14.03 Fall 2010 Problem Set 6

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Late problem sets are not accepted

1 Adverse Selection in Insurance (10 points)

Consider the following insurance market: there are 100 *risk-neutral* consumers, who have a probability X of getting ill. Where $X \sim U[\frac{1}{4}, \frac{3}{4}]$. $U[.,.]$ is the uniform distribution. X is not observable by the insurance company. If someone gets ill they lose \$1. If the individual has insurance, the insurance company pays \$1 to the ill person, so they are indifferent about getting ill if they have insurance. The consumers VNM utility function is $U(w) = w$. Then we can write the expected utility for an uninsured and insured consumer as:

$$\begin{aligned} U(X, \text{ no insurance}) &= X(-1) + (1-X) \times 0 = -X \\ U(X, \text{insurance}) &= X(-1+1-p) + (1-X)(-p) = -p, \end{aligned}$$

where p is the premium charged by the insurance company. Notice that p has to be the same for all consumers, since the insurance company has no information on the different risks for different people.

1. Define the set of people who would buy insurance if the premium were p. Is this set more or less risky than the full population of 100 consumers?

2. What is the expected probability of illness for those people who buy insurance when the insurance premium is equal to p?

   Hint if $X \sim U[a, b]$, then

   $$E(X|X > T) = \frac{1}{b-T} \int_T^b x dx$$

3. Is your answer to the last part greater or smaller than the premium? Explain.

4. Is there any price that the insurance company could charge where it would make positive profits? Is there any price it could charge where it would break even?

2 Advertising as a signal (12 points)

It is sometimes argued firms use advertising as a signal. The Regal Corporation produces a good for which there are 100 risk-neutral, rational consumers, each of whom would be willing to either buy 1 good at price 10 if the quality of the good is high and zero otherwise (that is, the good is worth 0 if its quality is low). Regal's marginal cost of producing a unit is 6 (regardless of quality). Consumers initially think the quality of the good is equally likely to be high or low. The market is only open for one period. (After which, the world ends)

1. Given consumers' beliefs, will it be profitable for Regal to produce any units?

   Consider next a case where the market is open for two periods (after which, the world ends). If a consumer purchases the product from Regal in the first period, she knows the quality of the product in the first *and* second periods.

2. Assume consumers believe incorrectly that Regal is a high quality firm but in reality it is a low quality firm. How much profit would Regal make in period 1? How much profit would Regal make in the second period?

3. Assume now that Regal is a high quality firm. Regal wants to send a signal to the consumers that it is a high quality firm. It will do this by advertising. The content of the advertising is uninformative *but* consumers can infer the amount that Regal has spent on advertising. Regal spends money on advertising to signal its high quality to consumers. How much money should Regal spend on advertising to signal to consumers that it is a high quality firm? And what will its profits be? [Hint: Recall that if consumers purchase the good in the first period, they will know its quality in the second period.]

4. Now assume that the market will be open for three periods (prior to the world ending). How much should Regal spend in the first period such that it is able to signal it is high quality? What are Regal's profits in this case?

5. In the real world, some firms offer unconditional money back guarantees on their products. These guarantees are expensive to offer since some consumers will return products in non-saleable condition, even if the products were of good quality when they were sold (e.g., a consumer will buy a pair of shoes, wear them on a date, and then return them to the seller for a full refund). Why, in light of the example above, would it be sensible for certain firms to offer these guarantees?

3 Adverse Selection in the Labor Market (12 points)

Suppose that there are two types of workers in the labor force, low ability (Low) and high ability (High) workers, where the fraction of High workers is λ . A worker knows her type. If High workers are employed by a firm they can produce θ_H units of output in one year. If Low workers are employed, they produce θ_L units of output per year, where $\theta_H > \theta_L > 0$. If a worker of any ability level does not work for a firm, she can work at home and, although less productive, she can still produce $c\theta_i$ per year, where $c < 1$, and i is either H or L, for High and Low workers respectively.

Each worker works for just one year and then she retires. Assume that one unit of output,either produced at home or in a firm, sells for \$1.

1. Suppose for now that a firm can observe the ability of the workers it hires. Further assume that there are two perfectly competitive firms that use the same production technology and that employ only labor. The profit that a firm makes for each worker it hires is therefore equal to the value of the output produced by that worker's type minus the wage paid to that worker. Perfect competition implies that the expected profit made on each worker has to be zero in equilibrium.

