# 14.127 Behavioral Economics (Lecture 2) 

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### 0.1 Cumulative PT

- Remind from last lecture: for continuous gambles with distribution $f(x)$
EU gives:

$$
V=\int_{-\infty}^{+\infty} u(x) f(x) d x
$$

PT gives:

$$
\begin{aligned}
V & =\int_{0}^{+\infty} u(x) f(x) \pi^{\prime}(P(g \geq x)) d x \\
& +\int_{-\infty}^{0} u(x) f(x) \pi^{\prime}(P(g \leq x)) d x
\end{aligned}
$$

- Alternatively, we can write it as Riemann-Stieltjes integral

$$
\begin{aligned}
V & =-\int_{0}^{+\infty} u(x) d \pi(1-P(g<x)) \\
& +\int_{-\infty}^{0} u(x) d \pi(P(g \leq x))
\end{aligned}
$$

- This simplifies to PT for two outcome gambles. Indeed, it is selfevident in the Riemann-Stieltjes form.


## 1 The endowment effect - a consequence of PT

- Lab experiment, Kahneman, Knetsch, Thaler, JPE 1990.
- Half of the subjects receives an MIT apple, and the other half receives $\$ 10$.
- Then willingness to pay WTP for the apple is elicited from subjects with money, and willingness to accept WTA is elicited from subjects with mugs.
- In EU we have $W T P=W T A$ (modulo wealth effects, which are small)
- In simplified (linear) PT value getting an apple and lose $\$ x$ is

$$
V=u(\text { apple })+u(-x)=A-\lambda x
$$

(note-there are mental accounting ideas plugged in here that is we process apple and money on separate accounts).

- Thus, in PT, one accepts when

$$
A-\lambda x \geq 0
$$

so that

$$
W T A=\frac{A}{\lambda}
$$

- In simplified (linear) PT value losing an apple and gaining $\$ x$ is

$$
V=u(- \text { apple })+u(x)=-\lambda A+x
$$

(note, once more time we process apple and money on separate accounts).

- Thus, in PT, one pays when

$$
-\lambda A+x \geq 0
$$

so that

$$
W T P=\lambda A
$$

- Thus, PT gives stability to humane life, a status quo bias.


### 1.1 Endowment effect experiment with mugs

- Classroom of one hundred. Fifty get the mug, fifty get $\$ 20$.
- One does a call auction in which people can trade mugs.
- Trading volume - "rational" expectation would be that the average trading volume should be $\frac{1}{2} 50=25$. Everybody has a valuation, and probability that someone with valuation higher than the market price is $\frac{1}{2}$.
- If $\mathrm{WTP}<\mathrm{WTA}$ then the trading volume is lower than $\frac{1}{2}$.
- In experiments, the trading volume is about $\frac{1}{4}$.


### 1.2 Open questions with PT

### 1.2.1 Open question 1: Narrow framing

- $N$ independent gambles: $g_{1}, \ldots, g_{N}$
- For each $i$ do you accept $g_{i}$ or not?
- In EU call $a_{i}=1$ if accept $g_{i}$ and $a_{i}=0$ otherwise. Your total wealth is

$$
W_{0}+a_{1} g_{1}+\ldots+a_{N} g_{N}
$$

and you maximize

$$
\max _{a_{1}, \ldots, a_{N}} E u\left(W_{0}+a_{1} g_{1}+\ldots+a_{N} g_{N}\right) .
$$

- In PT we have at least two possibilities
- Separation: $a_{i}=1$ iff $V^{P T}\left(g_{i}\right)$.
- Integrative: solve $\max _{a_{1}, \ldots, a_{N}} V^{P T}\left(a_{1} g_{1}+\ldots+a_{N} g_{N}\right)$.
- Separation is more popular, but unlikely in for example in stock market, or venture capital work.
- KT don't tell whether integration or separation will be chosen. That is one of the reasons PT has not been used much in micro or macro.
- How to fix this problem?
- Integration as far as possible subject to computational costs.
- Natural horizon between now and when I need to retire.
- Do what makes me happier, max (separation, integration). That would be an appealing general way to solve the problem.
* Problem, each everyday gamble is small against the background of all other gambles of life.
* So, an EU maximizer would be locally risk neutral.
* And also a PT maximizer would be locally risk neutral whenever he or she accpets integrationist frame.
1.2.2 Horizon problem - a particular case of the framing problem
- Stock market.
- Yearly values
standard deviation $\sigma T^{\frac{1}{2}}=20 \%$ per year where $T \simeq$ 250days,

$$
\text { mean } \mu T=6 \% \text { per year. }
$$

- Daily values

$$
\begin{aligned}
\sigma & =\frac{20 \%}{250^{\frac{1}{2}}} \\
\mu & =\frac{6 \%}{T}
\end{aligned}
$$

