1.225J (ESD 225) Transportation Flow Systems

Lecture 10

Control of Isolated Traffic Signals

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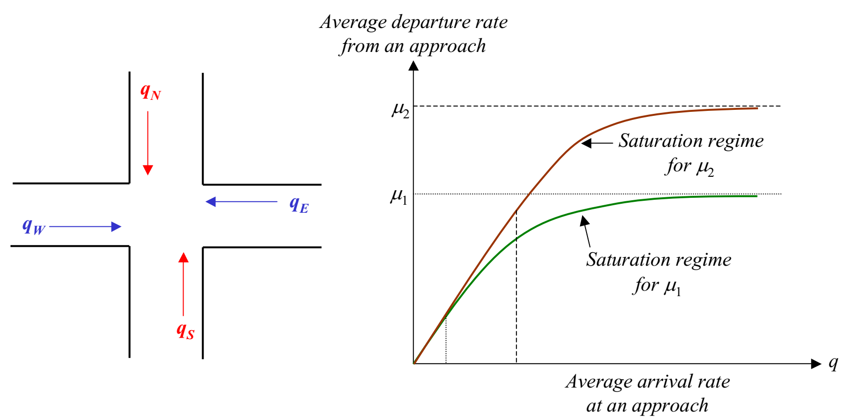
Lecture 10 Outline

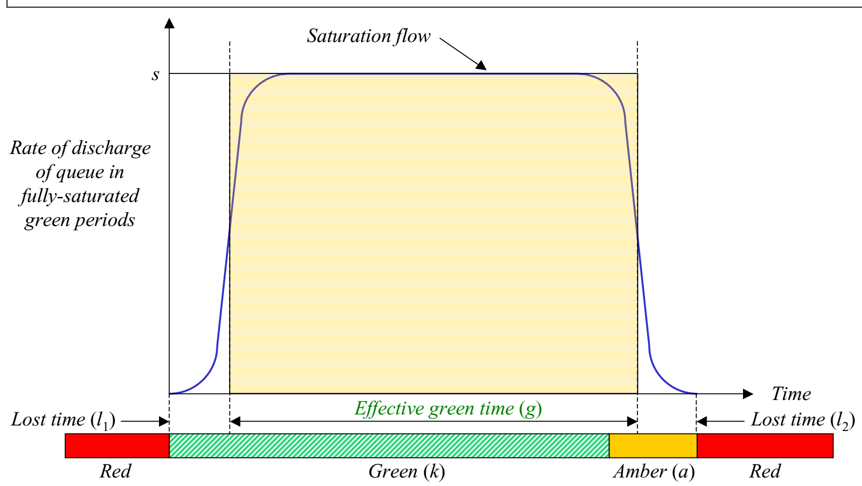
- Isolated saturated intersections
- Definitions: Saturation flow rate, effective green, and lost time
- Notation for an intersection approach variable
- Two assumptions for delay models
- Average delay per vehicle: deterministic term $\overline{W}_{q,A}$
- Average delay per vehicle: stochastic term $\overline{W}_{q,B}$
- Webster optimal green time settings: two approaches intersection and numerical example
- Webster cycle time optimization procedure
- Mid-day and evening-peak examples
- Lecture summary

Isolated Saturated Intersections



- ❑ An implication of saturation regime: need to efficiently allocate green times (g_N, g_S) and (g_E, g_W)

Saturation Flow, Effective Green, and Lost Time



- *Total lost time $l = l_1 + l_2$ (typically 2 sec)*
- *Green (k) + Amber (a) = Effective green time (g) + Total lost time (l) $\Rightarrow l = k + a - g$*
- *Effective green time (g) × Saturation flow (s) = Total vehicles discharged during ($k + a$)*

Notations for An Intersection Approach

- Webster
  - s : saturation flow rate
  - g : effective green time
  - c : cycle time
  - $\lambda = \frac{g}{c}$: fraction of effective green in cycle time

