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### 5.111 Principles of Chemical Science

Fall 2008

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Write your name below. Do not open the exam until the start of the exam is announced. The exam is closed notes and closed book.

1. Read each part of each problem carefully and thoroughly.
2. Read all parts of each problem. MANY OF THE LATTER PARTS OF A PROBLEM CAN BE SOLVED WITHOUT HAVING SOLVED EARLIER PARTS. However, if you need a numerical result that you were not successful in obtaining for the computation of a latter part, make a physically reasonable approximation for that quantity (and indicate it as such) and use it to solve the latter parts.
3. A problem that requests you to "calculate" implies that several calculation steps may be necessary for the problem's solution. You must show these steps clearly and indicate all values, including physical constants used to obtain your quantitative result. Significant figures must be correct.
4. If you don't understand what the problem is requesting, raise your hand and a proctor will come to your desk.
5. Physical constants, formulas and a periodic table are given on the last page. You may detach this page once the exam has started.

Suggested time

1. 12 minutes (22 points)
2. 10 minutes (16 points) $\qquad$
3. 19 minutes (38 points) $\qquad$
4. 9 minutes
(24 points) $\qquad$

## Total (100 points)

Name $\qquad$

## 1. (22 points) The photoelectric effect

A beam of light with an intensity of 15 W is incident on a copper plate ( $\phi=7.43 \times 10^{-19} \mathrm{~J}$ ). Electrons with a minimum wavelength of $3.75 \times 10^{-10} \mathrm{~m}$ are ejected from the surface of the copper.
(a) (12 points) Calculate the frequency of the incident light.

K.E. of electron:
$\lambda=\frac{\mathrm{h}}{\mathrm{p}} \quad \mathrm{p}=\frac{\mathrm{h}}{\lambda}=\frac{6.626 \times 10^{-34} \mathrm{JS}}{3.75 \times 10^{-10} \mathrm{~m}} \quad \begin{aligned} & \mathrm{kgm}^{2} \mathrm{~s}^{-2} \\ & =1.7 \underline{6} 7 \times 10^{-24} \mathrm{kgms}^{-1}\end{aligned}$
$\mathrm{E}=\mathrm{p}^{2}=\frac{\left(1.7 \underline{6} 7 \times 10^{-24} \mathrm{kgms}^{-1}\right)^{2}}{2\left(9.109 \times 10^{-31} \mathrm{~kg}\right)} \quad=1.7 \underline{1} 38 \times 10^{-18} \mathrm{~J}$
$\mathrm{E}_{\mathrm{i}}=\phi+\mathrm{KE}$
$=0.74 \underline{3} \times 10^{-18}+1.7 \underline{1} 4 \times 10^{-18} \mathrm{~J}=2.4 \underline{5} 7 \times 10^{-18} \mathrm{~J}$
$\begin{aligned} & \mathrm{E}=\mathrm{h} \nu \\ & v=\frac{\mathrm{E}}{\mathrm{h}}\end{aligned} \quad \nu=\frac{2.4 \underline{\underline{5} 7 \times 10^{-18} \mathrm{~J}}}{6.626 \times 10^{-34} \mathrm{~J}} \quad=3.7 \underline{0} 8 \times 10^{15} \mathrm{~s}^{-1}$
$v=3.71 \times 10^{15} \mathbf{s}^{-1}$ or $3.71 \times 10^{15} \mathrm{~Hz}$

A beam of light with an intensity of 15 W is incident on a copper plate ( $\phi=7.43 \times 10^{-19} \mathrm{~J}$ ). Electrons with a minimum wavelength of $3.75 \times 10^{-10} \mathrm{~m}$ are ejected from the surface of the copper.
(b) (6 points) Calculate the maximum number of electrons that can be ejected by a 3.0 -second pulse of the incident light.

$$
1 \mathrm{~W}=1 \mathrm{~J} / \mathrm{s}
$$

$3.0 \mathrm{~s} \times \frac{1 \underline{5} \mathrm{~J}}{\mathrm{~s}} \times \frac{\text { (photon) }}{2.4 \underline{5} 7 \times 10^{-18} \mathrm{~J}}=1 . \underline{8} 3 \times 10^{19}$
E per photon calc. in part (a)

## $1.8 \times 10^{19}$ electrons

(c) (4 points) If a new light source $\left(\mathrm{E}_{\mathrm{i}}=7.19 \times 10^{-19} \mathrm{~J}\right)$ with an intensity of 35 W is incident on the copper surface, what is the maximum number of electrons that can be ejected from a 6.0 second pulse of light?

