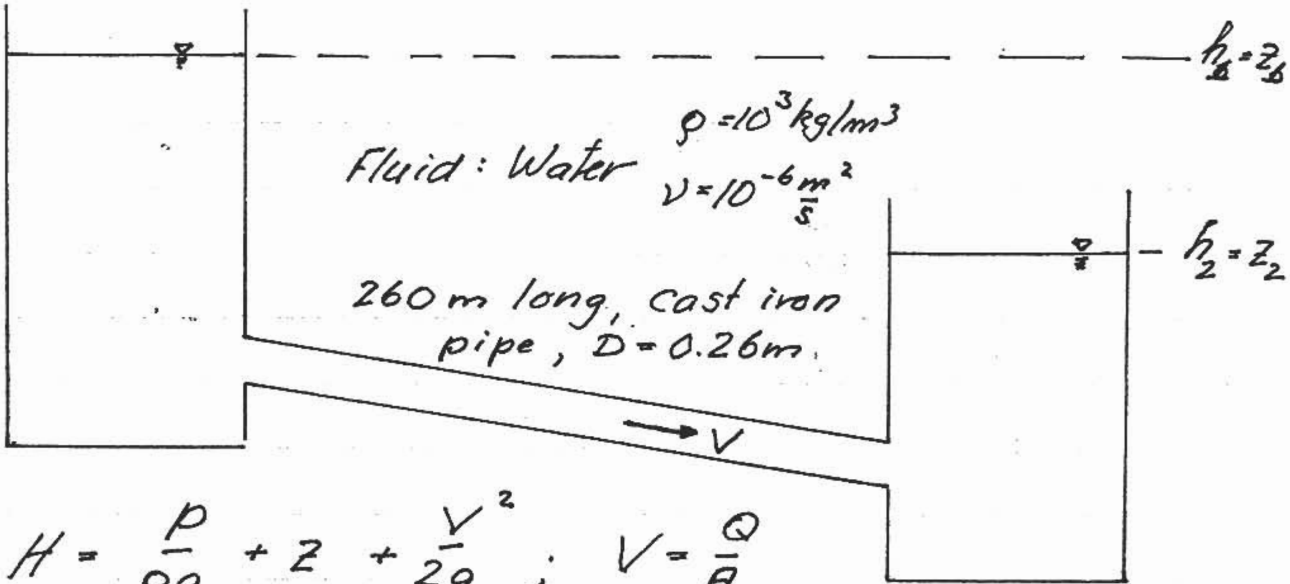
LECTURE #17

1.060 ENGINEERING MECHANICS II

ANALYSIS OF PIPE FLOW



$$H = \frac{p}{\rho g} + z + \frac{V^2}{2g} \; ; \; V = \frac{Q}{A}$$

Head in supply tank $= H_i = h_1$

Head in receiving tank $= H_{rec} = h_2$

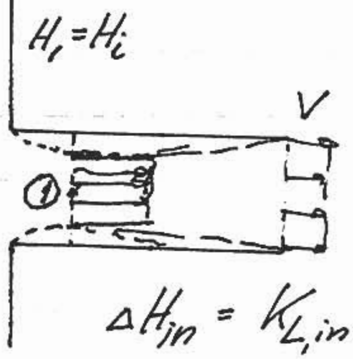
Total headloss in system $= \underline{\Delta H = h_1 - h_2}$

$$\Delta H = \sum \Delta H_f + \sum \Delta H_{minor}$$

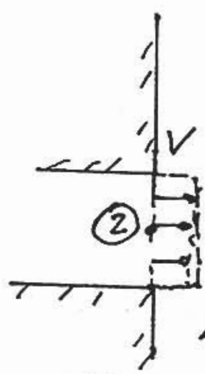
Here : $\Delta H_f = f \frac{\ell}{D} \frac{V^2}{2g}$ (only one pipe)

Inlet $\qquad \sum \Delta H_{minor} \qquad$ Outlet

$H_i = h_1$



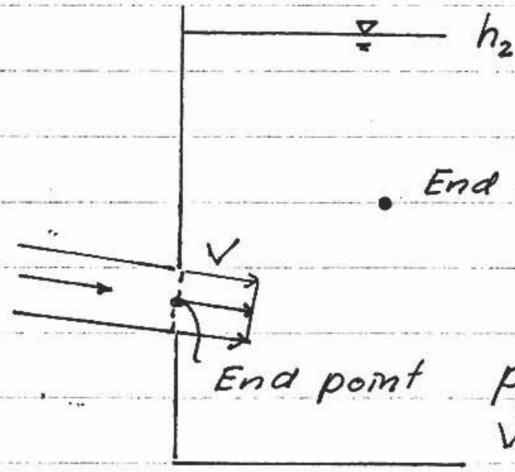
$$\Delta H_{in} = K_{L,in} \frac{V^2}{2g}$$



