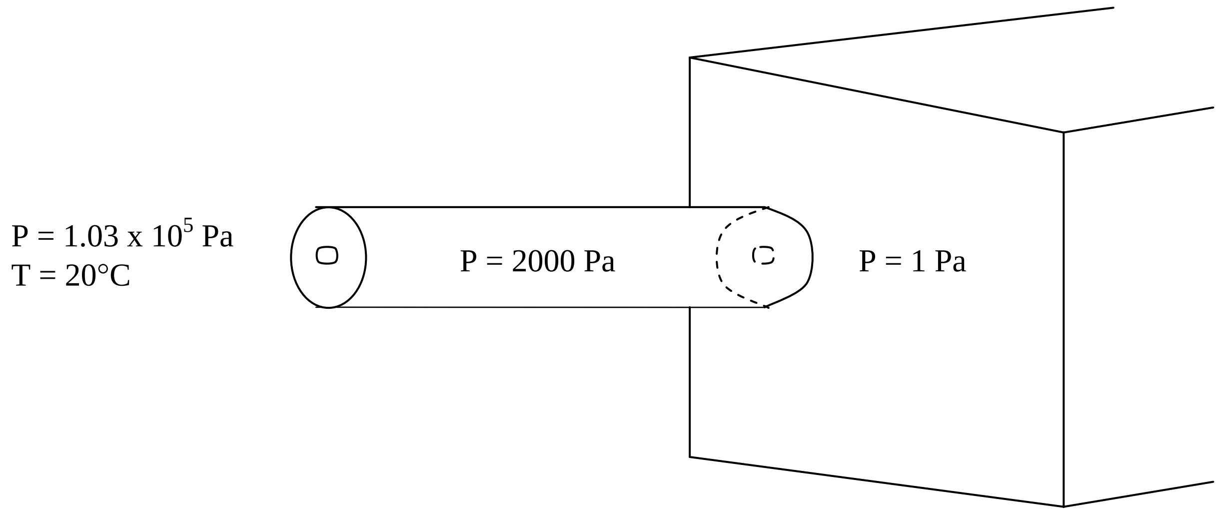


Figure 2

Problem 10

Ambient air is to be sampled into vacuum by a two step process. In particular, the plan is to use a cylindrical tube with "pin holes" at each end, as shown in the sketch:



The tube diameter is 2 cm and the desired flow rate is 2 std cm^3/sec.

What diameter would you recommend for each of the pin holes?