1.060 Engineering Mechanics II Spring 2006

Recitation 7 - Problems

April 6th and 7th

Problem 1

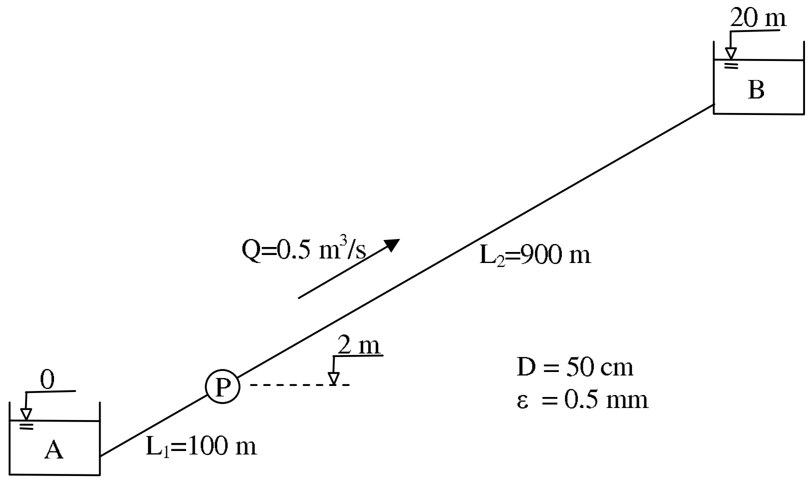


Figure 1: Pump impulsion in Problem 1.

A water flowrate $Q = 0.5\ m^3/s$ is being pumped from a large reservoir A, where the free surface elevation is $z = 0$, to a large reservoir B, where the free surface elevation is $z = 20\ m$. The pipe that connects the two reservoirs has diameter $D = 50\ cm$ and roughness $\epsilon = 0.5\ mm$. The pump is located at elevation $z = 2\ m$. The pipe length between A and the pump is $L_1 = 100\ m$ and the pipe length between the pump and B is $L_2 = 200\ m$. These characteristics are represented in Figure 1. Neglect minor losses.

a) Determine the pump head, H_p, necessary to pump the indicated flowrate.

b) If the required NPSH of the pump is 6 m, would you worry about cavitation in the pump? Assume that the temperature is 20°C.

c) Draw the energy grade line (EGL), specifying the relevant values.

d) The efficiency of the pump is $\eta = 0.8$ and the cost of electricity is 10 cents per kWh. How much does it cost to pump 1 m^3 (258.1 gallons) from A to B?

Problem 2

A sphere of diameter $D = 0.075\ m$ and weight $W = 1.37\ N$ is moving through air ($\rho_a = 1.2\ kg/m^3$, $\nu_a = 1.5 \cdot 10^{-5}\ m^2/s$) at a speed of $V_p = 42.5\ m/s$ (95 mph).

a) What is the kinetic energy of the sphere?

b) What is the Reynolds number upon which the sphere's coefficient, C_D, depends?

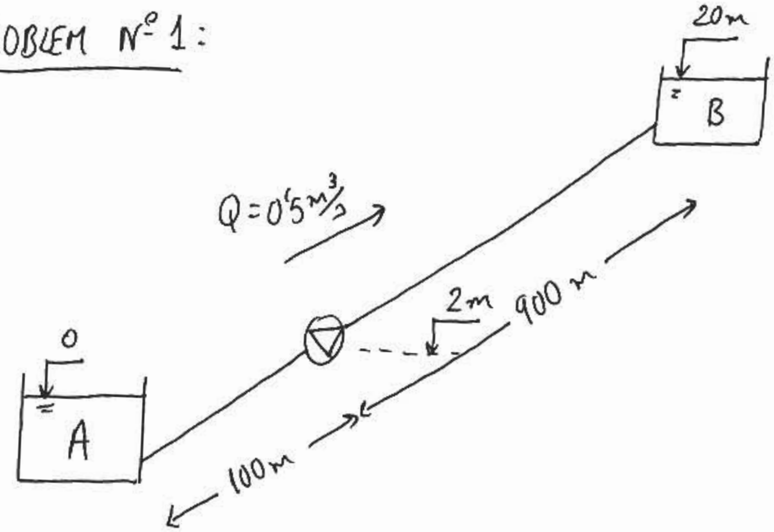
c) If $C_D = 0.3$, determine the drag force, F_D, acting on the sphere as it moves through the air.

d) Determine the rate at which the sphere loses kinetic energy due to the air resistance.

e) Assuming the rate of loss of kinetic energy determined in **(d)** to remain constant, estimate the velocity of the sphere 0.4 s after it was given the initial velocity $V_p = 42.5\ m/s$.

RECITATION 7 - SOLUTIONS

- PROBLEM Nº 1:



D = 0'5 m
ε = 5·10⁻⁴ m

a)

Apply Bernoulli between A and B:

$$H_A + H_p = H_B + \Delta H_{losses}$$

$H_A = 0$, $H_B = 20\ m$, $\Delta H_{losses} = \Delta H_f = f \frac{L}{D} \frac{V^2}{2g}$ (Minor losses neglected)

$$V = \frac{Q}{\pi D^2/4} = \frac{0'5}{\pi \cdot 0'5^2/4} = 2'55\ m/s \rightarrow Re = \frac{VD}{\nu} = \frac{2'55 \cdot 0'5}{10^{-6}} = 1'3 \cdot 10^6$$

$$\varepsilon/D = \frac{5 \cdot 10^{-4}}{0'5} = 10^{-3}$$

MOODY → $f = 0'020$

$$\Delta H_{f_1} = 0'020 \cdot \frac{100}{0'5} \cdot \frac{2'55^2}{2 \cdot 9'8} = 1'33\ m$$ (Headloss from A to pump)

$$\Delta H_{f_2} = 0'020 \cdot \frac{900}{0'5} \cdot \frac{2'55^2}{2 \cdot 9'8} = 11'94\ m$$ (Headloss from pump to B).

