PROBLEM SET 8 - SOLUTIONS

Comments on Problem Set 8

PROBLEM 1:

- Not many people got a good explanation for part (a). The assumption of straight surface is a good one because the difference between velocity heads over the crest and over the trough is small compared with the depths at these locations. For this reason, the influence of the change in velocity from crest to trough on the surface elevation is small. Note that the velocity heads being small compared to the depth is equivalent to the Froude number being much smaller than 1.

- In part (b), since the surface is straight, you can regard it as a "rigid lid" and apply the expression for expansion headloss we derived for pipes, i.e.,

$$\Delta H = (V_1 - V_2)^2/2g = 2.06E\text{-}3 \text{ m}. \quad (1)$$

Note that this expression is generally **not applicable** in open channel flow; this is an exception due to the validity of the rigid lid approximation.

- Some groups solved part (b) by saying that, given that the surface is straight and the horizontal difference from crest to trough is small, the elevation head difference between crest and trough is negligible. Thus, the expansion headloss becomes

$$\Delta H = H_1 - H_2 = (z_{01} + h_1 + V_1^2/(2g)) - (z_{02} + h_2 + V_2^2/(2g)) = (V_1^2 - V_2^2)/2g = 2.07E\text{-}2 \text{ m}. \quad (2)$$

This reasoning seems correct (and in fact it was given full grade). However, it yields a headloss one order of magnitude larger than (1). Therefore, I was puzzled. So I calculated the expansion using a more rigorous method. I applied conservation of linear momentum between the crest and the trough, similarly to how we derived the expression for the expansion headloss for pipes (Lecture 13). I have attached this more rigorous approach in an "addendum" at the end of the solutions. With this approach, I get ΔH =2.08E-3, which is similar to the result from (1).

What is the problem with (2)? As you can see in my detailed solution, the expansion needs some length to take place. Therefore, the elevation head difference between the sections before and after the expansion, while small, is not 0, as assumed in (2), but of comparable order to the velocity head difference.

- To obtain ε in part (e) you can do two things: Apply the relationship $n=0.038\ \varepsilon^{1/6}$ or go the long way and obtain ε using the Moody diagram (from knowledge of the average bottom shear stress). The first way is shorter, but trickier. Since ε = (n/0.038)^6, the value of ε is very sensitive to the value of n. Thus, if you use n=0.0242, as I did, you get ε = 6.7 cm. However, if you use n=0.02, you get ε = 2.1 cm !!

PROBLEM 2:

- Please take a careful look to the solution in part (c), since many groups had problems with this part. In the specific head vs. depth diagram, we have two different curves for sections 1-1 and 2-2, since their channel widths are different (remember that a curve in the specific head vs. depth diagram is valid for a fixed flowrate and for a fixed geometry of the channel). The curve for section 2-2 is located to the right of the curve for section 1-1 since, for a given h, the narrower section has a larger velocity and thus a larger specific head. Energy is conserved from 1 to 2 (short transition of converging flow), and thus the points 1 and 2 have the same energy (they are in a vertical line). Point 1 corresponds to subcritical flow (upper branch), while point 2 corresponds to critical flow. What happens if section 2-2 becomes narrower than the minimum value determined in part (b)? In that case, the curve corresponding to section 2-2 will be displaced even more to the right, and there would be no point 2 on the curve for section 2-2 with the same energy as point 1. Therefore, it would not be possible for the flow of Q=1 m^3/s to pass the narrowing under the current conditions. This is exactly what happens in Problem 5, part (d).

PROBLEM 3:

- Remember that we defined $S_0 = \sin\beta$ (and not $\tan\beta$). While $\sin\beta \approx \tan\beta$ for mild slopes, this is not the case in part (b) of this problem, where $S_0 = \sin\beta = 0.77 \neq 1.20 = \tan\beta$. Many people got confused with this.

- To solve part (c), you can apply conservation of momentum, which is the standard way of solving a hydraulic jump. Remember, however, that we derived a formula for calculating conjugated depths of an unassisted hydraulic jump in a rectangular channel (Lecture 28):

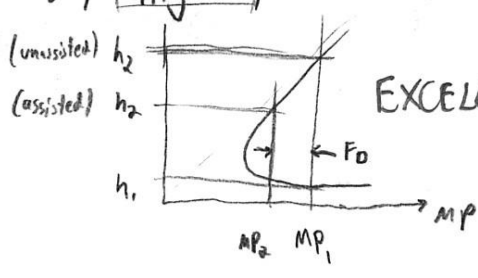
$h_2/h_1 = ½*(-1 + (1+8*Fr_1^2)^{1/2})$

This formula will be in the cheat sheet for test 3. If there is a hydraulic jump in the test (and if it is unassisted and happening in a rectangular channel), you will save time if you use this formula, rather than writing the momentum equation and solving it by iteration.

