14.381: Statistics

Fall, 2007

Midterm Examination

1. True, false, or uncertain? Explain your answer briefly.

   (a) If $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators for parameter θ, then $\frac{1}{3}\hat{\theta}_1 + \frac{2}{3}\hat{\theta}_2$ must be unbiased for θ.

   (b) Estimators obtained by the maximum likelihood method are unbiased.

   (c) If $X_1, X_2, \ldots, X_n$ are i.i.d. normal random variables $N(\mu, \sigma^2)$, then $\bar{X} \cdot 5$ is independent of $\sum_{i=1}^{n}(X_i - \bar{X})^2/20$, where $\bar{X}$ is the sample mean.

2. Let us have two independent random samples: $X_1, \ldots, X_n$ is a sample from $N(\mu_x, \sigma_x^2)$, and $Y_1, \ldots, Y_m$ is a sample from $N(\mu_y, \sigma_y^2)$.

   (a) Write down a joint pdf for $\{X_1, \ldots, X_n, Y_1, \ldots, Y_m\}$.

   (b) Find a 4-dimensional sufficient statistic.

   (c) Find the MLE of σ_x^2 and σ_y^2.

   (d) Assume for this question only that $\sigma_x^2 = \sigma_y^2 = \sigma^2$. Find the MLE for σ^2.

   (e) Find a LR test statistic for testing $H_0 : \sigma_x^2 = \sigma_y^2$.

   (f) Suggest an exact test for testing $H_0 : \sigma_x^2 = \sigma_y^2$. Hint: if $U \sim x_p^2$ and $V \sim x_q^2$ are independent, then $\frac{U/p}{V/q} \sim F_{p,q}$ (Fisher distribution)

   (g) Assume that $n = 100$, $m = 50$, $s_x^2 = 5$, $s_y^2 = 6$. Test the hypothesis $H_0 : \sigma_x^2 = \sigma_y^2$ using the test received in (f) at 95% level. Tables of quantailes of Fisher distribution are provided.

   (h) Test $H_0 : \sigma_x^2 = \sigma_y^2$ vs. $H_1 : \sigma_x^2 \neq \sigma_y^2$ by using an *asymptotic* LR test and the data as in (g).

   (i) Suggest a confidence set for $\beta = \frac{\sigma_x^2}{\sigma_y^2}$.

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