14.471: Fall 2012: Recitation 5: Consumption externalities and Imperfect Competitive Pricing

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Question: How to set corrective taxes when individuals differ in their per unit contribution to pollution, price-elasticity of demand and congestion elasticity of demand?

1 Main results

- In the lecture notes section "Pigouvian Taxation with many agents"? we obtained corrective taxes independent of elasticities assuming that the externality is just aggregate consumption ($\bar{x} = \sum x^h \pi^h$)
- Diamond (1973) :

1. First obtains that taxes should increase in elasticities of demand of relatively strong polluters when relaxing the aggregate consumption assumption ("we care more about dirty, fast cars")
2. Then shows that the optimal surcharge can be zero or negative despite the presence of diseconomies when relaxing the assumption that externalities are separable
3. Finally shows that the surcharge should decrease when strong polluters are price-insensitive and congestion-sensitive

2 Separable externalities

- Utility is additively separable $u^h(\alpha_1, \alpha_2, ..., \alpha_n) + \mu_h$. We assume that the marginal utility of own consumption is independent of the demand of others (i.e. for $i \neq h$)

$$\frac{\partial^2 U^h}{\partial \alpha_h \partial \alpha_i} = 0 \tag{1}$$

- Solving the consumer's problem:

$$\max_{\alpha_h} u^h(\alpha_1, \alpha_2, ..., \alpha_n) + \mu_h$$

gives a FOC $\frac{\partial U^h}{\partial \alpha_h} \leq p + t$ and demand which is only a function of the price to the consumer $\alpha_h^* = \alpha_h(p+t)$

- Note: Demand funciton which does not depend on demand of others is quite inappropriate for congestion problems
- Welfare as a function of the surcharge $W(t)$ is the sum over all the households of the utility over the studied good and the consumption of the quasilinear good (which equals the budget minus the expenditure on the studied good plus tax lump sum rebates) :

$$W(t) = \sum_h u^h(\alpha_1(p+t), ..., \alpha_n(p+t)) - p \sum \alpha_h(p+t) + \sum_h m_h$$

- Differentiating wrt to the surcharge, we have:

$$W'(t) = \sum_h \sum_i \frac{\partial U^h}{\partial \alpha_i} \alpha_i'(p+t) - p \sum \alpha_h'(p+t)$$

- Combining with the FOC gives:

$$W'(t) = \sum_h \sum_{i \neq h} \frac{\partial U^h}{\partial \alpha_i} \alpha_i' + t \sum \alpha_h'$$

- Check that $W'(0) > 0$ since utilities are decreasing in demand of others and since demand elasticity is negative
- $W'(t) = 0$ gives us a surcharge which equals a weighted average of externalities $\sum_{i \neq h} \frac{\partial u^h}{\partial \alpha_i}$ weighted by consumer demand derivatives $\alpha^{i'}$:

$$t* = \frac{-\sum_h \sum_{i \neq h} \frac{\partial u^h}{\partial \alpha_i} \alpha^{i'}}{\sum_h \alpha^{h'}} \tag{2}$$

  - Intuition: If by increasing tolls, you reduce miles of low-polluters (with a very high elasticity), then the surcharge is less usefull -> lower tax

3 Demand interactions

- Drop assumption (1) according to which externalities are separable
- Demands now affect each other: $\alpha_h^* = \alpha_h(p+t, \alpha_1(p+t), ..., \alpha_n(p+t))$.
  - "The more other drivers, the less I want to drive"
- Fixed point problem: Assume that these n equations have simulatenous non-negative solutions as a function of the price and that each demand is decreasing in price
- The FOC (2) is valid without the assumption of decreasing demand but the formula does not convey much general information since the weights do not necessarily lie in the unit interval
- Two-person example illustrates the lack of general info where optimal surcharge will be zero:
  - Assume utilities:

$$U^1(\alpha_1, \alpha_2) + \mu_1 = \alpha_1^{\frac{1}{2}} - \frac{1}{3}\alpha_2 + \mu_1$$

$$U^2(\alpha_1, \alpha_2) + \mu_2 = 0.3 \log(\alpha_2 + 0.9\alpha_1) - \alpha_1 + \mu_2$$