Webster	Meaning	Queueing Theory
q	arrival rate (veh/unit of time)	λ
λs	average flow rate at exit of an approach	μ
$x = \frac{q}{\lambda s}$	degree of saturation	$\rho = \frac{\lambda}{\mu}$

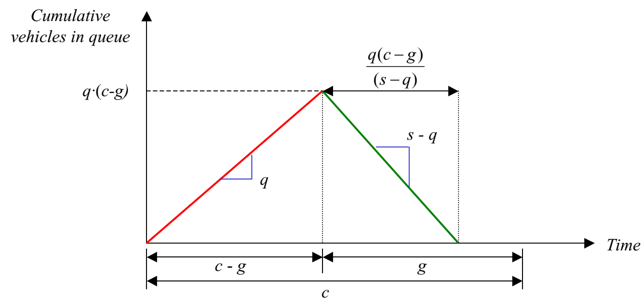
- $y = \frac{q}{s}$

Two Assumptions for Delay Models

- Assumption (A):
  - The interarrival times are constant (view arrivals as evenly spaced at rate q)
  - Service time is constant during effective green and zero in the rest of the cycle
  - Average waiting per vehicle is denoted by $\overline{W_{q,A}}$
- Assumption (B):
  - The interarrival times are exponentially distributed with rate q
  - Service time is constant with service rate λs
  - Average waiting per vehicle is denoted by $\overline{W_{q,B}}$
- Webster formula for total waiting time per vehicle:

$$d = \overline{W_{q,A}} + \overline{W_{q,B}} - correction\ \ factor\ obtained\ by\ simulation$$

Average Delay per Vehicle: Term $\overline{W}_{q,A}$



- Total waiting during c per approach:

$$\frac{1}{2}q(c-g)[(c-g)+\frac{q(c-g)}{s-q}]=\frac{q(c-g)^2}{2}\cdot\frac{s}{s-q}=\frac{q(c-g)^2}{2}\cdot\frac{1}{(1-\lambda x)}$$

- Total arrivals during cycle c: qc
- $\overline{W}_{q,A}=\left\{\frac{q(c-g)^2}{2}\cdot\frac{1}{(1-\lambda x)}\right\}\cdot\frac{1}{qc}=\frac{c}{2}\cdot\frac{(1-g/c)^2}{(1-\lambda x)}=\frac{c(1-\lambda)^2}{2(1-\lambda x)}$

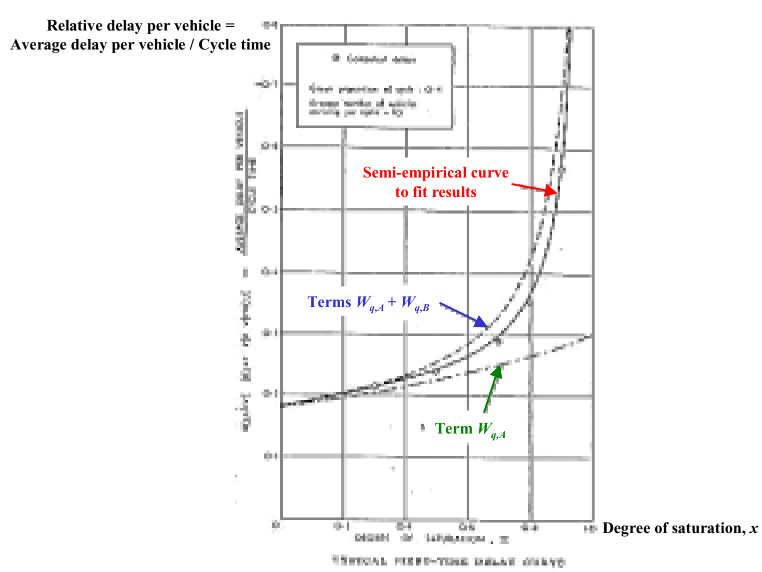
Average Delay per Vehicle: Term $\overline{W}_{q,B}$