   Determine the wage that the firms will offer to each type of worked in a perfect competitive equilibrium? Are workers willing to accept the job offers at those wages, or would they rather produce at home? Compute the total output in the economy (scaled by the number of workers) in one year.

2. Suppose from now on that there is asymmetric information. That is, firms have no way to know workers' ability. Since firms cannot distinguish between the two types of workers, they offer only one wage rate, which, by perfect competition, in equilibrium has to be equal to the expected productivity of the workers that accept the job offers. If the firms believed that only Low workers would accept the job offers, what wage should they pay? Would Low workers accept this wage, or would they rather work at home?

3. If the firm believed that both types of workers would accept the job offers, what would the offered wage be? Would Low workers accept this offer? Would High workers accept the offer? What would High workers' decision depend on?

4. If $\lambda = 0.5, c = 0.75, \theta_H = 4$, and $\theta_L = 1$ what is the equilibrium wage and what types of workers are employed in the firms in equilibrium?

5. Compute the total output in the economy if the equilibrium is the one obtained in part (4). Compare your result to the one you derived in part (1).

6. Explain how asymmetric information has affected the equilibrium in this economy.

4 Credit Crunch (30 points)

Definitions:

- Hard information: Information that can easily be contracted upon e.g., a credit score
- Soft information: Information that is not easily contracted upon and may require effort to learn

There are two time periods, $t \in \{0, 1\}$. In time period $t = 0$, households apply for a mortgage. If they obtain a mortgage they have to pay it back in $t = 1$. Households have a risky investment that either pays a high or low dividend $I \in \{H, L\}$. If $I = H$, they will pay back their mortgage. If $I = L$, they will default on the mortgage and the lender will receive zero. The amount of the loan is normalized to \$1 for all households:

Households vary in their propensity to default:

$$\Pr(\text{Default}|X_i) = \Pr(I = L|X_i) = \frac{1}{X_i} \quad \text{where} \quad X_i \sim U\left[\frac{4}{3}, 4\right],$$

and $U[\cdot, \cdot]$ denotes the uniform distribution.

We can think, for example, of X_i as the FICO score in the Keys et al. (QJE 2010) paper on your reading list.

1. Draw a plot with the probability of defaulting on the y-axis and the inverse probability of default (X_i) on the x-axis.

2. Assume the probability of default for each consumer, $1/X_i$, is observable to the lender, i.e. it is *hard information*. Assume the lender must charge an actuarially fair mortgage price, T, to each household—that is, the amount the lender expects to collect is exactly equal to the amount loaned. Calculate the payment the household must make if their investment comes out high ($I = H$) as a function of their observable signal (X_i).

3. Now assume that X is not observable but the lender knows the distribution of types in the economy. Therefore, the lender must charge the same premium to all borrowers. Assume that all consumers receive a loan. In this pooling equilibrium, what is value of the price T, such that the lender makes no profits?

   Hint 1: if Y is uniformly distributed on $[a, b]$ then for some function $g(\cdot)$:

   $$E(g(y)) = \frac{1}{b-a}\int_a^b g(y)dy$$

   Hint 2: $\int \frac{X-1}{X}dx = X - \ln X$

4. Let us return to the scenario where X_i is observable. Now assume there is an another dimension of heterogeneity Z_i: Z_i and X_i are independent of each other. We could think of Z_i as the particular career of the borrower. Some careers are more risky than others. Therefore, there is some other dimension which will affect the probability of defaulting as opposed to just the FICO score. Assume that this dimension Z_i is costlessly observable to the lender.

$$Z_i \sim U\left[-\frac{1}{4}, \frac{1}{4}\right] \quad (1)$$

$$P(\text{default}|X, Z) = \frac{1}{X+Z} \quad (2)$$

If the lender had to charge the same premium $T(X)$ as you calculated in part (1), for what values of Z_i would the lender be willing to lend?

5. Now assume the lender can sell all loans with $X_i > 2$ to a broker at time $t = 0.5$. In other words, the lender can securitize the loans and sell them on to a third party. The broker buys the loans from the lender at price $T(X) + 10$, hence the lender always prefers to sell the portfolio of mortgages for which $X_i > 2$. However, the lender cannot sell the loans with $X_i < 2$ to the broker. The lender must keep these loans.

   As in question (1) above, the lender must still charge the ex-ante expected break-even price to all borrowers who have $X_i \leq 2$. However, the lender can refuse to offer a loan.

