14.05 Lecture Notes

Labor Supply

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One-period Labor Supply Problem

- So far we have focused on optimal consumption and saving. Let us now shift the focus to labor supply. To do this within our micro-founded, neoclassical framework, we only need to introduce leisure as an additional good.

- You have previously studied the static labor supply problem of a household that lives only one period and decides how much labor to supply in that period. This looks as follows:

$$\max_{c,\ell} U(c, \bar{z} - \ell)$$

$$\text{s.t.} \quad c = a + w\ell$$

  where c is consumption, ℓ is labor supply, $\bar{z}$ parameterizes the overall time that is available for work or leisure, $z = \bar{z} - \ell$ is leisure, w is the wage, $w\ell$ is labor income, and a are assets or other sources of income.

- This can also be restated as

$$\max_{c,\ell} U(c,z)$$
$$\text{s.t.} \quad c + wz = a + w\bar{z}$$

The above underscores that the real wage w is the relative price of the leisure good over the consumption good, and that the "wealth" of the household includes both the endowment of consumption goods, a, and the value of the endowment of time, $\bar{z}$.

- Set up the Lagrangian and let μ be the Lagrange multiplier. Assuming an interior solution $(0 < z < \bar{z})$, the FOCs with respect to c and z (or ℓ) give, respectively,

$$\begin{aligned} U_c(c,z) &= \mu \\ U_z(c,z) &= \mu w \end{aligned}$$

The optimal consumption and labor supply decisions are then given by the solution of the above two FOCs along with the budget constraint.

- Combining the two FOCs we get

$$\frac{U_z(c,z)}{U_c(c,z)} = w$$

which simply says that the MRS between consumption and leisure should be equate with the relative price of leisure, which is the real wage w. Equivalently,

$$U_z(c,z) = wU_c(c,z)$$

which means that the disutility of an extra unit of labor is equated to the marginal utility of the extra consumption afforded by the income generated.

- Now suppose that the wage rate w increases. What happens to the optimal labor supply? Opposing income and substitution effects. The substitution effect (=the relative price of leisure went up) contributes to an increase in labor supply. The income effect (=the value of the endowment of time went up) contributes to a decrease in labor supply. The overall effect depends on weather income or substitution effects dominate. At the same time, consumption necessarily increases, because both effects work in the same direction.

- To see this more clearly, take the special case in which

  $$U(c, z) = \log c + \gamma \log z.$$

  Suppose further that the household has no resources other than labor income (meaning $a = 0$). Then, the budget implies $c = w\ell$ and therefore

  $$U(c, z) = \log w + \log \ell + \gamma \log(\bar{z} - \ell)$$

  It is then immediate that the optimal labor supply is given by $\ell = \ell^*$, where

  $$\ell^* = \arg\max_{\ell} \left\{\log \ell + \gamma \log(\bar{z} - \ell)\right\}$$

  is invariant to w. This is therefore an example in which income and substitution effects perfectly offset each other, making labor supply insensitive to wealth.

- But now suppose that $a > 0$. Then, it is easy to verify that ℓ now increases with w. Intuitively, now that the household has other sources of income, the wealth effect of a higher wage is relatively weaker, and the substitution effect dominates. So, the richer the household (in terms of non-labor income), the more elastic its labor supply might be.

Multi-period Labor Supply Problem

- Consider now a household that lives for two periods. To simplify the exposition, set $a_0 = 0$ and normalize $q_0 = 1$ (in which case $q_1 = 1/(1+R_1)$). Following similar steps as when we analyzed the optimal consumption-savings problem, the intertemporal problem of the household can now be formalized as follows:

$$\max U(c_0, z_0) + \beta U(c_1, z_1)$$

$$s.t. \quad c_0 + w_0 z_0 + q_1 c_1 + q_1 w_1 z_1 = x_0$$

where

$$x_0 = w_0 \bar{z} + q_1 w_1$$

Note that, for $t \in \{0, 1\}$, w_t is the relative price of leisure in period t relative to consumption in the *same* period, while q_1 is the relative price of consumption in $t = 1$ relative to $t = 0$. It follows that $q_1 w_1 / w_0$ is the relative price of leisure at $t = 0$ relative to leisure at $t = 0$.

- Let μ be, once again, the Lagrange multiplier. The FOCs now give

$$\begin{aligned} U_c(c_0, z_0) &= \mu && (1) \\ U_z(c_0, z_0) &= \mu w_0 && (2) \\ \beta U_c(c_1, z_1) &= \mu q_1 && (3) \\ \beta U_z(c_1, z_1) &= \mu q_1 w_0 && (4) \end{aligned}$$

  The optimal plan is given by the solution to the above FOCs together with the intertemporal budget constraint.

- From (1) and (2), we get

$$U_z(c_0, z_0) = w_0 U_c(c_0, z_0)$$

  and similarly from (1) and (2) we get

$$U_z(c_1, z_1) = w_1 U_c(c_1, z_1)$$

  This is the "static" optimality condition for labor supply that we have encountered before, now stated for each period.

- At the same time, combining (1) and (3) we get

$$U_c(c_0, z_0) = (1 + R)\beta U_c(c_1, z_1),$$

which is our familiar intertemporal Euler condition.

- So far nothing essential new. However, not that the labor supply and saving decisions are not disconnected. In particular, the household is now able to substituted leisure (and labor supply) intertemporally: if he wishes, he can work had in one period, take a vacation in the following period, and nevertheless sustain a high level of consumption in both periods by saving much of his first-period labor income. By the same token, a certain intertemporal optimality condition holds for leisure (or labor supply) just as for consumption. To see this, combine (2) and (4) to get

$$\frac{U_z(c_0, z_0)}{\beta U_z(c_1, z_1)} = \frac{w_0}{q_1 w_1}$$

This simply says that the MRS between leisure at $t = 0$ and leisure at $t = 0$ is equated with the relevant price ratio. Equivalently,

$$U_z(c_0, z_0) = \beta(1 + R)\frac{w_0}{w_1}U_z(c_1, z_1)$$

which looks like our familiar Euler condition for consumption, except for two differences: it regards leisure rather than the consumption of goods; and the relative wage ratio of the two periods shows up along with the interest rate.

- Think now of the relative substitution effects. What happens to labor supply in each period when R increases? When w_0 increases? When w_1 increases?

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