$$\underline{H_p} = H_B - H_A + \Delta H_{f_{A \to B}} = 20 - 0 + (1'33 + 11'94) = \underline{33'3\ m}$$

b)

$$NPSH = \frac{V^2}{2g} + \frac{p - p_{vapor,gauge}}{\rho g} = H - z_{pump} - \frac{p_{vapor,gauge}}{\rho g} =$$

(H ↑ head at the pump location)

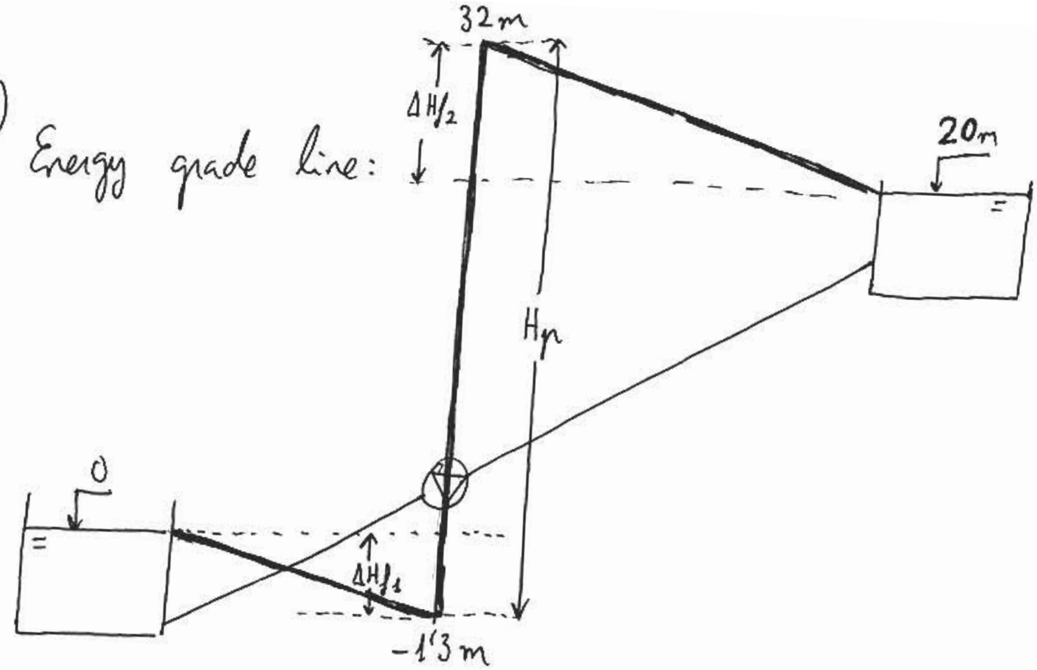
$$= (H_A - \Delta H_{f_1}) - z_{pump} - \frac{p_{vapor,gauge}}{\rho g} = (0 - 1'33) - 2 - \frac{(2338 - 101300)}{9800} =$$

(2338: 20°C)

$$= 6'77\ m > NPSH_{req} = 6\ m \Rightarrow \underline{\text{NO CAVITATION}}$$

c)

Energy grade line:



d)

$$BHP = \frac{\rho g Q H_p}{\eta} = \frac{9800 \cdot 0'5 \cdot 33'3}{0'8} = 163200 \text{ W} = 163 \text{ kW}$$

Volume lifted in 1 hour $= 0'5 \ m^3/s \cdot 3600 \ s/h = 1800 \ m^3/h$

$$\text{Cost per cubic meter} = \frac{163 \text{ kW} \cdot 10 \ ¢/\text{kWh}}{1800 \ m^3/h} = 0'91 \ \frac{\text{cents}}{m^3}$$

Problem No: 2

a)

$W = m \cdot g = m\,9.8 = 1.37\,N \Rightarrow m$ = mass of sphere = 0.14 kg

$\underline{E_{kin,o}} = \frac{1}{2} m V_p^2 = \underline{126}\,(Nm = Joule)$

b)

$\underline{Re} = D V_p / \nu_a = 0.075 \cdot 42.5 / (1.5 \cdot 10^{-5}) = \underline{2.1 \cdot 10^5}$

c)

$F_D = \frac{1}{2} \rho_a C_D A_\perp V_p^2$ (from cheat sheet) $= \frac{1}{2} \cdot 1.2 \cdot 0.3 \cdot \frac{\pi}{4} (0.075)^2 (42.5)^2$

$\underline{F_D = 1.44\,N}$ [notice $F_D \approx W$]

d)

Rate at which work is done against drag force =

$F_D \cdot V_p$ = Rate at which sphere losses energy

$\underline{\dot{E}_{loss}} = 1.44 \cdot 42.5 = \underline{61.0}\,(Nm/s = Joules/s = Watts)$

e)

Energy lost after 0.4s flight $= \dot{E}_{loss} \cdot 0.4 = 24.4$ Joules

Remaining $E_{kin} = E_{kin,o} - \dot{E}_{loss} \cdot 0.4 = 126 - 24.4 = 101.6$ Joules

$\frac{1}{2} m V_{0.4}^2 = \frac{1}{2} 0.14 \cdot V_{0.4}^2 = 101.6 \Rightarrow \underline{V_{0.4}} = (2 \cdot 101.6 / 0.14)^{1/2} = \underline{38.1\ m/s}$

[Note reduction in velocity = 42.5 − 38.1 = 4.4 m/s = 9.8 mph

Baseball rule of thumb: Pitch velocity crossing the plate is ~10mph slower than when it leaves the pitcher's hand].