- Interarrival times are exponentially distributed with rate q, and service times are deterministic with rate λs
- Average waiting time for *M/D/1* queueing system: $\frac{1}{2}\cdot\frac{\rho^2}{\lambda(1-\rho)}$
- $\overline{W}_{q,B}=\frac{1}{2}\cdot\frac{(q/\lambda s)^2}{q(1-q/\lambda s)}=\frac{1}{2}\cdot\frac{x^2}{q(1-x)}$

Webster's Average Delay Per Vehicle Model

- Average delay per vehicle: $d = \overline{W}_{q,A} + \overline{W}_{q,B} - correction\ \ term$
- $d = \dfrac{c(1-\lambda)^2}{2(1-\lambda x)} + \dfrac{x^2}{2q(1-x)} - 0.65\left(\dfrac{c}{q^2}\right)^{\frac{1}{3}} x^{(2+5\lambda)}$
- $\overline{W}_{q,A}$ dominates for very small values of x
- $\overline{W}_{q,B}$ dominates for large values of x ($x \rightarrow 1$)
- Small value of x is not an important case from an optimization standpoint
- Optimal green time setting problem: Find λ_E, λ_W, λ_N, and λ_S such that the total delay is minimum

Observed Delay vs. Webster's Model



"Optimal Settings": A Two Approaches Intersection

- $x_1 = \frac{q_1}{\lambda_1 s_1}, \quad x_2 = \frac{q_2}{\lambda_2 s_2}$
- Note:
  - (q_1, s_1) and (q_2, s_2) are given
  - $(\lambda_1 + \lambda_2)c = c$
  - if $x_1 \uparrow$, then $x_2 \downarrow$ and vice versa
- Total delay $\approx \overline{W}_{q,B}^{(1)} \cdot q_1 + \overline{W}_{q,B}^{(2)} \cdot q_2$

$$= \frac{1}{2}\sum_{i=1}^{2} \frac{x_i^2}{q_i(1-x_i)} \cdot q_i = \frac{1}{2}\sum_{i=1}^{2} \frac{x_i^2}{(1-x_i)}$$

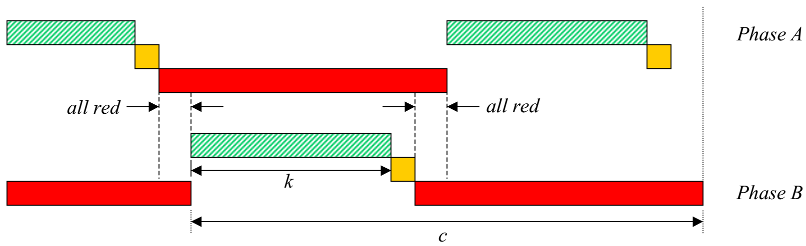
- Minimum total delay: Total delays are about the same on both approaches
  - $\frac{x_1^2}{1-x_1} = \frac{x_2^2}{1-x_2}$
  - $x_1 = x_2$
  - $\frac{\lambda_2}{\lambda_1}\left(= \frac{g_2/c}{g_1/c} = \frac{g_2}{g_1}\right) = \frac{q_2/s_2}{q_1/s_1}\left(= \frac{y_2}{y_1}\right)$

Numerical Example 1

- Saturation flow rate $s = 1800$ veh/hr for all arms (approaches)
  Lost time $L = 10$ sec
  Cycle length $c = 60$ sec
- $q_N = q_S = 600$ veh/hr; $q_E = q_W = 300$ veh/hr
- $y_N = y_S = \frac{600}{1800} = \frac{1}{3}; \quad y_E = y_W = \frac{300}{1800} = \frac{1}{6}$
- $y_{N-S} = \frac{1}{3}; \quad y_{E-W} = \frac{1}{6}$
- $\frac{g_{N-S}}{g_{E-W}} = \frac{1/3}{1/6} = 2$
- $g_{N-S} + g_{E-W} = 60 - 10 = 50$ sec
- $2g_{E-W} + g_{E-W} = 3g_{E-W} = 50 \text{ sec} \Rightarrow g_{E-W} = 50/3 \approx 17 \text{ sec}; \quad g_{N-S} \approx 33 \text{ sec}$

