

14.64

Problems Set One Solutions

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Borjas, Problem 4-4

This question asks how the wages of (presumably high-skilled) natives should change when there is a change in the quantity of available low-skilled (immigrant) labor assuming that natives and immigrants are complements. Whenever two factors of production are complementary, an increase in one increases the returns to another. So in this case, the illegal alien hiring penalties raises the cost of unskilled labor, lowering the amount of unskilled labor that is hired which reduces the returns to skilled labor, lowering their wage.

Borjas, Problem 4-6

Elasticity of demand for labor is -0.5 . This implies that a 1% increase in wages reduces employment by 0.5%. Alternatively (just flipping the fraction), an increase in employment by 1% reduces the wage offered by firms by 2%.

- (a) For now, treat each economy (North and South) as completely isolated and as separate countries and economies, essentially. Immigration has no effect on the North, so wages in the North don't change. Immigration in the South increases the population (and hence employment, since supply is perfectly inelastic) by 20,000, which represents a 5% growth in employment which (based on the above discussion) reduces wages by 10%, to \$13.50.
- (b) Since wages fall by \$1.50 in the South, 1,500 natives will migrate from the South to the North. This represents 0.25% employment growth in the North (1500/600000), which reduces wages by 0.5%, to \$14.93. Southern employment falls from its immediate post-migration level of 420,000 to 418,500 which represents a fall in employment of 0.36%, increasing wages by 0.72%, from \$13.50 to around \$13.60.

The ratio of wages in the North to the South is $\$14.93/\13.60 , or around 1.10. Wages are about 10% higher in the North after one year of migration.

(c) In the long run, people would migrate year after year (similar to b), as long as wages are higher in one place than the other. Eventually (in the long run), wages and employment would equate. You could solve for the final wage level by recognizing that employment would eventually be 510,000 in each area, and calculating what that would do to wages. Or, you could recognize that once we are thinking long-run, there is essentially no difference between the North and South (given that the labor demand elasticity is the same in each), so you can think of the North and South as a single economy that begins with an inelastically-employed population of 1,000,000 and experiences employment growth of 20,000 which represents growth of 2%, reducing wages by 4% from \$15.00 to \$14.40.

Borjas 2-3

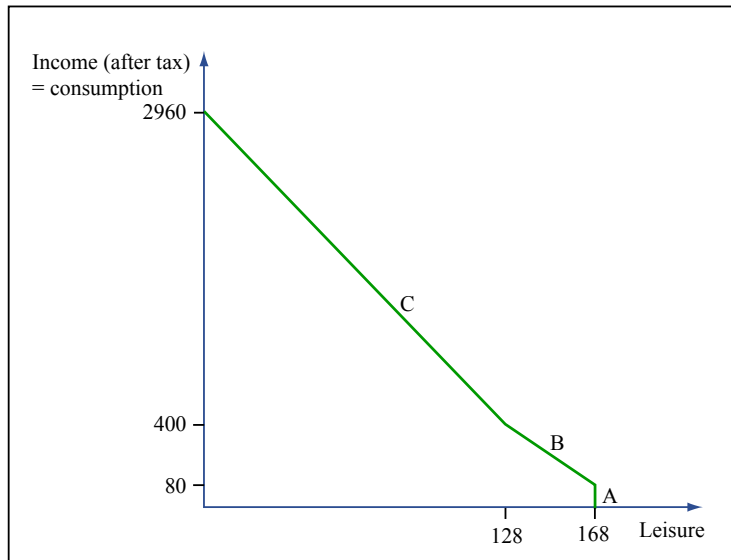


Figure by MIT OpenCourseWare.

A: Even if Tom works zero hours, he gets \$80.

B: For the 1st 40 hours, Tom earns $\underbrace{\$15}_{\text{wage}} - \underbrace{0.2 \cdot \$15}_{\text{tax}} - \underbrace{\$4}_{\text{child care}} = \$8 \rightarrow$ slope of segment B is $-\$8$.

