14.462
Problem Set 2

Problem 1

In this problem you will replicate Figures on pages 12 and 14 of the lecture notes (demand shocks, part I). Consider a stochastic growth model with preferences and technology given by

$$\begin{aligned} U(C_t, N_t) &= \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\eta} N_t^{1+\eta}, \\ A_t F(K_{t-1}, N_t) &= A_t K_{t-1}^{\alpha} N_t^{1-\alpha}. \end{aligned}$$

The process for A_t is as follows

$$A_t = e^{a_t},$$

$$a_t = \rho a_{t-1} + \epsilon_t.$$

Use parameters

$$\begin{aligned} \beta &= 0.99, \quad \delta = 0.025, \\ \eta &= 1, \quad \sigma = 1, \\ \alpha &= 0.36, \quad \rho = 0.95. \end{aligned}$$

You can use the Matlab package Dynare (http://www.cepremap.cnrs.fr/dynare/).
(i) Setup the planner problem and derive the first order conditions.
(ii) Derive impulse response functions for a, i, c, y, n for the model above.
(iii) Replace the technology process with

$$a_t = \rho a_{t-1} + \epsilon_{t-3}.$$

Derive impulse response functions for a, i, c, y, n for the new model.
(iv) Try to change the elasticity of intertemporal substitution σ and see how it affects equilibrium dynamics.
(v) (OPTIONAL) Introduce quadratic adjustment costs in labor inputs:

$$G(N_{t+1}, N_t) = \frac{\xi}{2} \left(\frac{N_{t+1} - N_t}{N_t} \right)^2.$$

Characterize the equilibrium dynamics for different values of ξ.

Problem 2

Consider an economy where productivity follows the process

$$x_t - x_{t-1} = \rho\left(x_{t-1} - x_{t-2}\right) + \epsilon_t.$$

Agents observe all past values $\{x_{t-1}, x_{t-2}, ...\}$ and a signal regarding the current shock

$$s_t = \epsilon_t + e_t.$$

Suppose consumers follow the forward-looking rule

$$c_t = \mathrm{E}\left[\sum_{j=0}^{\infty} \beta^j x_{t+j} | \mathcal{J}_t\right],$$

where $\mathcal{J}_t$ is the consumers' information set.

(i) Derive equilibrium consumption dynamics in terms of the shocks ϵ_t and e_t.

(ii) Suppose the econometrician information set at time t, $\mathcal{J}_t^E$, is given by $\{x_{t-1}, x_{t-2}, ...\}$ and $\{c_t, c_{t-1}, ...\}$. Write down a VAR representation for the joint behavior of x_{t-1} and c_t:

$$\begin{pmatrix} c_t \\ x_{t-1} \end{pmatrix} = \sum_{j=1}^{\infty} A_j \begin{pmatrix} c_{t-j} \\ x_{t-1-j} \end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \end{pmatrix}.$$

(Hint: careful when defining the innovation to the x_{t-1} equation, notice that $E\left[\epsilon_t | \mathcal{J}_t^E\right] \neq 0$). Argue that the econometrician can identify s_t but cannot separately identify ϵ_t and e_t from $\left(\eta_{1,t}, \eta_{2,t}\right)$.

(iv) Suppose now the econometrician information set is $\{x_t, x_{t-1}, ...\}$ and $\{c_t, c_{t-1}, ...\}$. Write down a VAR representation for the joint behavior of x_t and c_t

$$\begin{pmatrix} c_t \\ x_t \end{pmatrix} = \sum_{j=1}^{\infty} A_j \begin{pmatrix} c_{t-j} \\ x_{t-j} \end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \end{pmatrix}.$$

Discuss how an econometrician can impose identifying restrictions to estimate and recover the shocks e_t and ϵ_t from the innovations $\left(\eta_{1,t}, \eta_{2,t}\right)$.

Problem 3

Consider an economy populated by a continuum of households $[0,1]$ located on different islands.

Each household has an endowment $\bar{x} = 1$ of gold. Each household is made of a consumer and a producer. At the beginning of the day the producer sets the price p_i. Then the consumer i travels to an island j, randomly assigned. Then the preference shock α_i is realized. The consumer observes α_i and buys c_i units of the good produced in island j. At the same time the producer is selling y_i

to some other consumer. Then the consumer returns home and consumes the gold:

$$x_i = \bar{x} - p_j c_i + p_i y_i.$$

The central imperfection is that agents do not observe y_i (sales) at the time of making the purchases c_i.

Preferences are as follows

$$E\left[u\left(c_i, \alpha_i\right) + w\left(x_i\right) - v\left(n_i\right)\right]$$

where c_i is consumption, x_i

$$\begin{aligned} u\left(c_i, \alpha_i\right) &= \alpha_i c_i - \frac{1}{2} c_i^2 \\ w\left(x_i\right) &= x_i - \frac{1}{2} x_i^2 \end{aligned}$$

and $v\left(n_i\right)$ is a convex function.

The production function in each island is linear and given by:

$$y_i = n_i.$$

For simplicity, let c_i,x_i and n_i vary in $(-\infty, +\infty)$, and disregard all non-negativity constraints.

The preference shocks are generated by:

$$\alpha_i = \alpha + \epsilon_i$$

where α and ϵ_i are independent gaussian random variables with mean zero and variances σ_α^2 and σ_ϵ^2 and $\int \epsilon_i di = 1$.

Consider a symmetric equilibrium where $p_i = p$. For the purpose of this exercise we will fix p (i.e. disregard the optimality condition for prices at the beginning of the period).

(i) Write down the consumer first order condition and derive the optimal choice of c_i as a function of α_i and $E_i\left[y_i\right]$.

(ii) Show that p determines the degree of strategic complementarity in spending. Comment.

(iii) Find the equilibrium output y in the case of perfect information.

(iv) Go back to the case where agents only observe α_i. Find a linear equilibrium of the type

$$c = \psi \alpha$$

and show that –for a given value of p– ψ is larger for larger values of $\frac{\sigma_\alpha^2}{\sigma_\epsilon^2}$.

Consider the case where agents can observe both α_i and a public signal of the preference shock

$$s = \alpha + e$$

where e is gaussian, independent of α and ϵ_i, with mean zero and variance σ_e^2.

(v) Characterize an equilibrium of the type

$$c = \psi_a \alpha + \psi_s s$$

(Please use the notation: $E[\alpha|\alpha_i, s] = \beta_\alpha \alpha_i + \beta_s s$)

(vi) Show that –for a given value of p– the economy is very responsive to the public signal shock e when β_s is large and β_α is small.

(vii) Comment on the welfare implications, is the presence of the signal s always desirable?

Problem 4

Consider the version of the Lucas (1972) model derived in class.

(i) Derive an expression for the constant ξ or (which is the same) for the average price level $\bar{p}$. (Hint: you can take unconditional expectations on both sides of the labor supply equation to get

$$E[N_{i,t}] = E\left[\frac{P_{i,t}}{P_{j,t+1}}(1 + x_{t+1})\right],$$

substitute the equilibrium prices...)

(ii) Study the effect of changing σ_ϵ^2 on average labor supply and average output, interpret.

(iii) (OPTIONAL) Consider a planner who uses a utilitarian welfare function (i.e. who maximizes $E\left[\int C_{i,t}di - \frac{1}{2}\int N_{j,t}^2 dj\right]$ each period). What is the level of σ_ϵ^2 that maximizes welfare?