- Assume that a PT agent follows the rule: "accept if $\frac{\text { Risk premium }}{\text { St. dev. }}>$ $k^{\prime \prime}$ (PS1 asks to show existence of such an PT agent).
- So, a PT agent with yearly horizon invests if

$$
\frac{6 \%}{20 \%}>k^{*}
$$

- A PT agent with daily horizon invests if

$$
\frac{\mu}{\sigma}=\frac{.024}{1.3} \simeq .01 \ll k^{*}
$$

- This is not even a debated issue, because people don't even know how to start that discussion
- Kahneman says in his Nobel lecture that people use "accessible" horizons.
* E.g. in stock market 1 year is very accessible, because mutual funds and others use it in their prospectuses.
* Other alternatives - time to retirement or time to a big purchase. or "TV every day".
- In practice, for example Barberis, Huang, and Santos QJE 2001 postulate an exogenous horizon.


### 1.2.3 Open question 2: Risk seeking

- Take stock market with return $R=\mu+\sigma n$ with $n \sim N(0,1)$.
- Invest proportion $\theta$ in stock and $1-\theta$ in a riskless bond with return 0 .
- Total return is

$$
\theta R+(1-\theta) 0=\theta(\mu+\sigma n) .
$$

- Let's use PT with $\pi(p)=p$. The PT value is

$$
V=\int_{-\infty}^{+\infty} u(\theta(\mu+\sigma n)) f(n) d n
$$

- Set $u(x)=x^{\alpha}$ for positive $x$ and $-\lambda|x|^{\alpha}$ for negative $x$.
- Using homothecity of $u$ we get

$$
\begin{aligned}
V & =\int_{-\infty}^{+\infty}|\theta|^{\alpha} u(\mu+\sigma n) f(n) d n \\
& =|\theta|^{\alpha} \int_{-\infty}^{+\infty} u(\mu+\sigma n) f(n) d n
\end{aligned}
$$

- Thus optimal $\theta$ to equals 0 or $+\infty$ depending on sign of the last integral.
- Why this problem? It comes because we don't have concave objective function. Without concavity it is easy to have those bang-bang solutions.
- One solution to this problem is that people maximize $V^{E U}+V^{P T}$.


### 1.2.4 Open question 3: Reference point

- Implicitly we take the reference point to be wealth at $t=0$. Gamble is $W_{0}+g$ and

$$
V^{P T}=V^{P T}\left(W_{0}+g-R\right)
$$

- But how $R_{t}$ evolves in time?
- In practice, Barberis, Huang, and Santos QJE 2001 (the most courageous paper) postulate some ad hoc exogenous process. People gave them the benefit of a doubt.


### 1.2.5 Open question 4: Dynamic inconsistency

- Take a stock over a year horizon. Invest 70\% on Jan 1st, 2001.
- It's Dec 1, 2001. Should I stay invested?
- If the new horizon is now one month, I may prefer to disinvest, even though on Jan 1, 2001, I wanted to keep for the entire year.
- By backward induction, Jan 1 guy should disinvest!


### 1.2.6 Open question 5: Doing welfare is hard

- Why? Because it depends on the frame.
- Take $T=250$ days of stock returns $g_{i} \sim N\left(\mu, \sigma^{2}\right)$. Integrated $V^{P T}\left(\sum g_{i}\right)=V^{I}$ and separated $V^{P T}=V^{S}$.
- The cost of the business cycle (Lucas). Suppose $c=$ average monthly consumption. Assume simple consumption shocks:

$$
c_{t}=c+\varepsilon_{t}
$$

with normal iid $\varepsilon_{t}$.

- What is PT reference point? Take $R_{t}=c=0$.
- With PT integrated over one year

$$
V^{P T}\left(\sum \varepsilon_{t}\right)=V^{P T}\left(12^{\frac{1}{2}} \sigma_{\varepsilon} n\right)=\left(12^{\frac{1}{2}} \sigma_{\varepsilon}\right)^{\alpha} V^{P T}(n)<0
$$

- With segregated PT

$$
V^{P T}=12 \sigma_{\varepsilon}^{\alpha} V^{P T}(n)
$$

- Which frame is better?


### 1.3 Next time

- Lucas calculation of costs of business cycle. In practice people care about business cycles, and election are decided on those counts.
- Problem Set - next time. One question - try to circumvent one of the problems.
- Readings on heuristics and biases, the Science 74 KT article and Camerer's paper from the syllabus.