C: After 40 hours, Tom earns $\underbrace{\$30}_{\text{wage}} - \underbrace{0.2 \cdot \$30}_{\text{tax}} - \underbrace{\$4}_{\text{child care}} = \$20 \rightarrow$ slope of segment C is $-\$20$.

Borjas 2-6

a.

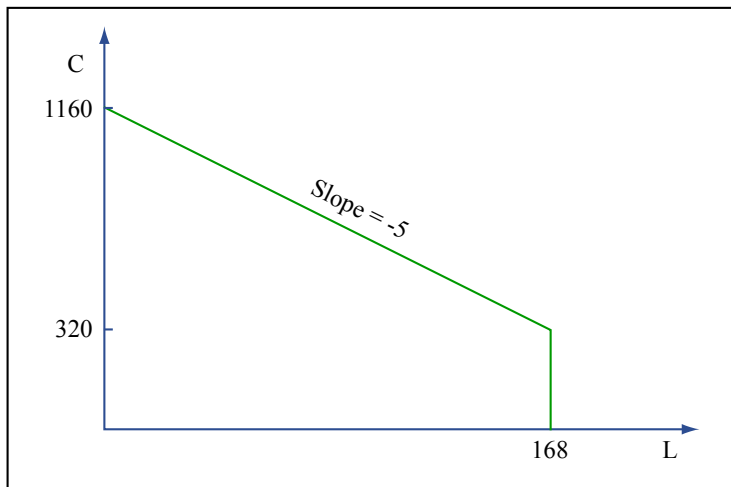


Figure by MIT OpenCourseWare.

b. The $MRS = -\frac{dC}{dL}\bigg|_{U=U^*} = \frac{\partial U/\partial L}{\partial U/\partial C} = \frac{C-20}{L-80} = \frac{460}{20} = 23$

Here $L = 100$. At this point, $C = 1160 - 5L = 1160 - 500 = 660$

c. Her reservation wage is the wage she must be paid to be just indifferent between working and not working when $L = 168$: the MRS at $L = 168$. Here $C = 320$ (just her welfare benefits). $MRS = \frac{C-200}{L-80} = \frac{320-200}{168-80} = \1.36

d.

$$\max_{C,L} (C - 200)(L - 80)$$

$$\text{s.t. } C = 1160 - 5L$$

$L \leq 168 \rightarrow$ forget about this constraint at first. If the optimum we find without imposing this restriction has $L \leq 168$, then we don't need to impose it.

$$\mathcal{L} = (C - 200)(L - 80) + \lambda(C - 1160 + 5L)$$

$$\frac{\partial \mathcal{L}}{\partial C} = L - 80 + \lambda = 0 \quad (1) \Rightarrow L - 80 = -\lambda$$

$$\frac{\partial \mathcal{L}}{\partial L} = C - 200 + 5\lambda = 0 \quad (2) \Rightarrow C - 200 = -5\lambda$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = C - 1160 + 5L = 0 \quad (3) \Rightarrow C = 1160 - 5L$$

Combining (1) and (2): $\frac{C-200}{L-80} = 5$

Substituting in from (3): $\frac{1160-5L-200}{L-80} = 5 \Rightarrow 1160 - 5L - 200 = 5L - 400$

$L = 136; C = 480.$

Since $L = 136 < 168$, $L \leq 168$ is satisfied.

Borjas, Problem 2-8

A diff-in-diff estimator for the effect of the NIT is:

$$[Y_{2000,NIT} - Y_{1999,NIT}] - [Y_{2000,TANF} - Y_{1999,TANF}]$$

We can represent these outcomes in two tables: one for whether the individual worked at all and one for total hours of work.