PROBLEM 4:

- Some of your answers for part (d) and (e) were unclear or wrong. Please read the solutions. Perhaps the clearest way to explain these parts is by using the MP-vs.-h and the E-vs.-h diagram, as done by Joseph, Yun, Piotr and Calvin. I have reproduced their answer here:

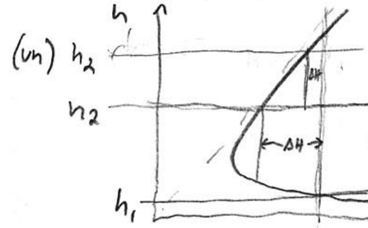
d) [Higher], we can see this by examining the MP vs. h diagram:



EXCELLENT! The MP difference in the assisted jump causes the height to be lower. ✓

2/2

e) Part (d) shows that h_2 is greater. Examining E vs. h:

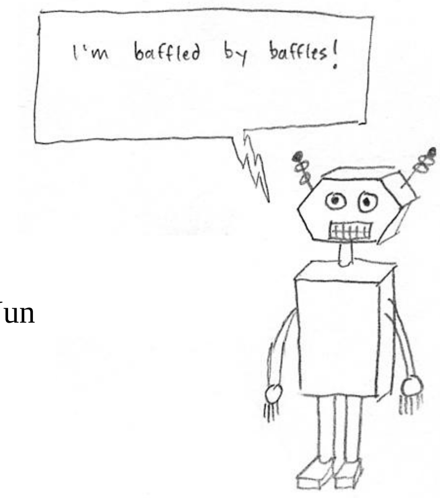


VERY GOOD! From this diagram, we can see that the headloss for the unassisted case will be [lower] than for the assisted case.

2/2

The unassisted jump has a larger MP_2, equal to MP_1, since there is no baffle to provide an extra force to assist MP_2 in balancing MP_1. Since section 2 has subcritical flow, a larger MP_2 implies a larger h_2, since the elevation term (P) is the largest component of MP in subcritical flow. This fact is clearly represented in the MP-vs.-h diagram.

Now, since the unassisted jump has a larger h_2, and since the energy in subcritical flow is mainly due to elevation, E_2 will be larger in the unassisted jump and consequently $\Delta H=E_1-E_2$ will be smaller. This is shown in the E-vs.-h diagram.



Reproduced from Joseph, Piotr, Yun and Calvin's solution

PROBLEM 5:

- In part (d), the flow is not able to pass the cofferdam. Many of you solved this part by repeating the procedure for part (c). Since now your equation for h_2 had no real positive solutions, you concluded that the flow was not able to pass. This procedure is right. However, I recommend you take a look at the solution for a more conceptual (and probably faster) way of solving this part.

- In part (d) the flow is not able to pass the cofferdam. So what happens? Only part of the flow will be able to pass; water will accumulate before the cofferdam, thus increasing the value of h_1, until the energy at 1 becomes as large as the critical energy at section 2, which is the minimum energy necessary to pass the cofferdam section.

PROBLEM 6:

- Note that, in the E-vs.-h diagram in part (1) (and in the MP-vs.-h diagram in part (2)), the trajectories from A to B (or from C to D) are unknown, and we don't actually expect them to be straight lines from the initial to the final point. We just know the position of the initial and the final points; the trajectories in between are unknown curves.

