

14.471: Fall 2012: PS3

October 18, 2012

1. Commodity Taxation

Consider an economy consisting of identical individuals with preferences given by

$$U(x_1, x_2, l) = \alpha_1 \log(x_1) + \alpha_2 \log(x_2) + (1 - \alpha_1 - \alpha_2) \log(1 - l).$$

Individuals supply labor, l , at a market wage of unity, and purchase goods 1 and 2, both of which have fixed producer prices equal to 1. The government's revenue requirement is R , measured in the fixed producer prices.

(a) Assume that the government is restricted to using linear commodity taxes on goods 1 and 2 to raise revenue. Set up the Ramsey optimal tax problem for this economy, and find the optimal tax rates on goods 1 and 2 as a function of R and the individual preference parameters.

(b) Obtain expressions for the marginal utility of money of the individual, α , and the marginal cost of government revenue, λ . How do changes in the government revenue requirement affect these costs and the difference between the two?

2. Capital Taxation

This question asks you to numerically explore the welfare implications of capital taxation in the Ramsey setup discussed in class. There is a representative agent with preferences

$$u(c, l) = \frac{1}{1 - \sigma} \left[c^\theta (1 - l)^{1 - \theta} \right]^{1 - \sigma}$$

over consumption c and labor l . The discount factor is β . Technology is Cobb-Douglas with

$$F(k, l) = Ak^\alpha l^{1 - \alpha},$$

and capital k depreciates at a rate δ . **Perform your calculations for $\sigma = 2$, $\sigma = 3$, and $\sigma = 4$.**

(a) Consider a steady state competitive equilibrium of this economy where the tax on capital is $\kappa = 0$ and there is also no tax on labor income. What is the steady state gross interest rate R ? Using the steady state resource constraint and the first order conditions from the consumer's utility maximization problem and the firm's profit maximization problem, find 4 equations that implicitly determine the steady state wage w , consumption c , labor supply l , and capital stock k as a function of the parameters. Calibrate the economy by choosing parameter values for θ , A , α , δ , and β that imply empirically reasonable values for the endogenous variables in the steady state (refer to Cooley (1995), chapter 1). Solve your equations for 1). c , l , k , and w .

(b) Using the parameters found in (a), compute the steady state for capital taxes $\kappa \in [-0.5, 0.5]$, assuming that the tax revenue is returned in a lump sum fashion to consumers. Plot steady state output, capital,

consumption, and labor $y(\kappa)$, $k(\kappa)$, $c(\kappa)$, and $l(\kappa)$ as a function of the capital tax κ . Compute the value of the steady state as

$$\tilde{V}(k(\kappa)) = \frac{1}{(1-\beta)(1-\sigma)} \left[c(\kappa)^\theta (1-l(\kappa))^{1-\theta} \right]^{1-\sigma}$$

for given κ . We want to evaluate the welfare cost of this capital tax distortion by comparing this to the value that consumers would obtain if the capital tax were set to zero. A naive approach to this would be to compare $\tilde{V}(k(\kappa))$ with $\tilde{V}(k(0))$. Compute $\lambda(\kappa)$ such that

$$\frac{1}{(1-\beta)(1-\sigma)} \left[((1+\lambda(\kappa))c(\kappa))^\theta (1-l(\kappa))^{1-\theta} \right]^{1-\sigma} = \tilde{V}(k(0)).$$

Plot $\lambda(\kappa)$. How does your result depend on σ ? Interpret.

(c) The conclusion in (b) ignores the transition from the capital stock $k(\kappa)$ to $k(0)$ once the capital tax κ is reduced to zero. The value function accounting for this solves the Bellman equation

$$V(k) = \max_{c,l,k'} u(c,l) + \beta V(k')$$

subject to

$$c + k' = F(k,l) + (1-\delta)k.$$

Solve numerically for $V(k)$ and evaluate it at $k(\kappa)$. Again, find $\lambda(\kappa)$ such that

$$\frac{1}{(1-\beta)(1-\sigma)} \left[((1+\lambda(\kappa))c(\kappa))^\theta (1-l(\kappa))^{1-\theta} \right]^{1-\sigma} = V(k(\kappa)).$$

Plot $\lambda(\kappa)$. How does your result depend on σ ? How does it compare to your result in (b)? Interpret.

3. Pigouvian taxation, pollution and jobs

Please read the paper Shimer (2012) “A Framework for Valuing the Employment Consequences of Environmental Regulation”.

- a) Map the model(s) into an Arrow-Debreu Walrasian economy and then apply the general results on Pigouvian taxation that we discussed in class
- b) Compare the results from a) with the results in Shimer (2012) and discuss.
- c) Compare the results from a) with the current EPA practices for valuing job losses of regulation and discuss.

4. Short questions

Are the following statements true, false or uncertain? Please explain your answer.

1. The Pigouvian principle does not apply with heterogeneous agents.
2. The fundamental problem of distortive taxation is that lump-sum taxes are unavailable.
3. Suppose preferences over consumption and leisure are Cobb-Douglas. Then a proportional tax on labor, used to finance government consumption, does not affect labor supply. Thus taxation in this case is non-distortionary.

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