Employment Rate

	1999	2000	Difference (2000-1999)
TANF	1,217/4,392	1,568/4,392	0.080
NIT	131/468	213/468	0.175
Difference (NIT-TANF)	0.003	0.098	0.095

Hours Per Capita

	1999	2000	Difference (2000-1999)
TANF	15,578/1,217	20,698/1,568	0.400
NIT	1,638/131	2,535/213	-0.602
Difference (NIT-TANF)	-0.297	-1.299	-1.02

The NIT increased the employment rate of public assistance recipients by 9.5%. It decreased the weekly average hours worked among recipients who worked positive hours by 1.02 hours.

Borjas 2-14

a.

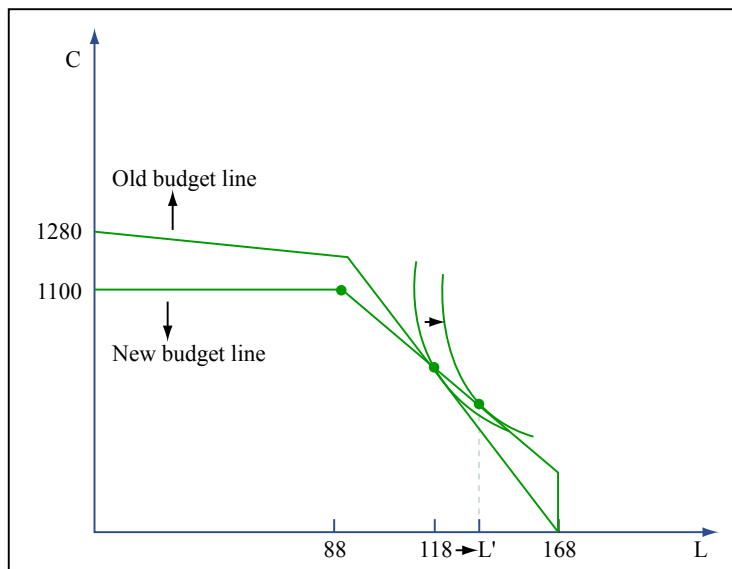


Figure by MIT OpenCourseWare.

Hourly wage rate:

$$\text{Before the NIT: } \underbrace{20}_{\text{wage}} - \underbrace{0.2 \cdot 20}_{\text{tax}} = 16$$

