14.126 Game theory
Problem Set 5

The due date for this assignment is Monday May 7.

1. Consider a two-player Bayesian game in which the payoffs are as in the following table

	a	b
a	θ, θ	$\theta - 1, 0$
b	$0, \theta - 1$	$0, 0$

Each player i observes a signal $x_i = \theta + \varepsilon\eta_i$ where $\varepsilon \in (0,1)$ and θ_1, η_1 and η_2 are normally and independently distributed random variables. Unlike in the example considered in the class, assume that players have differing beliefs about $(\theta_1, \eta_1, \eta_2)$. According to each player i, $\theta \sim N(\mu_i, 1)$, $x_i \sim N(0,1)$, and $x_j \sim N(\beta, 1)$, where μ_1 and μ_2 may differ and β can be non-zero.

   (a) Compute the set of (interim) rationalizable actions for each type.

   (b) What happens to the set of rationalizable actions as $\varepsilon \to 0$?

2. Two firms, namely 1 and 2, are in a generalized Cournot competition. The goods they sell are "price-theory" complements, such as sugar and coffee. Simultaneously, each firm i produces q_i at total cost $C(q_i)$ and sells at market-clearing price $\theta P(q_i, q_j)$ where q_j is the supply of the other firm and θ is a demand parameter. (The firms are symmetric.)

   (a) Find conditions on P and C and on the set of possible supplies under which there exists a weakly increasing strategy profile $q^* : (x_1, x_2) \mapsto (q_1^*(x_1), q_2^*(x_2))$ such that for any family of rationalizable strategies q_i^ε, $\lim_{\varepsilon \to 0} q_i^\varepsilon(x_i) = q_i^*(x_i)$ almost everywhere.

   (b) Take $C(q_i) = cq_i$ for some $c > 0$ and $P(q_i, q_j) = q_i^{-\alpha} q_j^{\beta}$ for some $\alpha, \beta \in (0,1)$ with $\alpha > \beta$. Compute q^*.

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