## 114.126 (Game Theory) Final Examination

**Instructions**: This is an open-book exam – you may consult written material but you may not consult other humans. There are five questions, weighted equally. You have 48 hours to complete the exam from the time you first open the envelope. When you have finished, place your answers in the envelope and return it to Professor Yildiz.

Please begin your answer to each question on a new page.

**114.126 (Game Theory) Final Examination**

**Instructions**: This is an open-book exam – you may consult written material but you may not consult other humans. There are five questions, weighted equally. You have 48 hours to complete the exam from the time you first open the envelope. When you have finished, place your answers in the envelope and return it to Professor Yildiz.

Please begin your answer to each question on a new page.

1. $(N, v)$ is a simple side-payment game, where $N$ is the set of natural numbers $\{1, 2, ...\}$, and $S$ is a winning coalition if and only if its complement is a finite set. Prove that $(N, v)$ has no stable sets.

2. A finite number $n$ $(\geq 2)$ of citizens is eligible to vote in an election between candidate $B$ and candidate $G$. Each citizen $i$ prefers one or the other candidate strictly; the identity of $i$'s preferred candidate is a random draw, with probability $1/2$ each for $B$ and $G$, independent of all others' draws. Each citizen $i$ also incurs an idiosyncratic cost $c_i$ of voting, so that $i$'s utility is:

   1 if $i$'s preferred choice wins and $i$ does not vote;

   $1 - c_i$ if $i$'s preferred choice wins and $i$ votes;

   0 if $i$'s preferred choice loses and $i$ does not vote;

   $-c_i$ if $i$'s preferred choice loses and $i$ votes.

   The $c_i$ are drawn independently and identically according to a common density $f$ having support $[\underline{c}, \overline{c}]$, where $0 \leq \underline{c} < \overline{c} \leq 1$. All citizens privately realize their own choices of candidate and idiosyncratic cost, then they simultaneously and independently decide whether and for whom to vote. (The common prior is of course, common knowledge.)

   1. Say all you can about the existence and uniqueness of a symmetric Bayes-Nash equilibrium (i.e., a Bayes-Nash equilibrium in which each citizen uses the same strategy).

2. If $f$ is the uniform density on $[0,1]$, compute all the symmetric Bayes-Nash equilibria that you can.

3. Another symmetric game played by a finite number $n$ ($\geq 2$) of players. Each player has one potential move to make; the options are whether to make it at all, and, if so, when. The possible moving times are $\{0,1,2,...\}$. If $i$ moves at $t$, then $i$ has no further decisions to make after $t$. At each time $t$, decisions of those who are still eligible to move are made simultaneously with complete knowledge of all prior decisions.

   Payoffs: There is a discount parameter $\delta$ ($0 \leq \delta < 1$), a quota parameter $m$ ($m$ is a natural number less than $n$) and a moving cost parameter $c$ ($0 < c < 1$). If $i$ never moves or if she moves only after $m$ others have already moved, her payoff is 0. Otherwise, if she moves at $t$, her payoff is

   $-\delta^t c$ if the total number of movers ever in the game is less than $m$;

   $1 - \delta^t c$ if the total number of movers in the game is at least $m$ and the quota is reached (i.e., the $m$th move occurs) after $t$.

   $\left(1 - \delta^t c\right) s/k$ if the number of movers at $t$ is $k$, the quota is reached at $t$, and $s$ ($\leq k$) movers were needed at $t$ to exactly reach the quota.

   Find all subgame-perfect equilibria as functions of the parameters $n, m, \delta$, and $c$.

4. There are three players, 1,2, and 3, and three states, $a$, $b$, and $c$. The information partitions of players 1,2, and 3 are $\{\{a\},\{b\},\{c\}\}$, $\{\{a,b\},\{c\}\}$, and $\{\{a\},\{b,c\}\}$, respectively. There is a (random) variable $G$ such that $G(a) = G(b) = -1$, and $G(c) = 2$. Is it common knowledge at state $a$ that $G$ is $-1$?

5. In problem 4, assume that players have a common prior, according to which each state is equally likely. Under this information structure, players play the following game. First 1 chooses between Left and Right. If he chooses Left, the game ends, yielding the payoff vector (1,1,1). If he chooses Right, then 2 is to choose between Left and Right. If 2 chooses Left, the game ends, when the payoff vector is (0,2,0). If 2 chooses Right, then 3 is to

choose between Left and Right, ending the game. If 3 chooses Left, the payoff vector is $(2, 0, G)$; if he chooses Right the payoff vector is (3,3,0). Confining to state $a$, describe all sequential equilibria.

MIT OpenCourseWare
http://ocw.mit.edu

14.126 Game Theory
Spring 2010

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.