# 14.127 Behavioral Economics (Lecture 2)

Xavier Gabaix

February 12, 2004

# 0.1 Cumulative PT

Remind from last lecture: for continuous gambles with distribution f(x)
EU gives:

$$V = \int_{-\infty}^{+\infty} u(x) f(x) dx,$$

PT gives:

$$V = \int_0^{+\infty} u(x) f(x) \pi' (P(g \ge x)) dx$$
$$+ \int_{-\infty}^0 u(x) f(x) \pi' (P(g \le x)) dx$$

• Alternatively, we can write it as Riemann-Stieltjes integral

$$V = -\int_0^{+\infty} u(x) d\pi (1 - P(g < x))$$
$$+ \int_{-\infty}^0 u(x) d\pi (P(g \le x))$$

• This simplifies to PT for two outcome gambles. Indeed, it is selfevident in the Riemann-Stieltjes form.

# 1 The endowment effect – a consequence of PT

- Lab experiment, Kahneman, Knetsch, Thaler, JPE 1990.
  - Half of the subjects receives an MIT apple, and the other half receives \$10.
  - Then willingness to pay WTP for the apple is elicited from subjects with money, and willingness to accept WTA is elicited from subjects with mugs.
- In EU we have WTP = WTA (modulo wealth effects, which are small)

• In simplified (linear) PT value getting an apple and lose x is

$$V = u$$
 (apple) +  $u(-x) = A - \lambda x$ 

(note—there are mental accounting ideas plugged in here that is we process apple and money on separate accounts).

• Thus, in PT, one accepts when

$$A - \lambda x \ge \mathbf{0}$$

so that

$$WTA = \frac{A}{\lambda}.$$

• In simplified (linear) PT value losing an apple and gaining x is

$$V = u (-apple) + u (x) = -\lambda A + x$$

(note, once more time we process apple and money on separate accounts).

• Thus, in PT, one pays when

$$-\lambda A + x \ge \mathbf{0}$$

so that

$$WTP = \lambda A.$$

• Thus, PT gives stability to humane life, a status quo bias.

## **1.1 Endowment effect experiment with mugs**

- Classroom of one hundred. Fifty get the mug, fifty get \$20.
- One does a call auction in which people can trade mugs.
- Trading volume "rational" expectation would be that the average trading volume should be  $\frac{1}{2}50 = 25$ . Everybody has a valuation, and probability that someone with valuation higher than the market price is  $\frac{1}{2}$ .
- If WTP<WTA then the trading volume is lower than  $\frac{1}{2}$ .
- In experiments, the trading volume is about  $\frac{1}{4}$ .

# **1.2 Open questions with PT**

#### **1.2.1** Open question 1: Narrow framing

- N independent gambles:  $g_1, ..., g_N$
- For each i do you accept  $g_i$  or not?
- In EU call  $a_i = 1$  if accept  $g_i$  and  $a_i = 0$  otherwise. Your total wealth is

$$W_0 + a_1 g_1 + \ldots + a_N g_N$$

and you maximize

$$\max_{a_1,...,a_N} Eu \left( W_0 + a_1 g_1 + ... + a_N g_N \right).$$

• In PT we have at least two possibilities

- Separation: 
$$a_i = 1$$
 iff  $V^{PT}(g_i)$ .

- Integrative: solve  $\max_{a_1,\ldots,a_N} V^{PT} (a_1g_1 + \ldots + a_Ng_N)$ .
- Separation is more popular, but unlikely in for example in stock market, or venture capital work.
- KT don't tell whether integration or separation will be chosen. That is one of the reasons PT has not been used much in micro or macro.
- How to fix this problem?

- Integration as far as possible subject to computational costs.
- Natural horizon between now and when I need to retire.
- Do what makes me happier, max (separation, integration). That would be an appealing general way to solve the problem.
  - \* Problem, each everyday gamble is small against the background of all other gambles of life.
  - \* So, an EU maximizer would be locally risk neutral.
  - \* And also a PT maximizer would be locally risk neutral whenever he or she accpets integrationist frame.

#### 1.2.2 Horizon problem — a particular case of the framing problem

- Stock market.
  - Yearly values

standard deviation  $\sigma T^{\frac{1}{2}} = 20\%$  per year where  $T \simeq 250$  days,

mean 
$$\mu T = 6\%$$
 per year.

