1.225J (ESD 205) Transportation Flow Systems

Lecture 1

Cumulative Plots & Time-Space Diagrams

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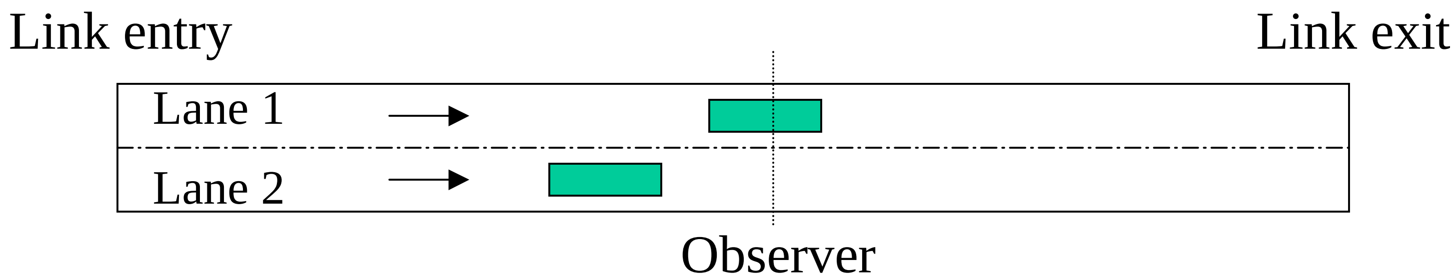
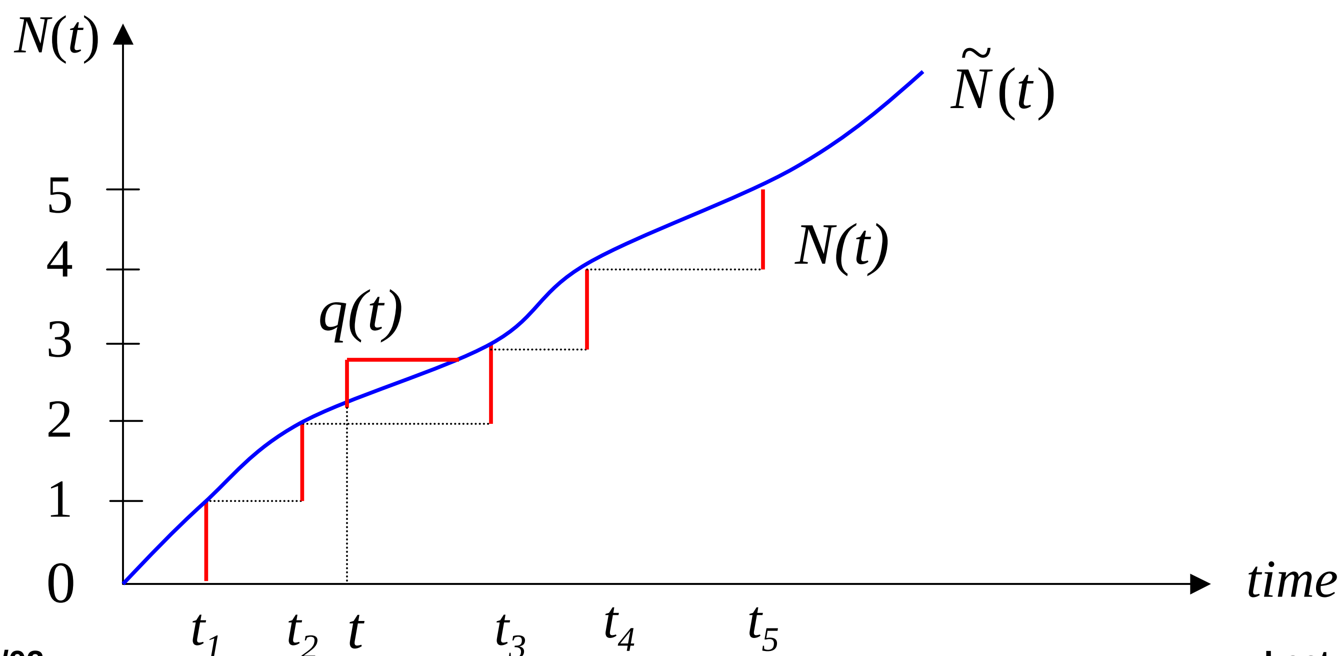
Cumulative Plots

- Observer: count the total number of vehicles, *N(t)*, that passed in front of him/her during time interval *[0,t]*.

