12.524 Problem set 2.
Due 25 October 2005

1. Tensor transformations.

a.) Write a tensor equation for the traction on a plane whose normal makes an angle θ with respect to the greatest principal compressive stress. The plane is always coaxial with the intermediate principal stress.

b.) How is this equation related to Mohr's circle?

2. Faults and friction.

Assume that a rock mass is subjected to external loads sufficient to cause failure by faulting on one or more fault surfaces. Assume that on any fault that yields, the shear stress and the normal stress are related by a friction law:

$$\sigma_\tau=\mu\sigma_n+\sigma_o.$$

where μ is the coefficient of friction and σ_o is the cohesion strength of the fault.

a.) Use the results of the problem above to plot the states of stress in the rock mass that are permissible. Use the greatest and least compressive stresses, σ1 and σ3, as the abscissa and ordinate, respectively.

b.) Now assume that many faults of random orientations exist. All follow the same friction law. Suppose the greatest and least principal stress are σ1, and σ3 and that the deviatoric stress is slowly increased from zero, starting from an isotropic state of stress. What is the orientation of the first plane to slip?

c.) Does this orientation change with changes in mean stress?

d.) Express the friction law in terms of the principal stresses, ∍:σ1-σ3=f(σ3). If the coefficient of friction is 0.60 and the cohesive strength is about 60 MPa, plot Byerlee's law (the friction law) as a function of depth for rocks with a density of 2.54g/cm3.

3. Airy stress function solution of externally loaded hole.

Consider the problem of the externally loaded cylindrical hole (refer to L9b). Recall that the boundary condition at r=a (if no internal pressure).

$$\sigma_{rr}=\sigma_{r\theta}=0$$

Boundary condition at r=b>>a

$$\sigma_{xx}=S,\ \sigma_{yy}=0$$

a.) Use problem 1 to convert the second boundary condition to cylindrical coordinates: $\sigma_{rr}(r,\theta)\big|_{r>>a}=\frac{S}{2}+\frac{S}{2}\cos 2\theta,\ \sigma_{r\theta}(r,\theta)\big|_{r>>a}=-\frac{S}{2}\sin 2\theta$ (0.1)

where θ = 0, π for x, -x directions respectively.

b.) Break problem into two separate boundary value problems, where forces at infinity are

$$\begin{matrix} S/2 & 0 & 0 \\ 0 & S/2 & 0 \\ 0 & 0 & 0 \end{matrix} \quad \textit{and} \quad \begin{matrix} S/2 & 0 & 0 \\ 0 & -S/2 & 0 \\ 0 & 0 & 0 \end{matrix}$$

Suppose that the Airy stress potential is separable and has the form $\phi = f(r)\cos 2\theta$ where $f(r) = Ar^2 + Br^4 + C\frac{1}{r^2} + D$. Show that this function is a solution of the biharmonic.

c.) Use the Airy stress function to obtain the radial principal stress for the first set of boundary conditions, $\sigma_{rr}(r) = \frac{-a^2 S}{2r^2} + \frac{S}{2}$

d.) Finally, derive the remainder of the solution (Eqs. 9.7)

$$\boxed{\begin{aligned} \sigma_{rr}(r) &= \frac{S}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{S}{2}\left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right)\cos(2\theta) \\ \sigma_{\theta\theta}(r) &= \frac{S}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{S}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos(2\theta) \\ \sigma_{r\theta}(r) &= -\frac{S}{2}\left(1 - \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right)\sin(2\theta) \end{aligned}} \tag{0.2}$$