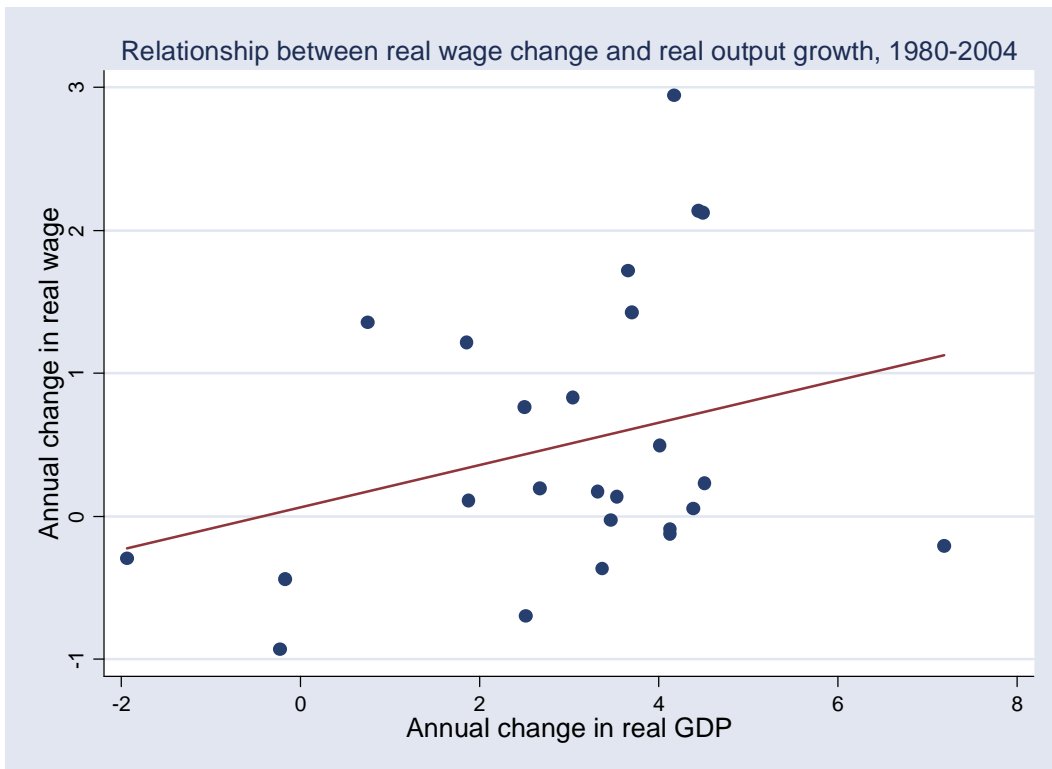
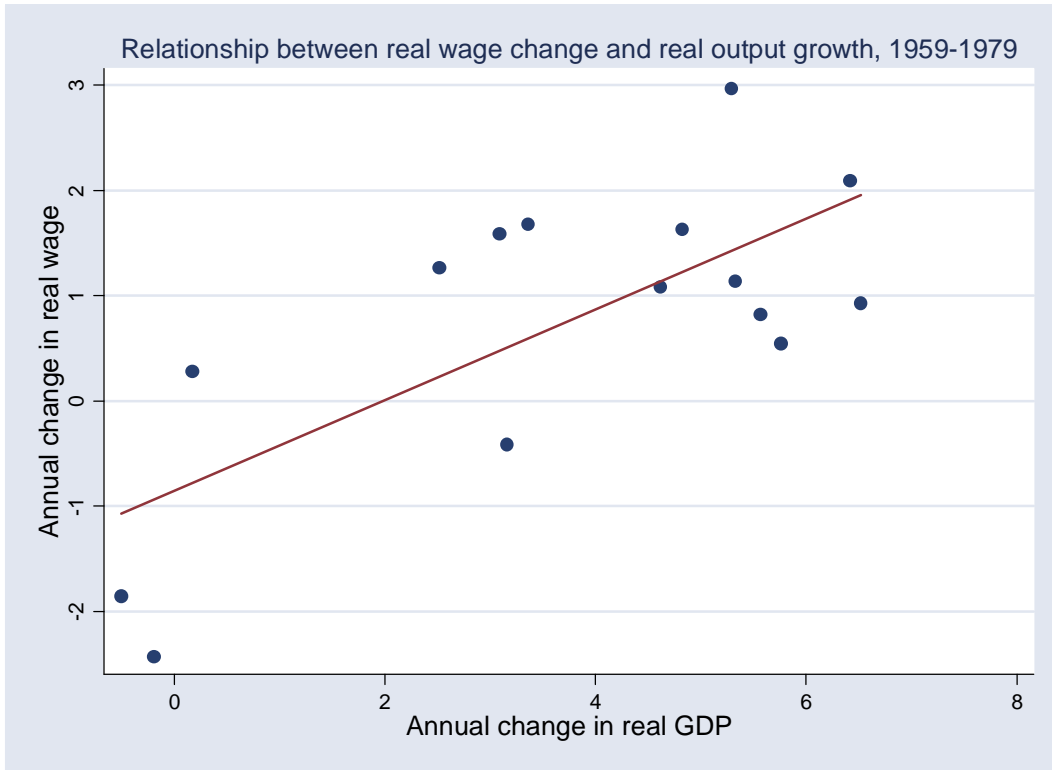
$$\text{With the NIT: } \underbrace{20}_{\text{wage}} - \underbrace{0.5 \cdot 20}_{\text{tax}} = 10$$

b. See the graph. Without the NIT, her indifference curve is tangent to her “old” budget line at $L = 118$. Her indifference curve is tangent to her budget line under the NIT at $L = L' > 118$. Since she has more leisure under the NIT she works fewer hours.

c. Her utility will be greater. She has the option of choosing the same bundle of C and L she chose before the NIT once the NIT is in place. Since she doesn't, here utility must be higher under the NIT.