$H_{rec} = h_2$

$$\Delta H_{out} = \frac{V^2}{2g}$$

Choice "End" point

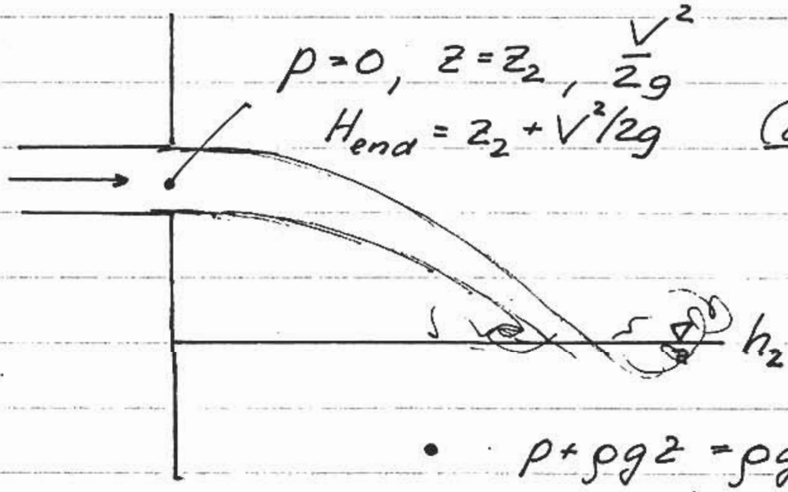


End point $H_2 = h_2$

End point $\left.\begin{array}{l} p_2 + \rho g z_2 = \rho g h_2 \\ V^2/2g = \text{vel. head} \end{array}\right\} H_2 = h_2 + \frac{V^2}{2g}$

Difference is (of course) the velocity head $\frac{V^2}{2g}$ which is the head loss of the outflow into a large reservoir!

But if outflow is free then what?



$p = 0, \; z = z_2, \; \frac{V^2}{2g}$

$H_{end} = z_2 + V^2/2g$ Correct

$p + \rho g z = \rho g h_2$

$H_{end} = h_2$ Incorrect

Difference $= (z_2 - h_2) + V^2/2g$ is NOT just the velocity head! Reason: Impact of free jet into pool causes dissipation.

Proper Choice: Take "end" at exit from pipe

If $H_{end} = h_2$ lowering pool level would increase $h_1 - H_2$ and increase flow which is ABSURD!

$$H_i = H_1 = h_1 = H_{rec} + \Delta H = h_2 + K_{L,in}\frac{V^2}{2g} + K_{L,out}\frac{V^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g}$$

$K_{L,in}$: 0.4 ~ 0.5 if sharp, ~ 0 if rounded

$K_{L,out} = 1$ regardless

$$h_1 - h_2 = \left(\sum K_L + f\frac{\ell}{D}\right)\frac{V^2}{2g}$$

$$V = \left\{2g(h_1 - h_2) \Big/ \left(\sum K_L + f\frac{\ell}{D}\right)\right\}^{1/2} \quad (1)$$

Assume rounded inlet conditions: $\sum K_L = K_{L,out} = 1$

$h_1 - h_2 = 1\ m$. BUT WE DON'T KNOW f!

$$f = f\left(Re = \frac{VD}{\nu},\ \underset{\varepsilon = 0.26mm,\ 10^{-3}}{\varepsilon/D}\right) \quad : \text{MOODY} \quad (2)$$

BUT WE DON'T KNOW V (until we know f)!

<u>Method 1</u>: Take "standard" $f = f^{(1)} = 0.02$, get $V = V^{(1)}$ from (1), then $f = f^{(2)}$ from Moody with $Re = Re^{(1)} = V^{(1)}D/\nu$. Go back to (1) etc until $V^{(n+1)} \sim V^{(n)}$

<u>Method 2</u>: Assume Fully Rough Turbulent Flow and get $f = f^{(1)} = f(Re \to \infty, \varepsilon/D)$ from Moody, then get $V = V^{(1)}$ from (1), now $Re = Re^{(1)} = V^{(1)}D/\nu$ & Moody gives $f = f^{(2)}$ etc.

