14.27 — **Economics and E-Commerce**	Fall '14
	Final Exam Solutions
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1. a) Some examples include making prices hard to find, offering several versions of a product differentiated on many dimensions, omitting required fees, taxes, shipping and handling when listing the prices. Many of these strategies could be cheaper to implement online because they involved one-time web design costs instead of ongoing training of sales personnel.

   b) • Google weighted bids by ad quality, mitigating the perennial negative externality arising from advertising

   • Google auctioned off all advertising slots on a page simultaneously in one auction instead of separately, likely resulting in more intense competition for the slots.

   c) If privacy legislation rendered internet display ads less effective, equilibrium prices for those ads would likely fall, display-ad-supported websites would likely earn less revenue, and the consumer experience on those websites would likely deteriorate as they are served less relevant ads. Some people also referred to differential effects across different types of websites and ads, as suggested by Goldfarb & Tucker.

   d) Mail order businesses are not required to collect sales tax on out-of-state transactions to states where they do not have physical presence. To the extent that consumers value this tax advantage, firms have an incentive to concentrate their operations in a small numbers of states, and, in particular, low tax and low population states. If this advantage disappears, firms' location decisions will be governed more by other business considerations, like lowering shipping times or having consumers think of them as "in state" or local.

2. I wanted to see answers where changing transportation costs for goods and people were discussed as a factor in the history of US mail order sales. Then I wanted to see specific examples of cost and quality advantages that e-retail has over catalog retail and how those advantages could affect the landscape of mail order retail.

3. a) Revenue = 0

   If the two advertisers know each others' valuations, they both know that advertiser A (higher valuation) will win slot C1 (the better slot) in the second round if both had refused to participate in the first round. Since advertiser A can only have one slot, he therefore will not bid on the C2 slot in the first round. So advertiser B is the only C2 slot bidder and therefore gets slot C2 at a price of 0. Given this, advertiser A is the only C1 slot bidder in the second round and therefore gets the C1 slot at a price of 0. Hence, the total revenue raised is 0. Notice that this is true under either the first or the second price auction.

   Unnecessary Note: The careful observer may have noticed that depending on the equilibrium concept used, (SPNE vs NE), and whether one allows the bidders strategy space to include "not participating" in a round as a distinct action separate from "bidding zero," and how any potential tie breaker rules are defined, you can create a (socially inefficient) equilibrium where A wins the C2 spot in the first round and B wins the C1 spot in the second. While this is true, it still raises 0 revenue under either a first or second price auction!

   b) As I'm sure you all noticed, this question was quite hard! I will approach this question using the Generalized Second Price auction framework that we covered in class. Recall that in the GSP the highest bid gets the C1 slot but pays the per click price of the second bidder. The second bidder will receive the C2 spot and will pay the per click bid of the third bidder. Since there are only two bidders in this example, the lower bidder will pay a price of zero.

To gain traction on the problem, we can first consider the utility of winning the C1 spot versus the C2 spot for each bidder as follows:

$$U_A(C1|b_B = b) = C1(v_A - b)$$
$$U_A(C2|b_B = b) = C2(v_A)$$
$$U_B(C1|b_A = b) = C1(v_B - b)$$
$$U_B(C2|b_A = b) = C2(v_B)$$

Setting the first two equations equal to one another gives us the price b that makes A indifferent from trying to stay in to win C1 or dropping out to win C2. Thus this is the highest bid A is willing to make in non-weakly dominated strategies. Similarly, setting the third and fourth equations equal gives the price b that makes B indifferent from trying to stay in to win C1 or dropping out to win C2. Thus, this is the highest bid B is willing to make in non-weakly dominated strategies. Hence, the most natural equilibrium to consider is, for $i = $ A, B:

$$b * (v_i) = v_i(1 - (C2/C1))$$

This equilibrium will raise total revenue equal to the number of clicks (C1) times the bid of the lower bidder, hence:

$$\text{Revenue } = C1(v_B(1 - (C2/C1))) = v_B(C1 - C2)$$

A few interesting notes: First, notice that if C2 = 0, ie. that slot C2 is worthless, than bidders are just bidding their valuation as they would in a standard second price auction.

Second, notice that if C2 = C1, so that slots are identical then the search engine gets 0 revenue as in part a.

Third, many students tried to use an adaptation of the first price auction to solve this problem, which is absolutely fine since it was unspecified in the question. However, the Generalized First Price auction, where the high bidder gets C1 and the second highest bidder gets C2, and both pay their bids is quite difficult to solve in this case. The reason for this is that, as we discussed in recitation, the GFP auction actually doesn?t have any pure strategy Nash equilibrium. For example, consider a proposed equilibrium with some $b_A > b_B > 0$, so that A wins the C1 slot and B the C2 slot. Then B would prefer to deviate to 0. He still would get the C2 slot but would pay less. But then A would prefer to deviate to ϵ. This would allow him to win the first object but pay less as well. But then, B would prefer to deviate to 2ϵ and take the better slot! This overcutting continues back and forth until the bids are high enough so that bidder B will again prefer to deviate back to zero accepting the C2 slot rather than fighting for the C1 slot. This cycle persists indefinitely!