$$
\mathrm{E}_{\mathrm{i}}<\phi \text { for copper }
$$

## zero

## 2. (16 points) One-electron atoms:

Consider a $\mathrm{Ca}^{19+}$ ion with its electron in the $5^{\text {th }}$ excited state. $\leftarrow \mathrm{n}=6$ state
(a) (12 points) Calculate the longest wavelength of light that could be emitted when the $\mathrm{Ca}^{19+}$ electron transitions to a lower energy state. Report your answer with three significant figures.

$$
\begin{gathered}
\text { longest } \lambda=\text { smallest } \mathrm{E} \\
\mathrm{n}_{\mathrm{i}}=6\left(5^{\text {th }} \text { excited state }\right) \\
\mathrm{n}_{\mathrm{f}}=5 \\
\nu=\mathrm{Z}^{2} \mathfrak{R} \quad\left[\frac{1}{\mathrm{n}_{\mathrm{f}}^{2}}-\frac{1}{\mathrm{n}_{\mathrm{i}}^{2}}\right]=(20)^{2} 3.2898 \times 10^{15} \mathrm{~s}^{-\mathrm{f}}\left[\frac{1}{25}-\frac{1}{36}\right] \\
v=1.608 \times 10^{16} \mathrm{~s}^{-1} \\
\mathrm{c}=\lambda v \quad \lambda=\frac{\mathrm{c}}{v}=\frac{2.9979 \times 10^{8} \mathrm{~m} / \mathrm{s}}{1.608 \times 10^{16} \mathrm{~s}^{-1}} \\
\lambda=1.864 \times 10^{-8} \mathrm{~m} \\
\lambda=\mathbf{1 . 8 6 \times 1 \mathbf { 1 0 } ^ { - 8 } \mathrm { m } \quad \text { or } 1 8 . 6 \mathrm { nm }}
\end{gathered}
$$

(b) (4 points) Suppose the same transition as in part (a) took place in a hydrogen atom. Would the wavelength of emission be longer than, shorter than, or the same as your answer to part (a). Very briefly explain why. (Note: This question does NOT require a calculation. Also, you do not need to use the answer to part (a) to answer this question.)
$\mathrm{n}=6$ to $\mathrm{n}=5$ in H atom
For H atom, $\mathrm{Z}=1$, so the $v$ of the emitted light would be lower. Since $v=c / \lambda$, the wavelength of the emitted light would be longer than the answer to part (a).

## 3. (32 points) Multi-electron atoms

(a) (16 points) An x-ray photoelectron spectroscopy experiment with an unidentified element, $\mathbf{X}$, displays an emission spectrum with four distinct kinetic energies: $5.9 \times 10^{-17} \mathrm{~J}, 2.53 \times 10^{-18} \mathrm{~J}, 2.59 \times 10^{-}$ ${ }^{20} \mathrm{~J}$, and $2.67 \times 10^{-20} \mathrm{~J}$. (Assume the incident light has sufficient energy to eject any electron in the atom.)
(i) (4 points) Name all of the possible ground state atoms that could yield this spectrum. 4 orbitals: 1s, 2s, 2p, 3s
$\mathbf{N a}$ or $\mathbf{M g}$
(ii) (8 points) Calculate the binding energy of an electron in the 2 p orbital of element $\mathbf{X}$ if the x -rays used for the spectroscopy experiment had an energy of $2.68 \times 10^{-16} \mathrm{~J}$.

2p orbital: $2^{\text {nd }}$ lowest IE, so $2^{\text {nd }}$ highest KE.

$$
\mathrm{KE}=2.53 \times 10^{-18} \mathrm{~J}
$$

$$
\left.\begin{array}{ll}
\mathrm{E}_{\mathrm{i}}=\mathrm{IE}+\mathrm{KE} \quad & \mathrm{IE}=\mathrm{E}_{\mathrm{i}}-\mathrm{KE} \\
\mathrm{IE}=2.6 \underline{8} \times 10^{-16} \mathrm{~J}-0.0253 \times 10^{-16} \mathrm{~J} \\
\mathrm{IE}=2.6 \underline{5} 47 \times 10^{-16} \mathrm{~J}
\end{array}\right] \quad \begin{aligned}
& \text { BE }=-\mathrm{IE}=-\mathbf{2 . 6 5 \times 1 0 ^ { - 1 6 }} \mathbf{J} \text { (also accept }-2.66 \times 10^{-16} \mathrm{~J} \text { ) }
\end{aligned}
$$