  - Now, require that the marginal utility equals the after-tax price and solve for the 2 demanded quantities and get that Mr.1 cares about the price whereas Mr.2 cares about congestion:

$$\alpha_1 = 0.25(p+t)^{-2}$$

$$\alpha_2 = \max\left\{0.3(p+t)^{-1} - 0.9\alpha_1, 0\right\}$$

  - Intuition: for small after tax prices $p + t < 1.5$
    - Demand by Mr. 1 explodes
    - Hence the demand by Mr. 2 hence decreases.
    - Thus the demand by Mr. 2 **increases** in its price ("Inelastic Mr. 2 wants a high price to kick Mr. 1 o ffthe road")

- Summing utilities net of production cost, we have the following welfare function:

$$W(t) = m_1 + m_2 + 0.3\log(0.3) - 0.3\log(p+t) + 0.1(p+t)^{-1} - 0.2(p+t)^{-2}$$

- For $p = 1$, this function achieves its maximum at $t = 0$ despite negative externalities!
  - Introducing a discharge decreases demand by Mr. 1 but increases demand by Mr. 2
  - Then, Mr. 1 would be hurt

4 An aggregator

- Assume now that demand $\alpha_h(p+t,\gamma)$ only depends on:
  - consumer price $p+t$
  - the externality "aggregator" $\gamma = \Gamma\left(\alpha_1, \alpha_2, ..., \alpha_3, ..., \alpha_h\right)$: "Externality affects everyone equally"
- Fixed point problem (1 equation, 1 unknown) to find the equilibrium level of congestion:

$$\gamma = \Gamma\left(\alpha_1(p+t,\gamma), \alpha_2(p+t,\gamma), , ..., \alpha_3(p+t,\gamma), , ..., \alpha_n(p+t,\gamma)\right)$$

- Impact of tax on pollution aggregate via Implicit Function Theorem (IFT)
- Applying IFT to: $\gamma - \Gamma\left(\alpha^1, \alpha^2, ..., \alpha^h, ..., \alpha^H\right) = 0 = G\left[\alpha^1, \alpha^2, ..., \alpha^h, ..., \alpha^H, p, t\right]$gives:

$$\frac{d\gamma}{dt} = -\frac{\frac{\partial G}{\partial t}}{\frac{\partial G}{\partial \gamma}} = \frac{\sum \Gamma_h \frac{\partial \alpha^h}{\partial t}}{1 - \sum \Gamma_h \frac{\partial \alpha^h}{\partial \gamma}} \tag{3}$$

  - Numerator: direct postive effect: higher taxes -> lower consumption -> lower pollution
  - Denominator: indirect negative feedback/"knock down"- effect of taxation on pollution
    - higher taxes -> lower consumption -> lower pollution -> higher consumption-> more pollution...
- Assume agents ingore their effect on pollution: $\gamma_h = 0$ (atomistic agents). Then we get FOC for individual demand $\frac{\partial U^h}{\partial \alpha_h} \leq p+t$
- We maximize welfare:

$$W(t) = \sum_h u^h(\alpha_h, \gamma) - p\sum \alpha_h + \sum_h m_h$$

- Let us compute $W'(t)$ and decompose the impact of tax on welfare into 3 terms:
  - Utility reduction from lower demand of taxed good (direct tax impact+ indirect tax -> pollution impact):

$$\sum_h \frac{\partial U^h}{\partial \alpha_h}\left[\frac{\partial \alpha^h}{\partial t} + \frac{\partial \alpha^h}{\partial \gamma}\frac{d\gamma}{dt}\right]$$

  - Utility gain from lower pollution:

$$\sum_h \frac{\partial U^h}{\partial \gamma}\frac{d\gamma}{dt}$$

  - Utility gain from higher consumption of residual good (income is stable):

$$-p\left[\frac{\partial \alpha^h}{\partial t} + \frac{\partial \alpha^h}{\partial \gamma}\frac{d\gamma}{dt}\right]$$

- Adding these 3 pieces gives equation (20) of the paper:

$$W'(t) = \sum_h \left(\frac{\partial U^h}{\partial \alpha_h} - p \right) \left[\frac{\partial \alpha^h}{\partial t} + \frac{\partial \alpha^h}{\partial \gamma} \frac{d\gamma}{dt} \right] + \frac{d\gamma}{dt} \sum_h \frac{\partial U^h}{\partial \gamma}$$