Lastly, while the problem asks you to find only one equilibrium that raised more revenue than in Question 3, Part a, there are many more equilibria that use weakly dominated strategies that you could have solved for. To characterize this set we appeal to the same logic above, except using inequalities, because the players are actually indifferent over many strategies (since by nature of the GSP you end up paying a price based on the other player's bid). Then any pair of bidding strategies such that the following three conditions hold simultaneously is actually a Nash Equilibrium:

$$b_A > v_B(1 - (C2/C1))$$
$$b_B < v_A(1 - (C2/C1))$$
$$b_a > b_B$$

Intuitively, the first equation says that bidder A bids high enough so that B doesn't want to overcut him to take slot C1. Similarly, the second equation says that bidder B bids low enough that bidder A

won't undercut him to try to take C2. The third equation requires bidder A to bid more than Bidder B since you will notice that the right hand side of equation 1 is actually less than the right hand side of equation 2. Finally, these equilibria will raise revenues equal to anywhere from $[0, v_A(C1 - C2)]$, depending crucially on the behavior of bidder B, since his bid determines the amount bidder A will actually be forced to pay.

c) If we use sequential second price auctions in this question then the answer is essentially identical to part b, and therefore better than part a. This serves as an example of how the order in which you auction of goods may matter significantly, given the stark difference in revenue for the seller seen between part a and part b.

The reason for the equivalence between the GSP and the sequential second price auctions is that a sequential second price auction with the valuable good first is essentially recreating the logic of the GSP, albeit in 2 steps. In the first auction the high bidder wins the C1 slot, and pays the second highest bid, just as in the GSP. In the second auction there is only one bidder remaining so he wins C2 at a price of 0, also as in the GSP. So the unique equilibrium in non-weakly dominated strategies is the same, and the remaining set of equilibrium that use weakly dominated strategies also exists, and they raise the same revenue as they each did before.

You could however, actually solve this question with a sequential first price auction in pure strategies. Again, there is a multiplicity of equilibria as anything of the following form will work:

$$b_A = b_B \in [v_B(1 - (C2/C1)), v_A(1 - (C2/C1))]$$

with a tie-breaking rule favoring bidder A. If you don't want to use that tie breaking rule then amend the above to $b_A = b_B + \epsilon$. Notice that the only equilibrium here that does not use weakly dominated strategies is $b_A = b_B = v_B(1 - (C2/C1))$ which raises the same revenue as the non-weakly dominated equilibria in the GSP or the sequential second price auctions.

4. a) This firm solves the following profit maximization problem:

$$\max_P \ (P - C)(Q(P))$$

$$\max_P \ (P - 5)(25 - P)$$

$$\max_P \ 30P - 125 - P^2$$

FOC: $30 = 2P^*$

$$P^* = 15$$

b) The firm can now identify the two groups of customers separately and so following our standard model of third degree price discrimination, the firm solves the following profit maximization problems, separately choosing P_E and P_{US} since their demands are independent:

$$\max_{P_E} \ (P_E - C)(Q_E(P))$$

$$\max_{P_E} \ (P_E - 5)925 - P_E)$$

$$\max_{P_E} \ 30P_E - 125 - P_E^2$$

FOC: $30 = 2P_E^*$

$$P_E^* = 15$$

and for the US:

$$\max_{P_{US}} \ (P_{US} - C)(Q_{US}(P_{US}))$$

$\max_{P_{US}} (P_{US} - 5)(20 - (P_{US}))$

$\max_{P_{US}} 25P_{US} - 100 - P_{US}^2$

FOC: $25 = 2P_{US}^*$

$P_{US}^* = 12.5$

c) Yes, this is our classic example of second degree price discrimination. The firm knows that there are two types of consumers but now it cannot identify them individually. This means that the firm will have to use a pricing scheme that causes individuals to "self-select" into the product bundle designed for them.

The firm will offer a menu of contracts of the form $(Q_{US}, T(Q_{US}))$ and $(Q_E, T(Q_E))$ to solve the following maximization problem:

$$\max (1/2)[T(Q_{US} + T(Q_E) - C(Q_{US} + Q_E)]$$

$$\text{s.t.: } U_{US}(Q_{US}) - T(Q_{US}) \geq 0$$
$$U_E(Q_E) - T(Q_E) \geq 0$$
$$U_{US}(Q_{US}) - T(Q_{US} \geq U_{US}(Q_E) - T(Q_E)$$
$$U_E(Q_E) - T(Q_E) \geq U_E(Q_{US}) - T(Q_{US})$$

The first two constraints are the individual rationality constraints, ensuring that each consumer actually will want to purchase the contract designed for it. The third and fourth constraints are the incentive compatibility constraints that ensure that each agent purchases the bundle targeted to him/her by the firm. Conditional on these four constraints all consumers will purchase in equilibrium and since this contract space is otherwise fully nonlinear, the firm will make the highest profits possible under this information structure of not being able to directly observe the consumer's type. Thus, this pricing plan must outperform setting a single price responding to aggregate demand. If you wanted, you could have solved the rest of this problem by solving for the utility functions by integrating the individual demands, and then working through the above maximization problem, but I didn't require that level of detail.

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