(iii) (4 points) Consider both the filled and unfilled orbitals of element $\mathbf{X}$. Determine the number of: total nodes in a 4d orbital:
$\mathrm{n}-1 \quad 4-1=3$ total nodes
angular nodes in the $2 \mathrm{p}_{y}$ orbital:
$\ell$ p orbitals have $\ell=1 \quad \mathbf{1}$ angular node
degenerate 5 p orbitals:
same energy $\left.\begin{array}{l}5 p_{x} \\ 5 p_{y} \\ 5 p_{z}\end{array}\right\} \quad 3$ orbitals
(b) (22 points) The first, second, and third ionization energies of phosphorus are $1011 \mathrm{~kJ} / \mathrm{mol}, 1903$ $\mathrm{kJ} / \mathrm{mol}$, and $2912 \mathrm{~kJ} / \mathrm{mol}$ respectively.
(i) (8 points) Calculate the effective nuclear charge ( $\mathrm{Z}_{\text {eff }}$ ) experienced by a 3 Ne electron in phosphorus.

$$
\begin{aligned}
& I E=-E_{n, l}=\frac{Z_{\text {eff }}^{2} R_{H}}{n^{2}} \quad Z_{\text {eff }}=\left[\frac{\mathrm{n}^{2} \mathrm{IE}}{\mathrm{R}_{\mathrm{H}}}\right]^{1 / 2} \\
& I E=\frac{1011 \mathrm{k} \mathrm{~J}}{\mathrm{~mol}} \frac{\mathrm{x} \frac{1000 \mathrm{~J}}{\mathrm{~kJ}} \frac{\mathrm{x} \frac{\mathrm{~mol}}{6.022 \times 10^{23}}}{}=1.67 \underline{8} 8 \times 10^{-18} \mathrm{~J}}{\mathrm{Z}_{\text {eff }}=\left[\frac{(3)^{2}\left(1.6788 \times 10^{-18} \mathrm{~J}\right)}{2.1799 \times 10^{-18} \mathrm{~J}}\right]} \quad 1 / 2=(6.9311)^{1 / 2} \\
& \mathbf{Z}_{\text {eff }}=2.633
\end{aligned}
$$

(ii) (4 points) Would it be expected that the minimum energy necessary to eject a 3s electron from phosphorus in a photoelectron spectroscopy experiment be larger, smaller, or the same as the $4^{\text {th }}$ ionization energy ( $\mathrm{IE}_{4}$ ) of phosphorus? Briefly explain your answer.
$[\mathrm{Ne}] 3 \mathrm{~s}^{2} 3 \mathrm{p}^{3} \longrightarrow \quad[\mathrm{Ne}] 3 \mathrm{~s}^{1} 3 \mathrm{p}^{3}+\mathrm{e}^{-}$
The E to eject a 3s electron from P would be smaller than the $4^{\text {th }}$ IE because there are more electrons in P , meaning more shielding and a lower $\mathrm{Z}_{\text {eff }}$.
(iii) (4 points) Which experience less shielding, 3s-electrons or 3p-electrons in phosphorus? Very briefly explain why.

3s 3 s e-s penetrate closer to the nucleus, so $Z_{\text {eff }}$ avg for $3 s>Z_{\text {eff }}$ avg for $3 p$.
(iv) (4 points) On the plot below, graph the radial probability distribution for a phosphorus 3p orbital with a solid line. Label the $\mathrm{r}_{\mathrm{mp}}$, and point to each node with an arrow. Label the axes, but do not include numbers or units.

(v) (2 points) Is the $r_{m p}$ for a hydrogen 3p orbital longer or shorter than the $r_{m p}$ for a 3p phosphorus orbital? Very briefly explain why.
$\mathrm{r}_{\mathrm{mp}}$ for a H 3 p orbital is longer because $\mathrm{Z}_{\text {eff }}$ is smaller for the H atom (because Z is less).
4. (24 points) Periodic trends and miscellaneous short answer
(a) (5 points) Consider the second ionization energies $\left(\mathrm{IE}_{2}\right)$ for the following $3^{\text {rd }}$ row elements: $\mathrm{Si}, \mathrm{S}$, $\mathrm{Mg}, \mathrm{Al}$.
(i) Which has the highest $I E_{2}$ ?

## S

(ii) Which has the third highest $\mathrm{IE}_{2}$ ?