PROBLEM SET 8 – SOLUTIONS

– PROBLEM Nº 1

a) Over a crest: $h_c = 4'5\ m$

$$V_C = \frac{q}{h_C} = \frac{5}{4'5} = 1'11\ m/s$$

$$\frac{V_C^2}{2g} = 0'063\ m$$

Over a trough: $h_T = 5'5\ m$

$$V_T = \frac{q}{h_T} = \frac{5}{5'5} = 0'909\ m/s$$

$$\frac{V_T^2}{2g} = 0'042\ m$$

The maximum change in velocity scale is about $0'021$ m, negligible in comparison with the elevation term. Therefore, we don't expect significant changes in elevation due to changes in velocity and the assumption of straight surfaces is a fairly accurate one.

b) Since the surface is nearly straight, we can assume it to behave as a "rigid lid". This enables us to estimate the headloss in the sudden expansion using the expression derived for pipes:

$$\underline{\underline{\Delta H_{EXP}}} = \frac{(V_C - V_T)^2}{2g} = \frac{(1'11 - 0'909)^2}{2 \cdot 9'8} = \underline{\underline{2'06 \cdot 10^{-3}\ m}}$$

c)

$$\underline{\underline{S_f}} = -\frac{dH}{dx} = -\frac{\Delta H}{\Delta x} = \frac{\Delta H_{EXP}}{\lambda} = \frac{2'06 \cdot 10^{-3}}{30} = \underline{\underline{6'87 \cdot 10^{-5}}}$$

According to the assumption of straight surface, uniform steady flow implies: $\underline{S_f}$ = slope of the surface = $= \overline{S}_0$ = average slope of the bottom.

d)

Manning $\rightarrow$ $q = \bar{h}\,\frac{1}{n}\,\bar{Rh}^{2/3}\,\bar{S_0}^{1/2}$

$5 = 5\,\frac{1}{n}\,5^{2/3}\,(6'87\cdot 10^{-5})^{1/2} \Rightarrow \underline{\underline{n = 0'024}}$ (S.i.)

e) In R.T. flow

$n = 0'038\,\varepsilon^{1/6} \Rightarrow \underline{\underline{\varepsilon}} = \left(\frac{n}{0'038}\right)^6 = \underline{\underline{6'7\ \text{cm}}}$

(S.i.)

- PROBLEM N° 2:

a)

$Fr_{c,1} = \frac{Q/(h_{c,1}\,b_1)}{\sqrt{g\,h_{c,1}}} = 1 \Rightarrow h_{c,1} = \left(\frac{Q}{b_1\sqrt{g}}\right)^{2/3} = \left(\frac{1}{1'5\sqrt{9'8}}\right)^{2/3} = 0'356\ m$

$h_1 = 0'6\ m > h_{c,1} \Rightarrow$ $\underline{\underline{\text{SUBCRITICAL FLOW}}}$

b)

Limiting case: $h_2 = h_{c,2}$ (Critical flow downstream)

Continuity: $V_2 = Q/(b_2, h_2)$

Short transition of converging flow $\Rightarrow$ Conservation of energy between 1 and 2 $\Rightarrow$ $E_1 = E_2$

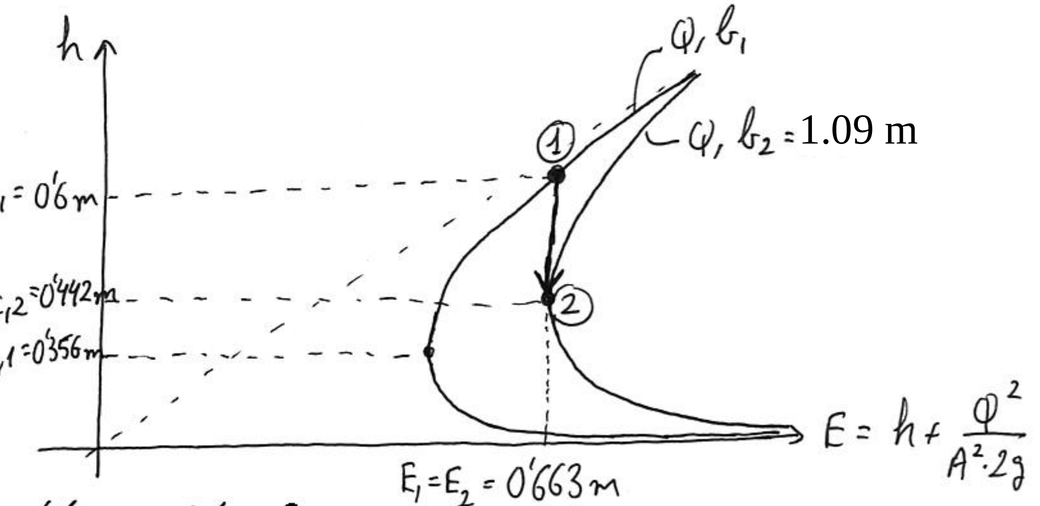
$E_1 = h_1 + \frac{V_1^2}{2g} = 0'6 + \frac{[1/(0'6\cdot 1'5)]^2}{2\cdot 9'8} = 0'663\ m$

