Final homework assignment

Problem 1. Temperature and salinity eddy fluxes.

The tracer conservation equation is

$$\frac{\partial Q}{\partial t} + \nabla.(\mathbf{u}Q) = \kappa_Q \nabla^2 Q \tag{1}$$

where Q is any tracer, and κ_Q is the molecular diffusivity of that tracer.

(a) Assuming that Q can be split into a large scale component $\overline{Q}$ and a small scale component Q', so that $Q = \overline{Q} + Q'$ and $\overline{Q'} = 0$, and similarly for the velocity field, write down equations for the time-evolution of the large scale temperature $\overline{T}$ and salinity $\overline{S}$.

(b) Assume that the small-scale dynamics influence the large scale fields only through vertical fluxes, which can be parameterized in terms of diffusion down the large scale gradient with eddy diffusivity κ_T^* for temperature and κ_S^* for salinity. Rewrite the equations from (a) incorporating this parameterization of the small-scale fluxes.

(c) Now combine both equations from (b) to form one equation for the large scale density $\overline{\rho} = \beta\overline{S} - \alpha\overline{T}$, and show that the small scale fluxes of density can again be written in terms of diffusion in the direction of the large scale density gradient, with eddy diffusivity

$$\kappa_\rho^* = \frac{\kappa_T^* R_\rho - \kappa_S^*}{R_\rho - 1} \tag{2}$$

where $R_\rho = \frac{\alpha \partial\overline{T}/\partial z}{\beta \partial\overline{S}/\partial z}$.

(d) For warm salty water overlying cold fresh water, small-scale fluxes of salt and temperature may be due to either turbulent mixing, or salt-fingering. In the turbulent regime, salt and heat are mixed equally efficiently. In the salt fingering regime $(\alpha\overline{w'T'})/(\beta\overline{w'S'}) \approx 0.7$. Find κ_ρ^*/κ_S^* under these two circumstances. Comment on the sign of κ_ρ^*

Problem 2. Stokes drift.

a) Surface gravity waves have $\psi \simeq \cos k(x - ct)\exp(kz)$. Find the particle trajectories at lowest order

$$\frac{\partial}{\partial t}(\boldsymbol{X} - \boldsymbol{x}_0) = \boldsymbol{u}(\boldsymbol{x}_0, t)$$

and then write the next order equations and find the mean displacement over a wave period.

b) Suppose the particle is constrained to a fixed depth z_0. What is its drift then? What if it's a fixed distance below the free surface?