   What is the probability that an individual with $X_i > 2$ will receive a loan?

   What is the probability that an individual with $X_i \leq 2$ will receive a loan?

6. Now let us look at the borrowers with $X_i \approx 2$. Denote $X^+ \equiv 2 + \epsilon$, $X^- \equiv 2 - \epsilon$ where ϵ is a very small number. Therefore X^+, are the borrowers with scores just slightly higher than 2.

   What is the probability of default for X^+? What is the probability of default *for those who obtain a loan* and X^-? Explain the intuition for why these differ.

   Hint:

$$\text{if } Z \text{ is } U[a, b] \text{ then } E(g(Z)|Z > \kappa) = \frac{1}{b-\kappa}\int_{\kappa}^{b} g(Z)dZ$$

   Empirical Part

   We wish to test whether the lender's ability to securitize the loans will affect its screening standards. In particular, we want to estimate the effect of securitization on default probabilities for a given sample of loans.

   We take a sample of 20,000 people who were offered loans, both securitized and not securitized. Assume you have data on whether or not the borrowers defaulted on their loans, their credit score X and whether or not their loan was securitized. Let $Y_i = 1$ if the person defaulted on the loan and 0 otherwise. As in the model above, assume that loans are securitized for $X_i > 2$ and not securitized for $X_i \leq 2$.

7. Your data shows that people whose loans were securitized were less likely to default on their loans. Specifically:

$$E[Y|S = 1] = .4, E[Y|S = 0] = .6$$

   Explain why a simple comparison of $E[Y|S = 1]$ and $E[Y|S = 0]$ would not provide a valid estimate of the effect of securitization on screening practices. Would you expect -0.2 to be an overestimate or underestimate of the effect of securitization on screening practices, and why?

8. What sign do you expect the true effect of securitization on default probabilities to be, if the lender does not screen the borrowers on Z_i when $X_i > 2$?

9. Draw a graph with X_i on the horizontal axis and the probability of default Y on the vertical axis. Plot the relationship you would expect to see between X and Y if (1) securitization does not affect screening practices; and (2) securitization *does* affect screening practices. *Note: This does not have to be drawn to scale.*

10. How would you calculate a regression discontinuity estimate, $\hat{T}$, of the effect of securitization on screening practices (measured as the probability of default)? Using appropriate causal notation, state the assumptions required for $\hat{T}$ to be a valid estimate of the effect of securitization on the probability of default. (You may refer to your diagram in part 9).

    Now consider whether each of the below effects will affect our estimation strategy, i.e. determine whether it will affect the assumptions necessary for the regression discontinuity estimate $\hat{T}$ to be a valid estimate. If it will affect our estimator, state the likely direction of bias.

11. Assume we do not have a sufficient number of borrowers on the borderline of $X_i = 2$, so that we are are forced to use the average probability of default of those with $X^- = [1.5, 2-\epsilon]$ and $X^+ = [2+\epsilon, 2.5]$. Will this affect our estimate of $\hat{H}$? Explain.

12. You learn that due to a technical error, 50% of people with $X_i = 2+\epsilon$, were screened by the lender and if their Z_i was sufficiently low, their loan was rejected. In other words their loans were treated as though they were X^-. In this case, do you think that $\hat{T}$ overstates, understates the effect of securitization on screening practices (or is it indeterminate)? Explain.

5 The mating behavior of bullfrogs: (12 points)

Male bullfrogs attract mates (females) by having the deepest croak. Consider a setting where there are 100 male bullfrogs, each with a different croak, and 100 female bullfrogs. Bullfrogs have the option to croak or not. On the croak-o-meter we find, remarkably, that each bullfrog has a different depth of croak, c. Specifically,

$$c_i \in C = [1, 2, 3 \ldots, 100]$$

Where $c = 100$ is the meanest, deepest croak of them all.

1. Let us assume only that those bullfrogs with $c_i > \bar{c}$ decide to croak out loud. What is the expected croakiness of a bull frog that does not croak out loud?

2. Given that bullfrogs would only be willing to croak if their croak is louder than the expected croak given not croaking, what would be the value of $\bar{c}$ in equilibrium? Which bullfrogs croak in equilibrium?

    Now consider the following example in light of the above question

3. The Commonwealth of Massachusetts is concerned that employers discriminate against job applicants who have a past history of arrest, even if those applicants had never been convicted of a crime. The employers' lobby argues that the Commonwealth has no reason for concern because criminal background checks are strictly voluntary; any job applicant who feels uncomfortable can simply decline the background check. Explain the flaw in this argument.