$E_2 = h_{c,2} + \frac{(Q/h_{c,2}b_2)^2}{2g} = h_{c,2} + \frac{(Q/h_{c,2}\,b_2)^2}{g\,h_{c,2}}\,\frac{h_{c,2}}{2} =$

$= h_{c,2} + \underbrace{Fr_{c,2}^2}_{=1}\,\frac{h_{c,2}}{2} = \frac{3}{2}\,h_{c,2} = 0'663\,m \Rightarrow h_{c,2} = 0'442\,m$

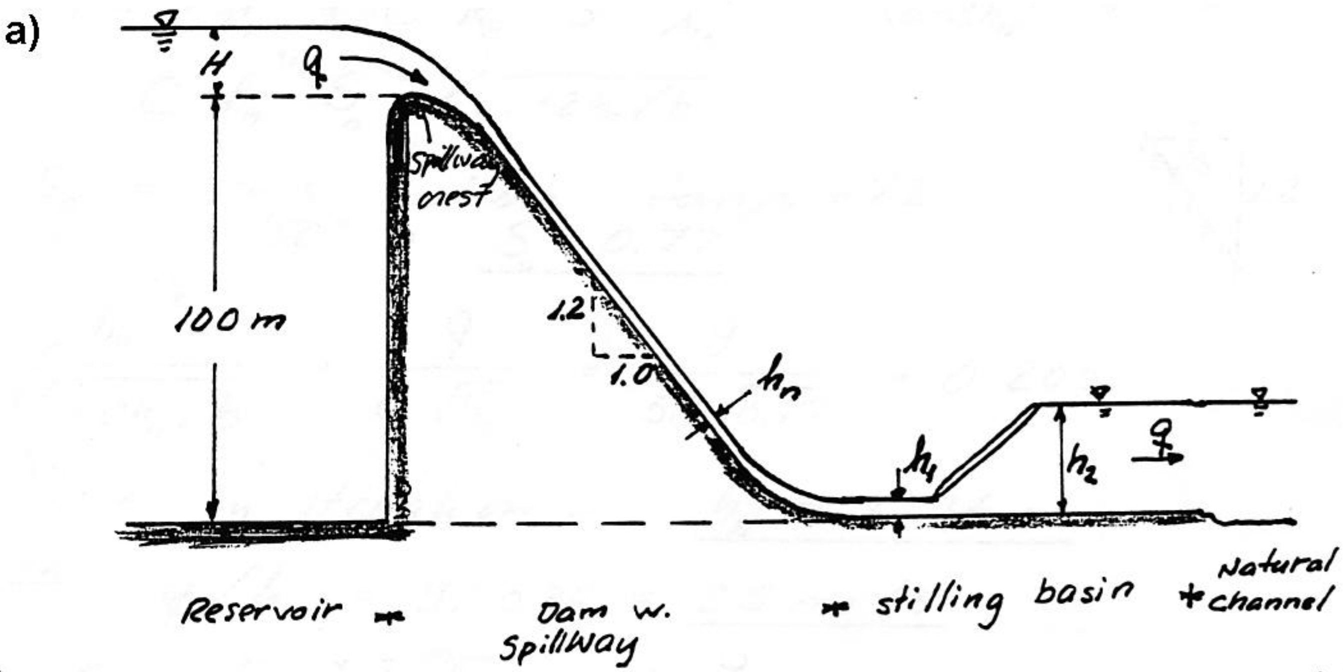
$h_{c,2} = \left(\frac{Q}{b_2\sqrt{g}}\right)^{2/3} = \left(\frac{1}{b_2\sqrt{9'8}}\right)^{2/3} = 0'442\,m \Rightarrow$ $\underline{\underline{b_{2,MIN} = 1.09\ m}}$

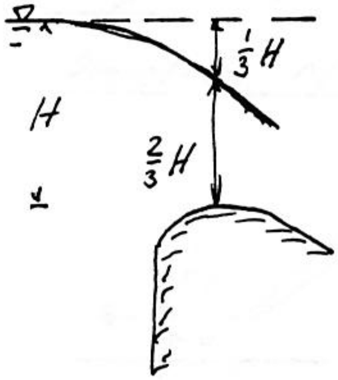
c)



Problem No: 3

a)