Cycle Time Optimization

- "Optimal" cycle: $c_o = \frac{1.5L+5}{1-y}$
- $y = \sum_{i=1}^{n} y_i$, n = number of phases (typically $n = 2$)
- $L = nl + R$, $\begin{cases} l = \text{average time lost per phase } (l \approx 2 \text{ sec}) \\ R = \text{all-red time } (R \approx 6 \text{ sec}) \end{cases}$
- Typically $L \approx 10$ sec
- Two phases:



Numerical Example 2: Mid-Day

- $s = 1600$ veh/hr in each direction ($N \to S$; $S \to N$; $E \to W$; $W \to E$)
  2 phases; all reds = 6 sec/cycle; lost time = 2 sec/phase
- $q_N = q_S = 600$ veh/hr; $q_W = 400$ veh/hr; $q_E = 300$ veh/hr
- $y_N = y_S = \frac{600}{1600} = \frac{3}{8}$; $y_W = \frac{400}{1600} = \frac{2}{8}$; $y_E = \frac{300}{1600} = \frac{3}{16}$
- $y_{N-S} = \frac{3}{8}$; $y_{E-W} = \frac{2}{8}$; $y = \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$; $L = 2 \cdot 2 + 6 = 10$ sec
- $c_o = \frac{1.5 \times 10 + 5}{1 - 5/8} = 53$ sec; optimal cycle
- $g_{N-S} \cong \frac{3}{5}(53-10) \approx 26$ sec; $g_{E-W} \cong \frac{2}{5}(53-10) = 17$ sec
- $x_N = x_S = 0.764$; $x_W = 0.779$; $x_E = 0.585$
- $\overline{W}_{q,N} = \overline{W}_{q,S} \cong 18.4$ sec; $\overline{W}_{q,W} \cong 28.6$ sec; $\overline{W}_{q,E} \cong 20.0$ sec
- $\overline{L}_{q,N} = \overline{L}_{q,S} \cong 3.07$ veh; $\overline{L}_{q,W} \cong 3.18$ veh; $\overline{L}_{q,E} \cong 1.67$ veh
- Total delay/hr $\cong 11$ hours

Numerical Example 2(cont.): Evening-Peak

- $s = 1600$ veh/hr in each direction $(N \to S; S \to N; E \to W; W \to E)$
  2 phases; all reds = 6 sec/cycle; lost time = 2 sec/phase
- $q_N = q_S = 800$ veh/hr; $q_W = q_E = 600$ veh/hr
- $y_N = y_S = \frac{800}{1600} = \frac{1}{2}$; $y_W = y_E = \frac{600}{1600} = \frac{3}{8}$
- $y_{N-S} = \frac{1}{2}$; $y_{E-W} = \frac{3}{8}$; $y = \frac{1}{2} + \frac{3}{8} = \frac{7}{8}$; $L = 2 \cdot 2 + 6 = 10$ sec
- $c_o = \frac{1.5 \times 10 + 5}{1 - 7/8} = 160$ sec; optimal cycle
- $g_{N-S} \cong \frac{4}{7}(160 - 10) \approx 86$ sec; $g_{E-W} \cong \frac{3}{7}(160 - 10) = 64$ sec
- $x_N = x_S = 0.93$; $x_W = x_E = 0.9375$
- $\overline{W}_{q,N} = \overline{W}_{q,S} \cong 62.0$ sec; $\overline{W}_{q,W} = \overline{W}_{q,E} \cong 88.3$ sec
- $\overline{L}_{q,N} = \overline{L}_{q,S} \cong 13.8$ veh; $\overline{L}_{q,W} = \overline{L}_{q,E} \cong 14.7$ veh
- Total delay/hr ≅ 57 hours

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