1.010 Uncertainty in Engineering
Fall 2008

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Homework Set #3
Due October 2, 2008 (in class)

1. The service stations along a highway are located according to a Poisson process with an average of 1 service station in 10 miles. Because of a gas shortage, there is a probability of 0.3 that a service station would have no gasoline available. Assume that the availabilities of gasoline at different service stations are statistically independent.

(a) What is the probability that there is *at most* 1 service station in the next 15 miles of highway?

(b) What is the probability that none of the next 3 stations has gasoline for sale?

(c) A driver on this highway notices that the fuel gauge in his car reads empty; from experience he knows that he can go another 15 miles. What is the probability that he will be stranded on the highway without gasoline?[1]

2. A random variable X has probability density function:

$$f_X(x) = \frac{a}{x}, \text{ for } 1 \le x \le 2$$
$$= 0, \text{ otherwise}$$

where α is a parameter.

a) Calculate α and plot $f_X(x)$

b) Find and plot the CDF of X.

[1] An important result for Poisson processes is that, if a Poisson process with rate λ is "thinned" randomly with probability p (meaning that each point of the process is eliminated with probability p independently of the other points), then the remaining points still form a Poisson process with a reduced mean rate of $(1-p)\lambda$. Apply this result to answer question 1(c).

3. A set of earthquake occurrence times (in years since the beginning of recording t =0) is given below. The mean recurrence rate λ may be estimated as the total number of events divided by the observation period, in this case λ=50/101.74≈0.5 events/yr.

Earthquake occurences									
no.	***t***	**no.**	***t***	**no.**	***t***	**no.**	***t***	**no.**	***t***
1	3.61	11	19.01	21	36.87	31	54.57	41	76.83
2	5.22	12	19.44	22	40.53	32	54.70	42	84.62
3	6.74	13	21.81	23	45.66	33	55.32	43	85.90
4	6.83	14	23.44	24	47.98	34	57.30	44	86.03
5	7.23	15	23.71	25	48.30	35	57.63	45	87.85
6	11.04	16	27.84	26	48.75	36	58.88	46	90.41
7	13.20	17	28.41	27	48.81	37	61.96	47	91.10
8	15.90	18	31.01	28	49.22	38	67.86	48	91.34
9	16.14	19	32.23	29	49.27	39	72.35	49	95.66
10	17.21	20	33.30	30	50.28	40	74.17	50	101.74

a) Construct a histogram of the earthquake interarrival time and compare with the exponential PDF with parameter λ.

b) Find the empirical distribution of N= number of earthquakes in T=4 years and compare it with the Poisson distribution with λT=4λ=2

c) Would you reasonably conclude that the occurrence of earthquakes follows a Poisson point process?