Rectangular channel: $Fr = \frac{V}{\sqrt{gh}}$

Critical Flow $\Rightarrow Fr = 1 = \frac{V_c}{\sqrt{gh_c}} \Rightarrow V_c = \sqrt{gh_c}$

Specific Energy:

$$E = \frac{V^2}{2g} + h = \frac{V_c^2}{2g} + h_c = \frac{h_c}{2} + h_c = \frac{3}{2} h_c$$

At crest of spillway (since flow from reservoir is a short transition with converging flow): $E = H = \frac{3}{2} h_c \Rightarrow h_c = \frac{2}{3} H$

$$q = V_c h_c = \sqrt{g h_c}\, h_c = \frac{2}{3}\sqrt{\frac{2}{3}}\,\sqrt{g}\, H^{3/2} = \underline{0.54\sqrt{g}\, H^{3/2}}$$

$H = 3.0\ m \Rightarrow \underline{q = 8.8\ m^3/s}$ per m $\Rightarrow \underline{Q} = q \cdot 10 = \underline{88\ m^3/s}$

b)

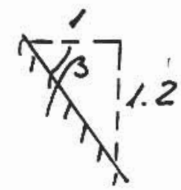
From Chezy's Equation we have

$$q = V h_n = C R_h^{1/2} S_o^{1/2} h_n = C\left(\frac{h_n b}{b+2h_n}\right)^{1/2} S_o^{1/2} h_n \Rightarrow$$

$$C h_n^{3/2} S_o^{1/2} \Big/ \sqrt{1+2h_n/b}$$

$S_o = \sin\beta$, where $\tan\beta = 1.2$

so $\beta = 50°$ ⇒ $\underline{S_o = 0.77}$



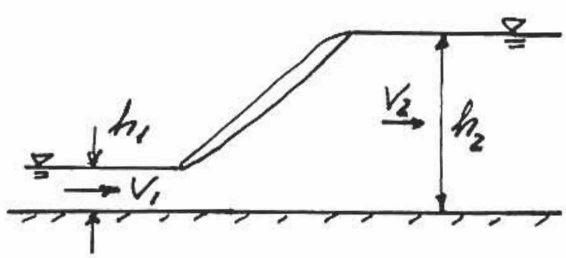
$$\frac{h_n^{3/2}}{\sqrt{1+2h_n/b}} = \frac{q}{C\sqrt{S_o}} = \frac{9}{50\sqrt{0.77}} = 0.205$$

Solve by iteration ⇒ $\underline{h_n = 0.36 \text{ m}}$

$\underline{V_n} = q/h_n = 9/0.36 = \underline{25 \text{ m/s}}$

$\underline{Fr_n} = V_n/\sqrt{g h_n} = \underline{13.3}$ -> **Supercritical flow.**

c)



Cons. of Volume:

$V_1 h_1 = V_2 h_2 = q$

$V_1 = q/h_1$, $V_2 = q/h_2$

Cons. of Momentum:

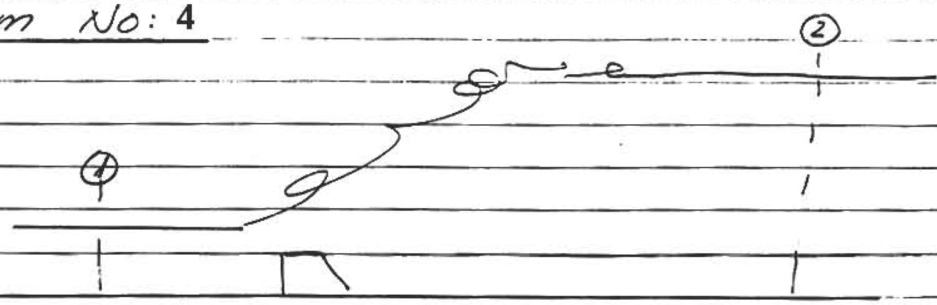
$$(MP)_1 = \rho V_1^2 h_1 + \frac{1}{2}\rho g h_1^2 = \rho \frac{q^2}{h_1} + \frac{1}{2}\rho g h_1^2 =$$

$$(MP)_2 = \rho V_2^2 h_2 + \frac{1}{2}\rho g h_2^2 = \rho \frac{q^2}{h_2} + \frac{1}{2}\rho g h_2^2$$

$$h_2^2 = \frac{2q^2}{g}\left(\frac{1}{h_1} - \frac{1}{h_2}\right) + h_1^2 = 16.5\left(2.5 - \frac{1}{h_2}\right) + 0.16$$