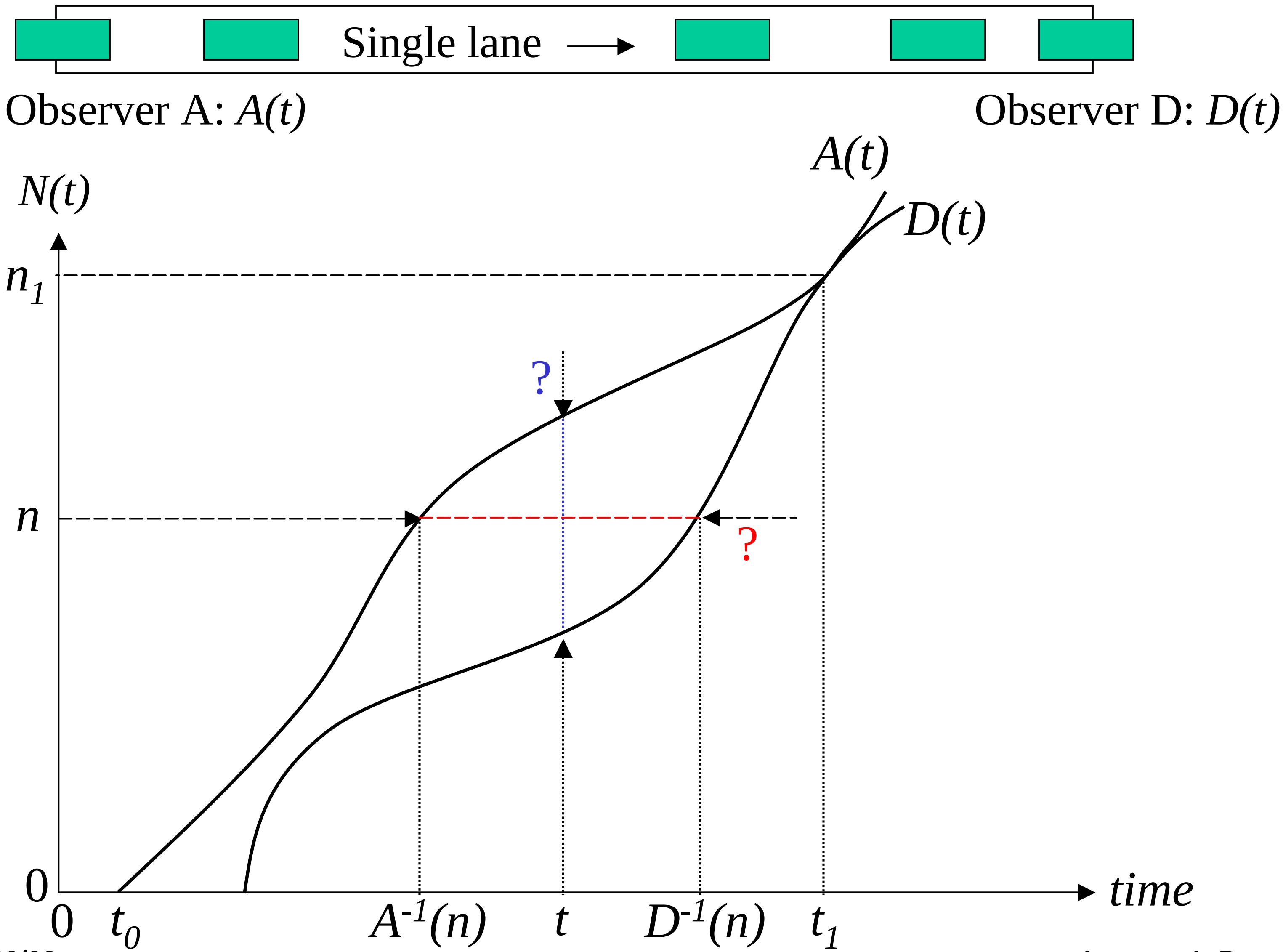




Observations on *N(t)*

- *N(t)* is a step function. (not smooth)
- $\tilde{N}(t)$ is a smooth approximation of *N(t)* $\Rightarrow$ $\dfrac{d\tilde{N}(t)}{dt}$ exists.
- $q(t) = \dfrac{d\tilde{N}(t)}{dt}$: instantaneous flow at time *t*.
- Average flow = $\dfrac{N(T) - N(0)}{T} \cong \dfrac{\tilde{N}(T) - N(0)}{T}$

Arrival-Departure Cumulative Plots



Accumulated Items: $Q(t) = A(t) - D(t)$?