- Daily values

$$\sigma = \frac{20\%}{250^{\frac{1}{2}}} \\ \mu = \frac{6\%}{T}$$

- Assume that a PT agent follows the rule: "accept if  $\frac{\text{Risk premium}}{\text{St. dev.}} > k$ " (PS1 asks to show existence of such an PT agent).
- So, a PT agent with yearly horizon invests if

$$\frac{6\%}{20\%} > k^*$$

- A PT agent with daily horizon invests if

$$\frac{\mu}{\sigma} = \frac{.024}{1.3} \simeq .01 << k^*$$

- This is not even a debated issue, because people don't even know how to start that discussion
- Kahneman says in his Nobel lecture that people use "accessible" horizons.

- \* E.g. in stock market 1 year is very accessible, because mutual funds and others use it in their prospectuses.
- \* Other alternatives time to retirement or time to a big purchase. or "TV every day".
- In practice, for example Barberis, Huang, and Santos QJE 2001 postulate an exogenous horizon.

- **1.2.3** Open question 2: Risk seeking
  - Take stock market with return  $R = \mu + \sigma n$  with  $n \sim N(0, 1)$ .
  - Invest proportion  $\theta$  in stock and  $1 \theta$  in a riskless bond with return 0.
  - Total return is

$$\theta R + (1 - \theta) \mathbf{0} = \theta (\mu + \sigma n).$$

• Let's use PT with  $\pi(p) = p$ . The PT value is

$$V = \int_{-\infty}^{+\infty} u \left(\theta \left(\mu + \sigma n\right)\right) f(n) dn$$

- Set  $u(x) = x^{\alpha}$  for positive x and  $-\lambda |x|^{\alpha}$  for negative x.
- $\bullet \ \mbox{Using homothecity of } u$  we get

$$V = \int_{-\infty}^{+\infty} |\theta|^{\alpha} u (\mu + \sigma n) f(n) dn$$
$$= |\theta|^{\alpha} \int_{-\infty}^{+\infty} u (\mu + \sigma n) f(n) dn$$

- Thus optimal  $\theta$  to equals 0 or  $+\infty$  depending on sign of the last integral.
- Why this problem? It comes because we don't have concave objective function. Without concavity it is easy to have those bang-bang solutions.

• One solution to this problem is that people maximize  $V^{EU} + V^{PT}$ .

#### **1.2.4** Open question 3: Reference point

Implicitly we take the reference point to be wealth at t = 0. Gamble is W<sub>0</sub> + g and

$$V^{PT} = V^{PT} \left( W_0 + g - R \right)$$

- But how  $R_t$  evolves in time?
- In practice, Barberis, Huang, and Santos QJE 2001 (the most courageous paper) postulate some ad hoc exogenous process. People gave them the benefit of a doubt.

#### **1.2.5** Open question 4: Dynamic inconsistency

- Take a stock over a year horizon. Invest 70% on Jan 1st, 2001.
- It's Dec 1, 2001. Should I stay invested?
- If the new horizon is now one month, I may prefer to disinvest, even though on Jan 1, 2001, I wanted to keep for the entire year.
- By backward induction, Jan 1 guy should disinvest!

#### **1.2.6** Open question 5: Doing welfare is hard

- Why? Because it depends on the frame.
- Take T = 250 days of stock returns  $g_i \sim N(\mu, \sigma^2)$ . Integrated  $V^{PT}(\sum g_i) = V^I$  and separated  $V^{PT} = V^S$ .
- The cost of the business cycle (Lucas). Suppose *c* =average monthly consumption. Assume simple consumption shocks:

$$c_t = c + \varepsilon_t$$

with normal iid  $\varepsilon_t$ .

• What is PT reference point? Take  $R_t = c = 0$ .

• With PT integrated over one year

$$V^{PT}\left(\sum \varepsilon_t\right) = V^{PT}\left(12^{\frac{1}{2}}\sigma_{\varepsilon}n\right) = \left(12^{\frac{1}{2}}\sigma_{\varepsilon}\right)^{\alpha}V^{PT}\left(n\right) < 0.$$

• With segregated PT

$$V^{PT} = 12\sigma_{\varepsilon}^{\alpha}V^{PT}(n)$$

• Which frame is better?

### 1.3 Next time

- Lucas calculation of costs of business cycle. In practice people care about business cycles, and election are decided on those counts.
- Problem Set next time. One question try to circumvent one of the problems.
- Readings on heuristics and biases, the Science 74 KT article and Camerer's paper from the syllabus.