Solve by iteration: $\underline{h_2 = 6.2 \text{ m}}$, $\underline{V_2 = \frac{q}{h_2} = 1.45 \text{ m/s}}$

Problem No: 4



a)

$$\underline{F_D} = \tfrac{1}{2}\rho\, C_D\, A_L\, V_1^2 = \tfrac{1}{2}\, 1000 \cdot 0.6 \cdot (0.5 \cdot 1)\, 30^2 = \underline{135\ kN/m}$$

b)

F_D acts on the fluid in same direction as M_2, therefore using momentum:

$$M_1 = \tfrac{1}{2}\rho g h_1^2 + \rho V_1^2 h_1 = M_2 + F_D = \tfrac{1}{2}\rho g h_2^2 + \rho V_2^2 h_2 + F_D$$

$$V_1 h_1 = V_2 h_2 \quad \text{so} \quad V_2^2 = V_1^2 (h_1/h_2)^2$$

$$\tfrac{1}{2}\rho g\, h_2^2 + \rho V_1^2 h_1^2 \frac{1}{h_2} = \tfrac{1}{2}\rho g h_1^2 + \rho V_1^2 h_1 - F_D = 769{,}900\ \frac{N}{m}$$

$$4{,}900\, h_2^2 + 900{,}000 \frac{1}{h_2} = 769{,}900 \Rightarrow h_2^2 + 183 \frac{1}{h_2} = 157.1$$

$\underline{h_2 = 11.91\, m}$ ($h_2 = 1.17\, m$ must be disregarded since it is supercritical).

$\underline{V_2 = 2.52\ m}$

c)

$$H_1 = V_1^2/2g + h_1 = 46.9\, m; \quad H_2 = V_2^2/2g + h_2 = 12.2\, m$$

$$\underline{\Delta H} = H_1 - H_2 = \underline{34.7\, m}$$

Rate of energy dissipation =

$$\rho g Q H_1 - \rho g Q H_2 = \rho g Q \Delta H = \underline{10.2 \cdot 10^6\ \text{Watt/m} = 13.7 \cdot 10^3\ \frac{HP}{m}}$$

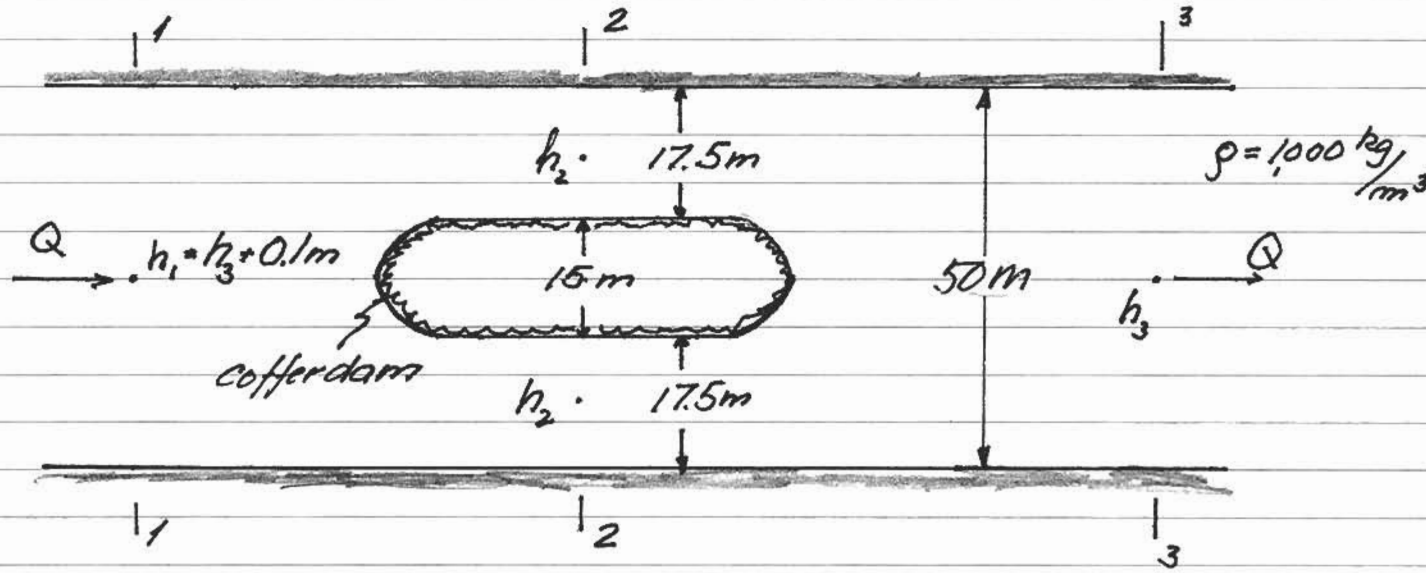
d)