- $Q(t)$: number of items (cars, planes) accumulated between the two observers.

- $$\begin{aligned} Q(t) &= Q(0) + [A(t) - A(0)] - [D(t) - D(0)] \\ &= (Q(0) + D(0) - A(0)) + A(t) - D(t) \\ &= A(t) - D(t) \qquad (\text{ if } Q(0)+D(0)-A(0) = 0) \end{aligned}$$

Waiting Under FIFO Order

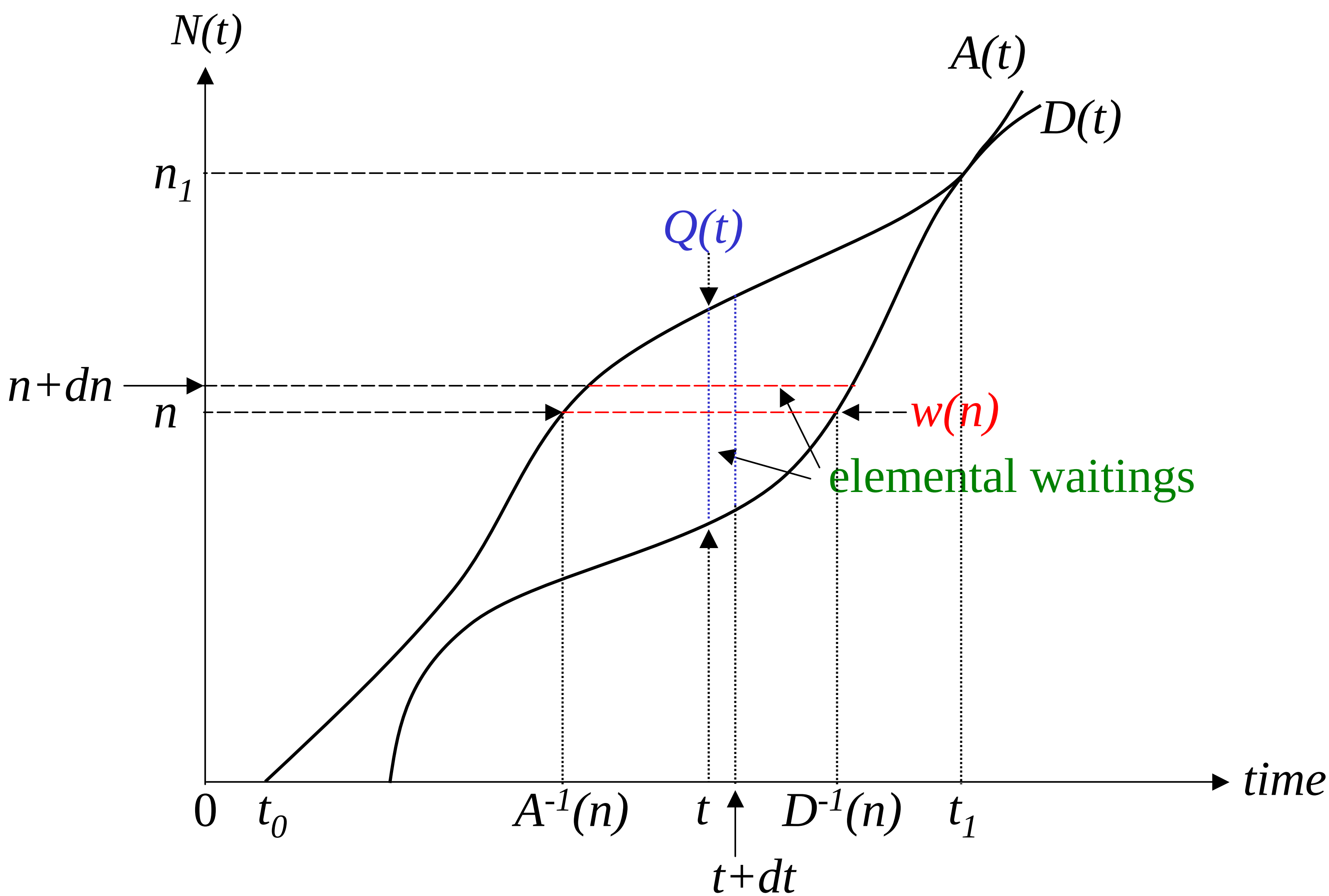
- Vehicles depart in the same order as they entered a link (i.e. segment of road) ≡ (First-In-First-Out) FIFO