6 Crying Fire (12 points)

Bert is moving into his new apartment and deciding on what to do about the heat in his apartment. Bert has VNM expected utility function:

$$U_B(\text{Heat,Income}) = -(\kappa - H)^2 + (1 - \Pr(\text{fire}))(I - R)^{0.5} + \Pr(\text{fire})\,(I - R - L)^{0.5},$$

where H =Heat, $I = Income = (16 + 5\kappa)$, R =Rent, $\kappa = 0.5$ is a parameter, and L is the amount of loss in the event of fire. The more heat Bert uses, the greater the probability of fire. Specifically, $\Pr(\text{fire}|H) = H$.

TO SIMPLIFY SOME OF THE MATHS, YOU CAN ASSUME $\kappa = 0.5$

Bert is risk averse and his landlord is risk-neutral. Therefore, the landlord initially proposes that she will cover the losses if there is a fire.

The Landlord's profit function is equal to the rent paid R minus a loss of $L = 5$ if there is a fire. This Loss is paid by the landlord, not by Bert, since the landlord is insuring Burt. Thus, the landlord's expected profits are:

$$E(\pi) = R - \Pr(\text{fire}) \times L$$

(Since the landlord is risk neutral, expected profits and actual profits can be treated identically. Over a sufficiently large number of years, expected and actual profits will be identical, though in the short term, they may sometimes be higher and sometimes lower.)

Assume further that the landlord has an outside option of $\bar{\pi} = 7$. That means that she will not accept profits lower than 7 because she could rent the apartment for some other purpose (e.g., a server farm) and earn profits of 7.

The timing of the action is as follows: (1) Landlord and Bert agree on a rent; (2) Bert uses heat; (3) With some probability there is a fire.

1. Under the initial agreement with the landlord providing full insurance, how much heat will Bert use? How much rent will he have to pay such that the landlord is willing to rent the property to him given the probability of fire? What is Bert's utility?

   HINT: To find the equilibrium, note that the landlord must calculate how much heat Bert will use and infer the subsequent probability of a fire before he decides how much rent to charge Bert.

2. Now assume the landlord is fed up with the Bert constantly burning the house down. He tells Bert he will lower the rent—**but** Bert has to pay for any losses caused by a fire. Therefore, the landlord will now charge Bert 7 in rent. How much heat will Bert use now? Is this more or less than before? What is Bert's resulting utility? Which of the two arrangements does he prefer?

3. Given your understanding of risk aversion, explain the intuition for what is happening in this equilibrium. (Your explanation should account for the fact that Bert is risk averse and the landlord is risk neutral.)

7 Application: Tyler, Murnane, and Willett (TMW) study (12 points)

As we discussed in class, Human Capital acquisition and Signaling are two competing explanations for why people with more education receive higher earnings. The Human Capital model posits that people acquire skills in school that make them more productive in their jobs, yielding higher wages.

1. At the top of page 435, TMW sketch in words the necessary conditions for a GED signaling equilibrium to hold. Write out the key equations for these conditions. If the GED were offered for free, was not unpleasant to take, and required no preparation time (perhaps because it measures 'ability' rather than knowledge), could the signaling equilibrium ever hold? Why or why not?

2. The TMW study provides evidence that the GED holds a pure signaling value of approximately $1,500 for people just above the passing score versus those just below the passing score. Does the TMW study therefore demonstrate that the human capital model of education as a productivity enhancing mechanism is incorrect? Please answer in a few precise sentences.

3. The U.S. Undersecretary of Education proposes the following policy to boost the low earnings of U.S. high school dropouts: the Department of Education will grant each high school dropout a GED immediately. In a few years time, this should raise their earnings by about $1,500, just as TMW demonstrate. Explain whether and why this policy is likely to yield the intended result.

4. The U.S. Federation of Picky Employers (FOPE) is distressed because its members can no longer determine which GED holders have actually passed the test and which were simply issued the certificate by the Department of Education. Of course, every GED holders knows whether or not he or she actually passed the test, but they all claim to have passed it. FOPE proposes charging a price P to each worker who would like to take the GED. Assume that the pure disutility of taking the test is \$100. FOPE's goal is to maximize profits for its members. What price should it charge for retesting each worker who volunteers to be tested? Does retesting raise, lower, or leave unaffected total social welfare, or is this indeterminant? Explain.

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