Computations for $h_1 - h_2 = 1m$

Cast Iron : Table 8.1 $\varepsilon = 0.26\ mm$

Rel. Roughness $= \varepsilon / D = 0.26mm / 0.26m = 10^{-3}$

If flow is R.T. $\Rightarrow$ Moody gives $f^{(1)} = 0.0196$

[very close to "standard" value of 0.02 used in Method 1 - here the two approaches are the "same"]

$$V^{(1)} = \frac{\sqrt{2g(h_1 - h_2)}}{\sqrt{1 + f^{(1)}(\ell / D)}} = \frac{\sqrt{2 \cdot 9.8 \cdot 1}}{\sqrt{1 + 0.0196 \frac{260}{0.26}}} = \frac{\sqrt{19.6}}{\sqrt{1 + 19.6}} = 0.98 \frac{m}{s}$$

minor loss is minor

Now $Re^{(1)} = V^{(1)} D / \nu = 0.98 \cdot 0.\cancel{26} / 10^{-6} = 2.5 \cdot 10^5$

and $\varepsilon / D = 10^{-3}$ gives

$f^{(2)} = 0.0209$

$$V^{(2)} = \frac{\sqrt{2g(h_1 - h_2)}}{\sqrt{1 + f^{(2)} \ell / D}} = \frac{\sqrt{19.6}}{\sqrt{1 + 20.9}} = 0.95 \frac{m}{s}$$

Now $Re^{(2)} = V^{(2)} D / \nu = 2.47 \cdot 10^5 \simeq 2.5 \cdot 10^5$ same as before. Therefore same $f^{(3)} = f^{(2)}$, and we're done !

$\underline{V = V^{(2)}} = 0.95\ m/s \Rightarrow Q = V \cdot A = V \frac{\pi}{4} D^2 = \underline{0.0504 \frac{m^3}{s}}$

If $h_1 - h_4 = 4m$! not $1m$ - quick estimate from above : $V \propto \sqrt{h_1 - h_2}$ assuming $f = f^{(2)} \simeq 0.02$

$\underline{V_4} = \sqrt{4/1}\, V_1 = \underline{1.9\ m/s} \Rightarrow \underline{Q \simeq 0.1\ m^3/s}$

$Re = 4.9 \cdot 10^5$; $\varepsilon / D = 10^{-3} \Rightarrow f = 0.0203 \simeq 0.0209$ ✓

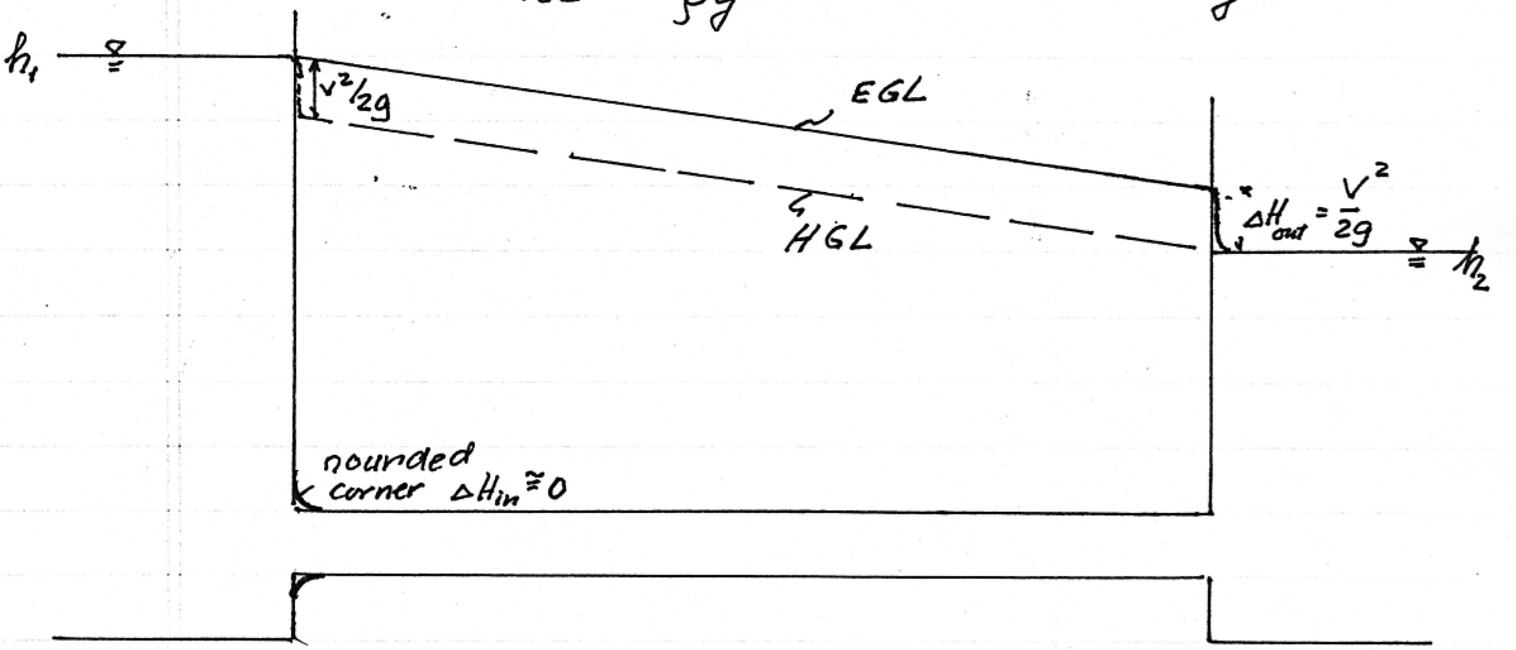
V_4 is correct.