- Item *n* is observed at the entrance of a link at time $A^{-1}(n)$.

- Item *n* is observed at the exit of a link at time $D^{-1}(n)$.

- Waiting time of the item *n*: $w(n) = D^{-1}(n) - A^{-1}(n)$

$Q(t)$, $w(n)$, and Elemental Waiting



Total Waiting Time

- Elemental waiting during *[t, t+dt]: Q(t)dt*

- Total waiting during $[t_0, t_1]$: $Area = \int_{t_0}^{t_1} Q(t)dt = \int_{t_0}^{t_1} (A(t) - D(t))dt$

- Elemental waiting during *[n, n+dn]: w(n)dn*

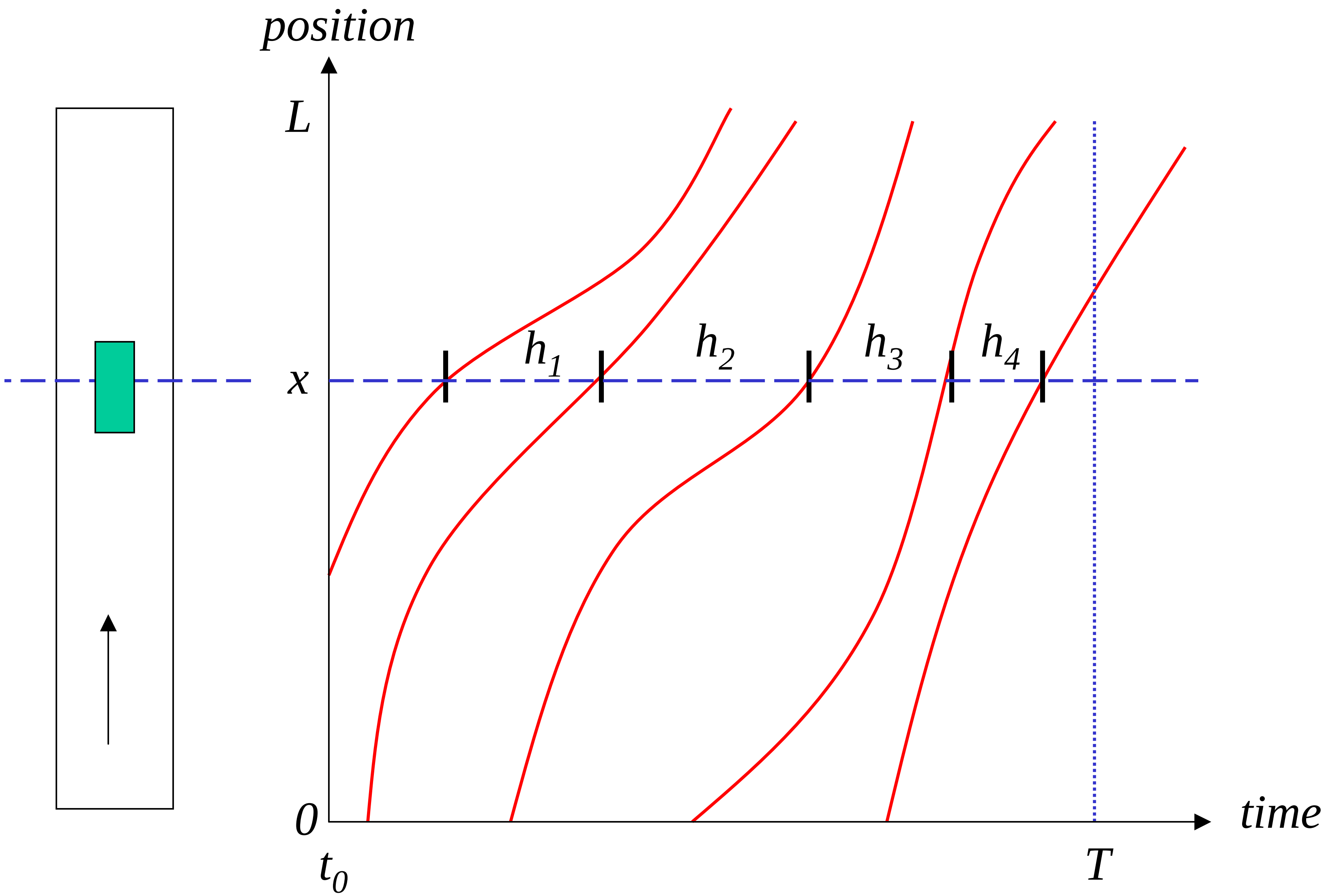
- Total waiting during $[0, n_1]$: $Area = \int_0^{n1} w(n)dn$

