Statistics and Estimation

Question 1: Using the data from problem set 2, estimate using your own estimator (not polyfit) the coefficients of the quadratic polynomial that best fit the data in a least squares sense.
(a) Write the equation for the polynomial. What are the observations and unknown parameters in the polynomial (5-points).
(b) Write the above set of equations in matrix form (5-points).
(c) Form the least squares estimator and solve for the coefficients. You may use the Matlab matrix inversion routine inv and/or a calculator matrix inversion to solve the system of equations (10 points).

Question 2: Estimate the standards deviation of the errors in the measurements using the differences between the observed values and the polynomial fit. (20-points)

Question 3: What is the probability that the 17th measurement (time 12hr 21m   26s Measurement 70° 58.4') differs from the polynomial fit due to random error assuming that the noise in the measurements is Gaussian distributed and the data standard deviation computed in question 2 (20-points).

Question 4: Estimate the standard deviation of the peak in the polynomial (i.e., its maximum value and the time at which the maximum occurs) based on the least squares estimate in Question 1 (20-points).

Question 5: Using the non-linear model for the observed double elevation angle as a function of the site latitude and longitude (and the Sun's declination and Greenwich hour angle which you can assume are known), rigorously estimate the latitude and longitude of campus. Approach the problem with the following steps:
   (a) Write the equation for the double-elevation as a function of latitude and longitude (5-pts)
   (b) Find the partial derivative of the double angle with respect to latitude and longitude (10-pts)
   (c) Find the differences between the measured double elevation angles and the values predicted from an a priori estimate of the latitude and longitude of 42.0 and 71.0 (5-pts)
   (d) Using the partial derivatives from (b) and the prefit residuals from (c), form the least squares solution for the estimates of the adjustments to the latitude and longitude. Since this is a non-linear problem, iterate your solution until the adjustments are small compared to the standard deviations of the estimates i.e., compute new residuals from the non-linear model with the new estimates of the latitude and longitude. Use the estimates if the standard deviations of sextant measurements from Q2 above. (10-pts)
   (e) Give the final estimates of the latitude and longitude and their standard deviations and correlation between the estimates. (10-pts)