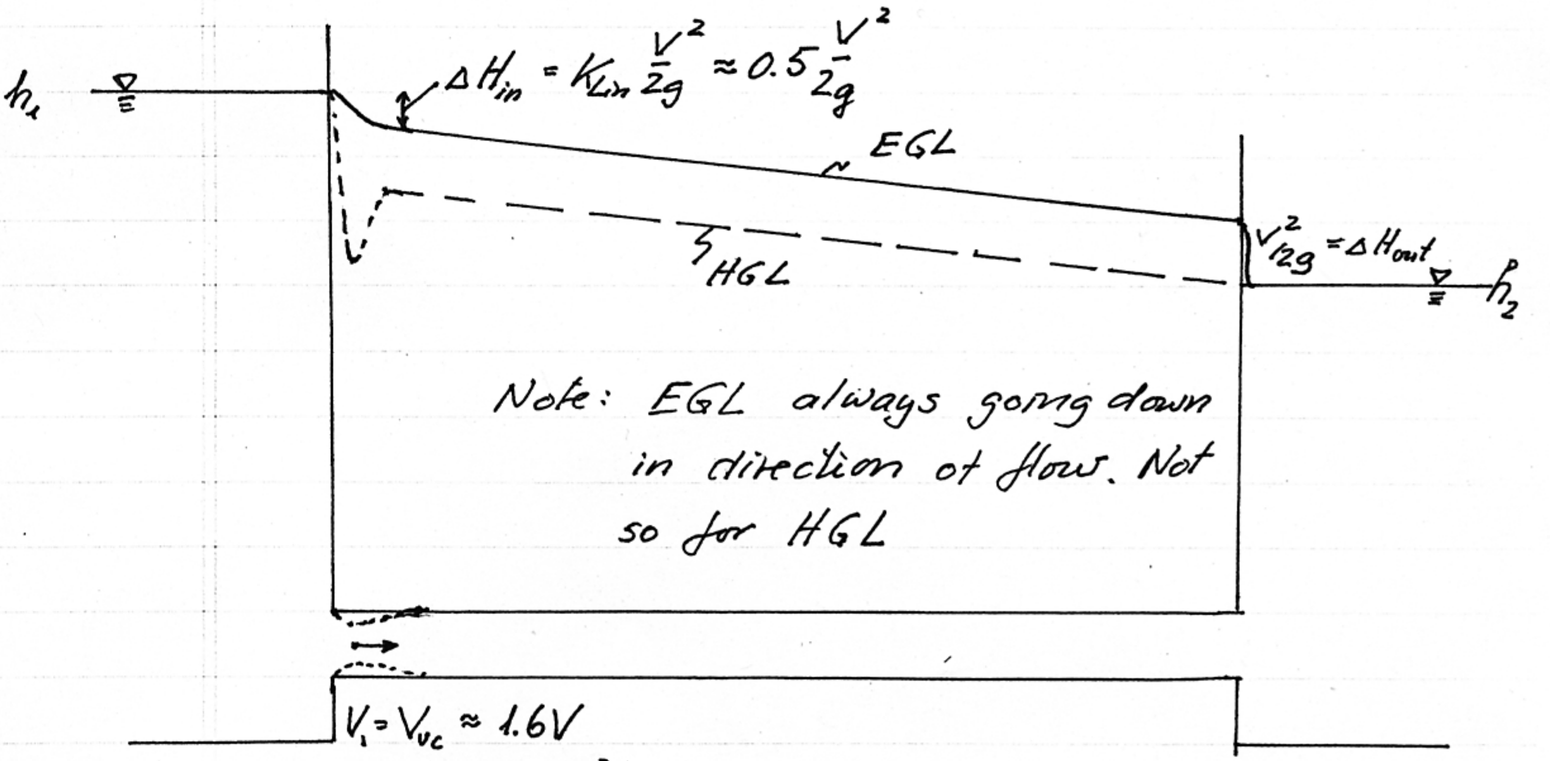
EGL : ENERGY GRADE LINE (TOTAL HEAD LINE) &

HGL : HYDRAULIC GRADE LINE (PIEZOMETRIC HEADLINE

$$EGL \; @ \quad z_{EGL} = H = \frac{p_{CG}}{\rho g} + z_{CG} + \frac{V^2}{2g}$$

$$HGL \; @ \quad z_{HGL} = \frac{p_{CG}}{\rho g} + z_{CG} = H - \frac{V^2}{2g}$$





Note: EGL always going down in direction of flow. Not so for HGL

$V_1 = V_{vc} \approx 1.6 V$

$V_1^2/2g = 2.6\, V^2/2g$