- Average wait suffered by vehicle arriving between t_0 and t_1

$$\overline{W} = \frac{Area}{A(t_1) - A(t_0)} = \frac{Area}{t_1 - t_0} \times \frac{t_1 - t_0}{A(t_1) - A(t_0)} = \overline{Q} \times \frac{1}{\overline{\lambda}}$$

$$\Rightarrow \quad \overline{Q} = \overline{\lambda} \times \overline{W} \quad \text{(Queuing formula)}$$

Time-Space Diagram: Analysis at a Fixed Position



Flows and Headways

- *m(x)*: number of vehicles that passed in front of an observer at position *x* during time interval *[0,T]*. (ex. *m(x)=5*)

- Flow rate: $q(x) = \frac{m(x)}{T}$

- Headway $h_j(x)$: time separation of consecutive vehicles

- Average headway: $$\bar{h}(x) = \frac{\sum_{j=1}^{m(x)} h_j(x)}{m(x)-1}$$

- What is the relationship between *q(x)* and $\bar{h}(x)$?

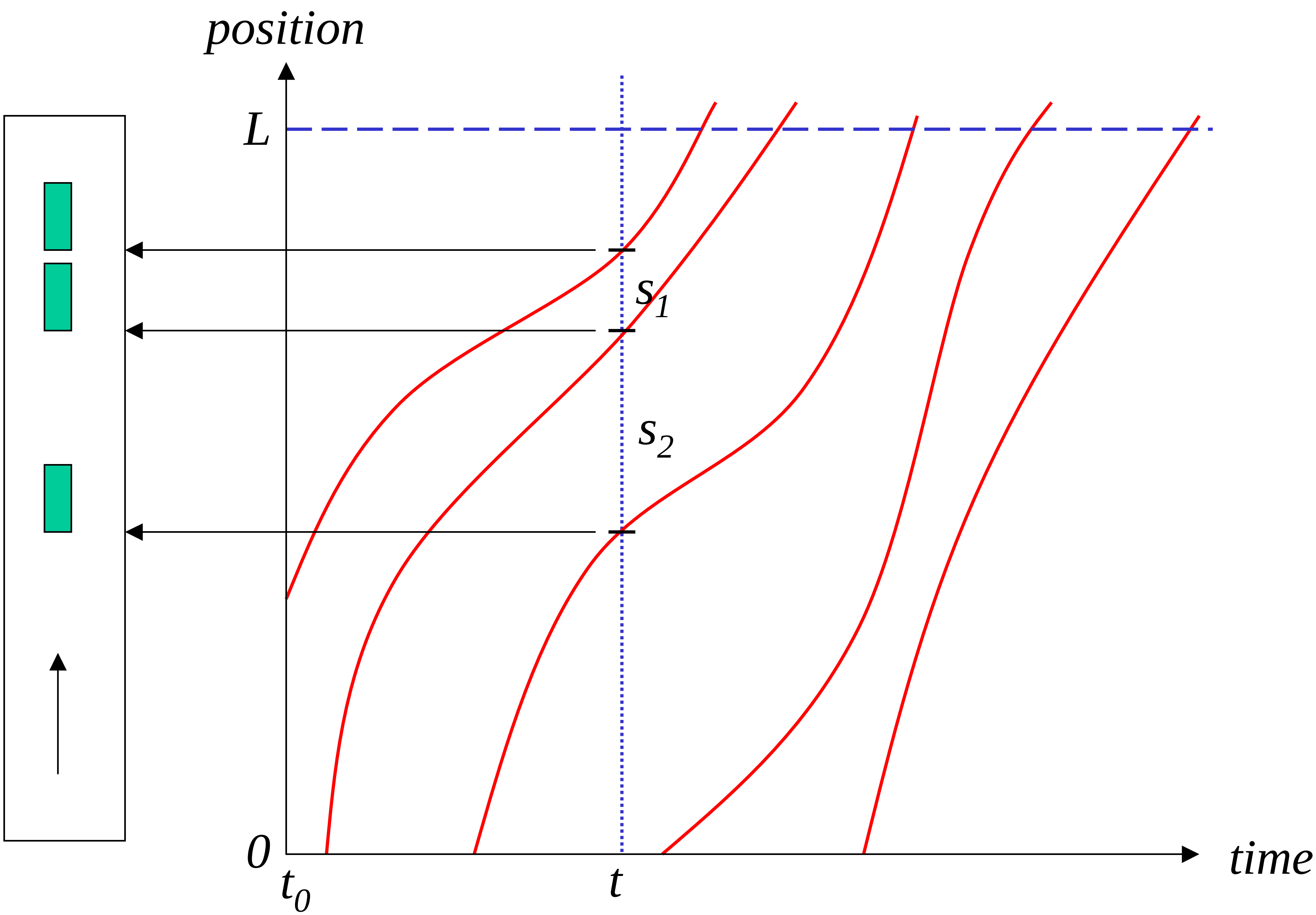
Flow Rate vs. Average Headway

- If T is large, $T \approx \sum_{j=1}^{m(x)} h_j(x)$
- Then, $\dfrac{1}{q(x)} = \dfrac{T}{m(x)} \approx \dfrac{\sum_{j=1}^{m(x)} h_j(x)}{m(x)} = \bar{h}(x)$

