14.54 International Economics
Handout 2

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1 A quasi-linear model

The home country produces and consumes two goods: A and B.

The preferences of the home consumer are:

$$c_A + u(c_B) - v(l),$$

where l is labor supply, u is a concave function and v is a convex function.

The technology is as follows: there is a fixed endowment of good A is

$$y_A = \bar{y}_A,$$

the production function for good B is

$$y_B = l.$$

The domestic equilibrium problem is the usual one: given p_A and p_B, derive the optimal behavior on the supply side and on the demand side, and then derive the net demand of imports/exports.

From now on we choose good A as the numeraire, i.e., we set

$$p_A = 1.$$

Begin with the supply side. Let w be the wage rate. The firms in the B sector maximize profits given p_B and w:

$$\max_{y_B, l} \quad p_B y_B - wl$$
$$s.t. y_B = l.$$

Therefore, to ensure that these firms are active we need

$$w = p_B,$$

and firms make zero profits.

Then look at the consumer behavior

$$\begin{array}{ll} \max & c_A + u(c_B) - v(l) \\ & s.t. c_A + p_B c_B = \bar{y}_A + wl \end{array}$$

we have optimal consumption of c_A, c_B and optimal labor supply

$$\begin{aligned} 1 &= \lambda, \\ u'(c_B) &= \lambda p_B, \\ v'(l) &= \lambda w. \end{aligned}$$

Substituting we obtain the following implicit expressions for the demand and supply of good B:

$$u'(c_B) = p_B \text{ demand of good } B \tag{1}$$
$$v'(y_B) = p_B \text{ supply of good } B \tag{2}$$

and we get the demand for imports of good B:

$$c_B - y_B = M(p_B) \equiv u'^{-1}(p_B) - v'^{-1}(p_B).$$

Notice that since u is concave and v is convex M is a decreasing function.

Suppose the supply of exports of good B by the rest of the world is given by the increasing function $X(p_B)$. Then the equilibrium price of good B is found from the condition:

$$M(p_B^*) = X(p_B^*).$$

From now on we assume that at the world equilibrium $M(p_B^*) < 0$ so that the labels 'imports' and 'exports' are right.

2 Consumer and producer surplus

Substituting c_A from the budget constraint the consumer utility in equlibrium can then be written as follows (remember that $w = p_B$)

$$U = \bar{y}_A + p_B y_B - p_B c_B + u(c_B) - v(y_B).$$

Since $\bar{y}_A$ is a constant we only worry about the other terms, which can be decomposed in two terms

$$\begin{aligned} \text{"consumer surplus"} &= u(c_B) - p_B c_B \\ \text{"producer surplus"} &= p_B y_B - v(y_B) \end{aligned}$$

(notice that here the labels "consumer surplus" and "producer surplus" are a bit confusing here because in fact both surpluses accrue to the same agent, the consumer, who is also a worker, i.e., a producer).

3 A Tariff

Now we introduce a tariff τ on the imports of good B. The domestic price of the good is

$$p_B = p_B^* + \tau,$$

while the world price is p_B^*.

The world equilibrium is given by

$$M\left(p_B^* + \tau\right) = X\left(p_B^*\right), \tag{3}$$

because domestic importers have to pay τ to the domestic government when they buy 1 unit of good B. Equation (3) gives a decreasing relation between τ and the world price p_B^* (you can prove that formally if you want). Knowing p_B^* you can find the domestic price p_B and you can derive c_B and y_B from relations (1) and (2).

The government obtains the revenue

$$\tau \cdot (c_B - y_B)$$

which is returned to consumers as a lump-sum transfer. The utility of consumers with the tariff is

$$U = \bar{y}_A + \underbrace{\left[u\left(c_B\right) - p_B c_B\right]}_{\text{consumer surplus}} + \underbrace{\left[p_B y_B - v\left(y_B\right)\right]}_{\text{producer surplus}} + \underbrace{\tau\left(c_B - y_B\right)}_{\text{gov't revenue}},$$

where

$$p_B = p_B^* + \tau. \tag{4}$$

Now we want to see the effects of changing the tariff τ on the consumers' welfare U :

$$\begin{aligned} dU &= u'\left(c_B\right) \cdot dc_B - p_B \cdot dc_B - dp_B \cdot c_B + \\ &\quad + p_B \cdot dy_B - v'\left(y_B\right) \cdot dy_B + dp_B \cdot y_B + \\ &\quad + d\tau \cdot \left(c_B - y_B\right) + \tau \cdot \left(dc_B - dy_B\right). \end{aligned}$$

Replacing the relations (1) and (2) and (4) this gives

$$dU = \underbrace{-\left(c_B - y_B\right) \cdot dp_B^*}_{\substack{\text{terms of trade gain} \\ >0}} + \underbrace{\tau \cdot \left(dc_B - dy_B\right)}_{\substack{\text{distortion} \\ <0}}.$$

When $d\tau > 0$ the first term is positive because $\frac{dp_B^*}{d\tau} < 0$, while the second term is negative because $dc_B - dy_B = M'\left(p_B^* + \tau\right)\left(\frac{dp_B^*}{d\tau} + 1\right) d\tau$ and $M' < 0$ and $\left(\frac{dp_B^*}{d\tau} + 1\right) > 0$ (we have shown above that M is a decreasing function, can you show that $\left(\frac{dp_B^*}{d\tau} + 1\right) > 0$?)

Example 1 *A country is importing 200 tons of good X, with a tariff of .5\$ per ton, the world price is 2\$ per ton. The tariff is increased to .6\$ per ton and, as a consequence the imports fall to 180 tons and the world price falls to 1.98\$. What is the approximate effect of the higher tariff on the welfare of the country's consumers?*

Answer:

$$dU = 200 \times .02 - .5 \times 20 = 4 - 10 = -6.$$

The back-of-the-envelope calculation is as follows:

- we are saving .02\$ on the 200 tons we were importing
- we are importing 20 tons less. Due to the existing tariff, the marginal value of a ton of X in the country is higher than the marginal value of a ton of X in the rest of the world. The difference between these marginal values (the distortion) is measured by the amount of the tariff, .5.

Exercise 2 *Redo the graphic derivation of consumer and producer surplus done in class. Introduce a small increase in τ and identify the two pieces in the expression above. Explain why the second term correspond to the "distortion" in the picture.*

Exercise 3 *Suppose that the demand and supply curves are linear. Use the numbers of the example and measure the exact value of the welfare loss associated to the increase in the tariff. (Answer: you should get $\Delta U = -7.2$)*