1 A Two Period Moral Hazard Model

- Rogerson (1985)
- moral hazard model
- two periods $t = 0, 1$
  - effort in first period e_0
  - consumption in both periods c_0, c_1
  - stochastic output in second period $y_1 = \theta_1$ with denisty $f(\theta_1|e_0)$
- separable utility

$$U(c_0) - h(e) + \beta \int U(c_1(\theta_1)) f(\theta_1|e_0)$$

- incentive compatible $\{c_0, e_0, c_1(\theta_1)\}$ requires:

$$U(c_0) - h(e) + \beta \int U(c_1(\theta_1)) f(\theta_1|e_0) \geq U(c_0) - h(e') + \beta \int U(c_1(\theta_1)) f(\theta_1|e')$$

- rewrite in terms of utility assignements: $u_t = U(c_t)$....

$$u_0 - h(e) + \beta \int u_1(\theta_1) f(\theta_1|e) \geq u_0 - h(e') + \beta \int u_1(\theta_1) f(\theta_1|e')$$

- planning problem

$$\min \left\{ C(u_0) + q \int [C(u_1(\theta_1)) - y_1(\theta_1)] f(\theta_1|e_0) \right\}$$

$$u_0 - h(e) + \beta \int u_1(\theta_1) f(\theta_1|e) = v_0$$

$$u_0 - h(e) + \beta \int u_1(\theta_1) f(\theta_1|e) \geq u_0 - h(e') + \beta \int u_1(\theta_1) f(\theta_1|e')$$

- here $q = R^{-1}$

1.1 Savings Distortions

- if agent could save at safe rate of return $R = q^{-1}$ then we would have

$$U'(c_0) = \beta R \int U'(c_1(\theta_1)) f(\theta_1 | e_0)$$

standard Euler equation.

- we will show this does not hold: there is a distortion in savings.

- fix e_0 and consider variations in consumption/utility:

$$\hat{u}_0 = u_0 - \beta\Delta$$
$$\hat{u}_1(\theta_1) = u_1(\theta_1) + \Delta$$

- no effect on utility or incentive constraint since:

$$u_0 - h(e') + \beta \int u_1(\theta_1) f(\theta_1 | e') = \hat{u}_0 - h(e') + \beta \int \hat{u}_1(\theta_1) f(\theta_1 | e')$$

for all e'

- we need to solve

$$\min_{\hat{u}_0, \hat{u}_1(\cdot), \Delta} \left\{ C(\hat{u}_0) + q \int C(\hat{u}_1(\theta_1)) f(\theta_1 | e_0) \right\}$$

$$\hat{u}_0 = u_0 - \beta\Delta$$
$$\hat{u}_1(\theta_1) = u_1(\theta_1) + \Delta$$

- substituting

$$\min_{\Delta} \left\{ C(u_0 - \beta\Delta) + q \int C(u_1(\theta_1) + \Delta) f(\theta_1 | e_0) \right\}$$

- note: similarity with savings problem (Δ looks like assets; $-C(-x)$ looks like the utility function)

- FOC is

$$C'(u_0 - \beta\Delta)\beta = q \int C'(u_1(\theta_1) + \Delta) f(\theta_1 | e_0)$$

this condition is necessary and sufficient for an interior: we can use this to solve for Δ

- if original allocation was optimal then $\Delta = 0$ and using that $C = U^{-1}$ we obtain

$$\frac{1}{U'(c_0)} = \frac{1}{\beta R} \int \frac{1}{U'(c_1(\theta_1))} f(\theta_1|e_0)$$

Inverse Euler equation

- since $\frac{1}{x}$ is convex we can apply Jensen's inequality
- if $\text{Var}[c_1(\theta_1)] > 0$ then

$$U'(c_0) < \beta R \int U'(c_1(\theta_1)) f(\theta_1|e_0)$$

agents are "savings constrained"

- wedge

$$U'(c_0) = \beta(1-\tau) R \int U'(c_1(\theta_1)) f(\theta_1|e_0)$$

is positive

$$\tau \geq 0$$

1.2 Mirrlees Model

- Mirrlees model [Golosov et al]
- work time at $t = 1$

$$u(c_0) + \beta \int [u(c_1(\theta_1)) - h(y_1, \theta_1)] f(\theta_1)$$

- same perturbations
- same optimality conditions: Inverse Euler equation
- true also for a mixed model of moral hazard and adverse selection where effort affects distribution of θ

2 General Horizon and Welfare

- Utility

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t[U(c_t)-h(y_t,\theta_t)]$$

- $\{\theta_t\}$ general stochastic process and private information

- again: rewrite in terms of u_t

- incentive constraint

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t[u(\theta^t)-h(y(\theta^t),\theta_t)]\geq\mathbb{E}\sum_{t=0}^{\infty}\beta^t[u(\sigma^t(\theta^t))-h(y(\sigma^t(\theta^t)),\theta_t)]$$

- Planner's net cost:

$$\mathbb{E}\sum_{t=0}^{\infty}q^t[C(u(\theta^t))-y(\theta^t)]$$

- variations as before: [Farhi-Werning]

$$\hat{u}(\theta^t)=u(\theta^t)+\Delta(\theta^{t-1})-\beta\Delta(\theta^t)$$

  and a "No Ponzi" condition

$$\lim\beta^t\mathbb{E}\Delta(\sigma^t(\theta^t))=0$$

- preserve utility and incentive compatibility since

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(\sigma^t(\theta^t))=\mathbb{E}\sum_{t=0}^{\infty}\beta^t\hat{u}(\sigma^t(\theta^t))$$

- hence, must minimize cost

$$\mathbb{E}\sum_{t=0}^{\infty}q^t[C(u(\theta^t)+\Delta(\theta^{t-1})-\beta\Delta(\theta^t))$$

- looks like savings problem:
  Farhi-Werning exploit this to solve this partial reform

- implementation: see slides

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