  $\Rightarrow \quad q(x) \approx \dfrac{1}{\bar{h}(x)}$ This is intuitively correct.

- *q(x)* is also called **volume** in traffic flow systems circles (i.e. 1.225)
- *q(x)* is also called **frequency** in scheduled systems circles (i.e. 1.224)

Time-Space Diagram: Analysis at Fixed Time



Density and Spacing

- $n(t)$: number of vehicles in a stretch of length L at time t.
- Density $k(t) = \frac{n(t)}{L}$
- $s_i(t)$: spacing between vehicle i and vehicle $i+1$.
- $L \approx \sum_{i=1}^{n(t)} s_i(t)$
- $$\frac{1}{k(t)} = \frac{L}{n(t)} \approx \frac{\sum_{i=1}^{n(t)} s_i(t)}{n(t)} = \bar{s}(t)$$
- $k(t) \approx \frac{1}{\bar{s}(t)}$ (Is this intuitive?)

Lecture 1 Summary

- Cumulative plots: *A(t), D(t), Q(t), w(n)* $\Longrightarrow$ $\overline{Q} = \overline{\lambda} \times \overline{W}$

- Time-Space Diagram: Analysis at a fixed position

$$\Longrightarrow \quad q(x) \approx \frac{1}{\overline{h}(x)}$$

- Time-Space Diagram: Analysis at a fixed time

$$\Longrightarrow \quad k(t) \approx \frac{1}{\overline{s}(t)}$$