- Rewriting this with the FOC gives:

$$W'(t) = t \left(\sum_h \frac{\partial \alpha^h}{\partial t} + \frac{d\gamma}{dt} \sum \frac{\partial \alpha^h}{\partial \gamma} \right) + \frac{d\gamma}{dt} \sum_h \frac{\partial U^h}{\partial \gamma}$$

- Setting W' equal to zero gives the optimal surchage:

$$t* = \frac{-\frac{d\gamma}{dt} \sum_h \frac{\partial U^h}{\partial \gamma}}{\sum_h \frac{\partial \alpha^h}{\partial t} + \frac{d\gamma}{dt} \sum \frac{\partial \alpha^h}{\partial \gamma}}$$

- Using (3) and rearranging gives:

$$t* = \left[-\sum \frac{\partial u^h}{\partial \gamma} \right] * \left[\frac{\sum \Gamma_h \frac{\partial \alpha^h}{\partial t}}{\sum \frac{\partial \alpha^h}{\partial t}} \right] * \left[\frac{\sum \frac{\partial \alpha^h}{\partial t}}{\sum \frac{\partial \alpha^h}{\partial t} - \triangle} \right]$$

where

$$\triangle = \sum \gamma_h \frac{\partial \alpha^h}{\partial t} \sum \frac{\partial \alpha^h}{\partial \gamma} - \sum \gamma_h \frac{\partial \alpha^h}{\partial \gamma} \sum \frac{\partial \alpha^h}{\partial t}$$

which can be rewritten as:

$$\triangle = \left(\sum \frac{\partial \alpha^h}{\partial t} \right) \left(\sum \frac{\partial \alpha^h}{\partial \gamma} \right) \left[\sum \gamma_h \frac{\frac{\partial \alpha^h}{\partial t}}{\sum_i \frac{\partial \alpha_i^h}{\partial t}} - \sum \gamma_h \frac{\frac{\partial \alpha^h}{\partial \gamma}}{\sum_i \frac{\partial \alpha_i^h}{\partial \gamma}} \right] \tag{4}$$

- The optimal tax rate thus equals the product of:
  - the value of a decrease in congestion $-\sum \frac{\partial u^h}{\partial \gamma}$
  - the average change in congestion from an increase in the tax via individual demand $\frac{\sum \Gamma_h \frac{\partial \alpha^h}{\partial t},}{\sum \frac{\partial \alpha^h}{\partial t},}$
  - weighting term adjusting for interactions $\frac{\sum \frac{\partial \alpha^h}{\partial t}}{\sum \frac{\partial \alpha^h}{\partial t} - \triangle}$
    * If $\triangle < 0$, then the weight is below 1 since $0 < \frac{\sum \frac{\partial \alpha^h}{\partial t}}{\sum \frac{\partial \alpha^h}{\partial t} - \triangle} < 1$
    * When is $\triangle < 0$? From (4), necessary and sufficient conditions are:

$$\sum \gamma_h \frac{\frac{\partial \alpha^h}{\partial t}}{\sum_i \frac{\partial \alpha_i^h}{\partial t}} - \sum \gamma_h \frac{\frac{\partial \alpha^h}{\partial \gamma}}{\sum_i \frac{\partial \alpha_i^h}{\partial \gamma}} < 0$$

    * This last condition is equivalent to (using $Cov(xy, y) = E(xy) - E(x)E(y)$):

$$Cov \left\{ \gamma_h, \frac{\frac{\partial \alpha^h}{\partial \gamma}}{\sum_i \frac{\partial \alpha_i^h}{\partial \gamma}} \right\} > Cov \left\{ \gamma_h, \frac{\frac{\partial \alpha^h}{\partial t}}{\sum_i \frac{\partial \alpha_i^h}{\partial t}} \right\}$$

* **Intuition**: the surcharge should be below the weighted average of externalities when strong polluters (large γ_h) have demands which are:
  - Congestion sensitive ($-\frac{\partial \alpha^h}{\partial \gamma}$ large) since a high tax would reduce congestion and attract polluters
  - Price insensitive ($-\frac{\partial \alpha^h}{\partial t}$ small) since a high tax would not discourage polluters much

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