Baffles help downstream M_2 to balance M_1. Less M_2 is needed and therefore a smaller h_2 when jump is baffle-assisted.

e)

We expect greater headloss with baffle present. The baffle results in skin friction and form drag which will increase the headloss.

Problem No: 5



a)

Using the momentum principle - assuming gravity force and frictional forces to balance each other - gives

$$\overrightarrow{MP_1} = \left(\tfrac{1}{2}\rho g h_1 + \rho\left(\frac{Q}{bh_1}\right)^2\right)bh_1 = \overrightarrow{MP_2} + \overleftarrow{F}_{D,fluid} =$$

$$\left(\tfrac{1}{2}\rho g h_3 + \rho\left(\frac{Q}{bh_3}\right)^2\right)bh_3 + \overleftarrow{F}_{D,on\ fluid}$$

With $b = 50\,m$, $Q = 1000\,m^3/s$, $h_3 = h_n = 6.56\,m$, and $h_1 = h_3 + 0.1 = 6.66\,m$ we get [SI units]

$$(32{,}634 + 9{,}018)\,50 \cdot 6.66 = (32{,}144 + 9{,}295)\,50 \cdot 6.56 + \overleftarrow{F}_{D,on\ fluid}$$

F_D (on cofferdam) = 278 kN acting → (downstream)

b)

$$V_1 = Q/(bh_1) = 1000/(50 \cdot 6.66) = 3.00\,m/s$$

$$V_3 = V_n = 3.05\,m/s$$

$$E_1 = h_1 + V_1^2/2g = 6.66 + 3^2/2 \cdot 9.8 = 7.12\,m$$

$$E_3 = h_3 + V_3^2/2g = 6.56 + (3.05)^2/2 \cdot 9.8 = 7.03(5)\,m$$

Headloss due to cofferdam $\cong E_1 - E_3 = 0.085 \approx 0.09\,m$

c)

Flow from 1-1 to 2-2 splits evenly (symmetry) and represents a short transition with a converging flow. Therefore no headloss from 1-1 to 2-2. So,

$$E_1 = 7.12\,m = E_2 = h_2 + \frac{\{(Q/2)/[(50-15)/2)h_2]\}^2}{2g} = h_2 + \frac{41.65}{h_2^2}$$

[in SI units], or

$$h_2 = 7.12 - \frac{41.65}{h_2^2}$$

Have to iterate and we start with $h_2^{(0)} = 6.6\,m$ [it must be smaller than h_1!

DONE!

$h_2^{(1)} = 6.16m \Rightarrow h_2^{(2)} = 6.02m \Rightarrow h_2^{(3)} = 5.97m \rightarrow h_2^{(4)} = 5.95m \Rightarrow h_2^{(5)} = 5.94\,m$

$\underline{h_2 = 5.94\,m}$ [Note: $Fr_2 = \frac{V_2}{\sqrt{gh_2}} = 0.63$ - quite large]

$\underline{V_2 = [Q/(b-15)]/h_2 = 4.81\,m/s}$

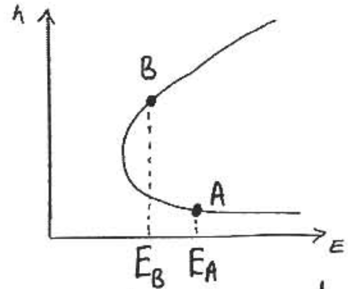
d)

If coffer dam were 20 m wide, the discharge per unit width at the narrowest section would be $q = Q/(b-20) = 33.3\,m^3/s/m$. Maximum discharge for given E corresponds to critical flow. With $E = E_1 = 7.12\,m \Rightarrow h_c = (2/3)E_1 = 4.75m$; $V_c = \sqrt{gh_c} = 6.82\,m/s$