## Si

(b) (5 points) Order the following atoms and ions in order of increasing atomic radius: $\mathrm{Cl}, \mathrm{Te}, \mathrm{Te}^{2-}, \mathrm{S}$. Note: use the < symbol for clarity.
$\mathrm{Cl}<\mathrm{S}<\mathrm{Te}<\mathrm{Te}^{2-}$
(c) (6 points) Give the electron configuration expected for the following atoms or ions. (You may use the noble gas configuration as a means to abbreviate the full configuration.)
(i) $\mathrm{Pb}(\mathrm{Z}=82)$

$$
[\mathrm{Xe}] 6 \mathrm{~s}^{2} 4 \mathrm{f}^{14} 5 \mathrm{~d}^{10} 6 \mathrm{p}^{2}
$$

(ii) $\mathrm{Mo}(\mathrm{Z}=42)$

$$
[\mathrm{Kr}] 5 s^{1} 4 \mathrm{~d}^{5} \quad \text { or } \quad[\mathrm{Kr}] 4 \mathbf{d}^{5} 5 s^{1}
$$

(iii) $\mathrm{Zr}^{+}(\mathrm{Z}=40)$
$\begin{array}{ll}\mathrm{Zr}:[\mathrm{Kr}] 5 \mathrm{~s}^{2} 4 \mathrm{~d}^{2} & {[\mathrm{Kr}] 4 \mathrm{~d}^{2} 5 \mathrm{~s}^{2}} \\ \mathrm{Zr}^{+}:[\mathrm{Kr}] 4 \mathrm{~d}^{2} 5 \mathbf{s}^{1} & \end{array}$
$\mathrm{Zr}^{+}:[\mathrm{Kr}] 4 \mathrm{~d}^{2} 5 \mathbf{s}^{1}$
(d) (4 points) In one sentence (or less!), briefly explain the physical interpretation of $\Psi^{2}$ for a hydrogen atom.
probability density of finding the electron
(e) (4 points) How many electrons in a single atom can have the following two quantum numbers: $\mathrm{n}=$ $7, \mathrm{~m}_{l}=-3$ ?

$$
\left.\begin{array}{rr}
\ell=7 \\
\mathrm{n}=7 \\
5 \\
4 & \mathrm{~m}_{\mathrm{e}}=-3 \\
3
\end{array} \quad-3 \begin{array}{l}
-3 \\
-3
\end{array}\right\} 4 \text { orbitals } \rightarrow \mathbf{8} \text { electrons }
$$

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$\begin{aligned} \mathrm{c} & =2.9979 \times 10^{\mathrm{b}} \mathrm{m} / \mathrm{s} \\ \mathrm{h} & =6.6261 \times 10^{-34} \mathrm{~J} \mathrm{~s}\end{aligned}$

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$\mathrm{N}_{\mathrm{A}}=6.022 \times 10^{23} \mathrm{~mol}^{-1}$
$m_{e}=9.1094 \times 10^{-31} \mathrm{~kg}$
$a_{0}=5.292 \times 10^{-11} \mathrm{~m}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}$
$\lambda=\frac{\mathrm{h}}{\mathrm{p}}$
$\mathrm{R}_{\mathrm{H}}=2.1799 \times 10-18 \mathrm{~J}$
$\mathfrak{R}=\mathrm{R}_{\mathrm{H}} / \mathrm{h}=3.2898 \times 10^{15} \mathrm{~Hz}$
$E=\frac{p^{2}}{2 m}$
$E_{n}=-\frac{Z^{2} R_{H}}{n^{2}}$
$\mathrm{E}_{\mathrm{nl}}=-\frac{\mathrm{Z}_{\text {eff }}{ }^{2} \mathrm{R}_{\mathrm{H}}}{\mathrm{n}^{2}}$
$1 \mathrm{~W}=1 \mathrm{~J} \mathrm{~s}^{-1}$
$1 \mathrm{~J}=1 \mathrm{kgm}^{2} \mathrm{~s}^{-2}$
$1 \mathrm{eV}=1.6022 \times 10^{-19} \mathrm{~J}$
for s wavefunction：
$R P D=4 \pi r^{2} \Psi^{2} d r$
for $n_{f}<n_{i} \ldots \ldots$
$v=\frac{\mathrm{Z}^{2} \mathrm{R}_{\mathrm{H}}}{\mathrm{h}}\left(\frac{1}{\mathrm{n}_{\mathrm{f}}^{2}}-\frac{1}{\mathrm{n}_{\mathrm{i}}^{2}}\right)$
for $n_{f}>n_{i} \ldots \ldots$
$v=\frac{\mathrm{Z}^{2} \mathrm{R}_{\mathrm{H}}}{\mathrm{h}}\left(\frac{1}{\mathrm{n}_{\mathrm{i}}^{2}}-\frac{1}{\mathrm{n}_{\mathrm{f}}^{2}}\right)$