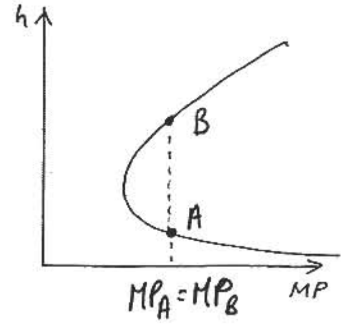
$q_c = \sqrt{gh_c}\,h_c = 32.4\,m^3/s/m < \frac{Q}{b-20} = 33.3\,m^3/s/m$: $\underline{\text{No Flow could not pass in this case}}$

- PROBLEM N° 6:

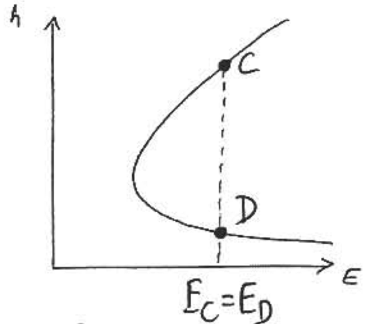
(1)



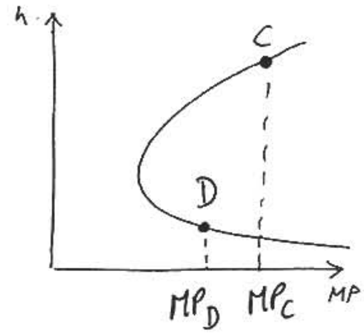
Specific energy of flow decreases from A to B.



(2)



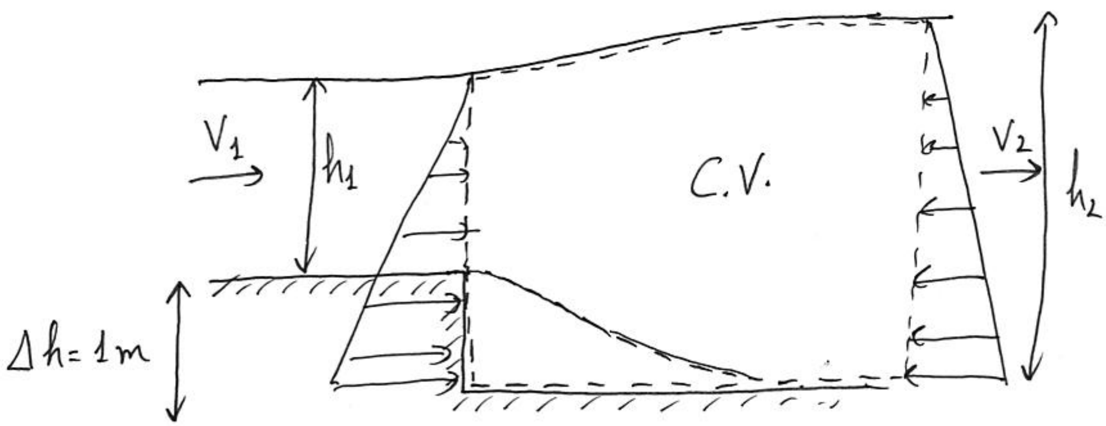
Specific energy is conserved from C to D



$MP_C = MP_D +$ force exerted by gate on fluid.

ADDENDUM:

MORE RIGOROUS SOLUTION TO PROBLEM 1, PART B



$h_1 = 4'5\,m$, $V_1 = 1'11\,m/s$

h_2, V_2? → Conservation of momentum in horizontal direction

$$\underbrace{\rho V_1^2 h_1}_{M_1} + \underbrace{\rho g \frac{(h_1+\Delta h)^2}{2}}_{P_1} = \underbrace{\rho V_2^2 h_2}_{M_2} + \underbrace{\rho g \frac{h_2^2}{2}}_{P_2}$$

$$31'38 = \frac{5'092}{h_2} + h_2^2$$

$$h_2 = \sqrt{31'38 - \frac{5'092}{h_2}} \Rightarrow h_2 = 5'519\,m,\ V_2 = 0'905\,m/s$$

$$\Delta H = H_1 - H_2 = \left(\Delta h + h_1 + \frac{V_1^2}{2g}\right) - \left(h_2 + \frac{V_2^2}{2g}\right) =$$

$$= 2'08 \cdot 10^{-3} \approx$$ ΔH_{EXP} CALCULATED IN PROBLEM 2