Problem B:

1) Using data from the Economic Report of the President:



And related regression output:

```
. reg ch_wages_real ch_realGDP if year<1980
```

Source	SS	df	MS	Number of obs =	15
Model	14.4489043	1	14.4489043	F(1, 13) =	13.69
Residual	13.7159996	13	1.0550769	Prob > F =	0.0027
Total	28.164904	14	2.01177886	R-squared =	0.5130
				Adj R-squared =	0.4756
				Root MSE =	1.0272

ch_wages_r~1	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
ch_realGDP	.4308531	.116427	3.70	0.003	.1793278 .6823784
_cons	-.8533103	.5086612	-1.68	0.117	-1.952206 .2455854

```
. reg ch_wages_real ch_realGDP if year>=1980
```

Source	SS	df	MS	Number of obs =	25
Model	1.87154841	1	1.87154841	F(1, 23) =	2.03
Residual	21.200494	23	.921760609	Prob > F =	0.1676
Total	23.0720424	24	.961335101	R-squared =	0.0811
				Adj R-squared =	0.0412
				Root MSE =	.96008

ch_wages_r~1	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
ch_realGDP	.1477099	.1036616	1.42	0.168	-.0667306 .3621503
_cons	.0633763	.3671484	0.17	0.864	-.696128 .8228807

(where ch_wages_real is annual fraction change in real wages, ch_realGDP is annual fraction change in real GDP)

So by comparing the two regressions, we can note two relevant facts:

- 1) the magnitude of the relationship between annual changes in real GDP and annual changes in real wages appears stronger in the earlier period than the later period
- 2) the variability in annual real wage changes is better explained by annual changes in real GDP in the earlier period (because the R-squared is much higher in the earlier period regression).

So this seems to suggest that the relationship between wage change and GDP change was stronger prior to 1980, as the claim in the problem suggests.

We might want to know whether the difference between the two coefficients (.431-.148=.283) is “statistically significant” (although statistical significance is perhaps irrelevant in this context because we have the entire population of data rather than a sample from a population – we’re not trying to use a sample regression to infer anything about a population regression). You might be tempted to try and construct a t-statistic for the difference between coefficients, but to do so you would need to somehow construct the standard error of the difference in coefficients, which would require the covariance of the coefficients, which is difficult – or impossible – to get.

An alternative is to run the following equation on the entire sample of data (1959 through 2004):

$$ch_wages_real_t = \alpha + \beta_1 ch_realGDP_t + \beta_2 ch_realGDP_t * yr1980_t + \beta_3 yr1980_t + \varepsilon_t$$

where yr1980 is a dummy variable recording a 1 if the observation is in 1980 or later, and a 0 if before 1980.

Now, testing whether β_2 is different from zero is EXACTLY the same as testing whether the coefficients on ch_realGDP are equivalent in the earlier regressions. The way to think about the above regression is that it's allowing some overall relationship to exist between annual GDP and wage changes over the entire sample (that's what β_1 is) as well as allowing the relationship to be different post-1980 (that's what β_2 is measuring).

You could think of it this way: how is real GDP change predicted to affect real wage change?

$$\frac{d(ch_wages_real_t)}{d(ch_realGDP_t)} = \beta_1 + \beta_2 * yr1980_t$$

Prior to 1980, a real GDP increase of 1%age point is predicted to increase real wages by β_1 %age points. Post-1980, the same change is predicted to increase real wages by $\beta_1 + \beta_2$. If $\beta_2 = 0$, then the relationship between real wage change and real GDP change is the same. Hence, running the above regression allows us to explicitly test whether the relationship of interest is different in one sample from another.

Here's the output from such a regression:

```
. reg ch_wages_real ch_realGDP ch_realGDP_80 yr1980
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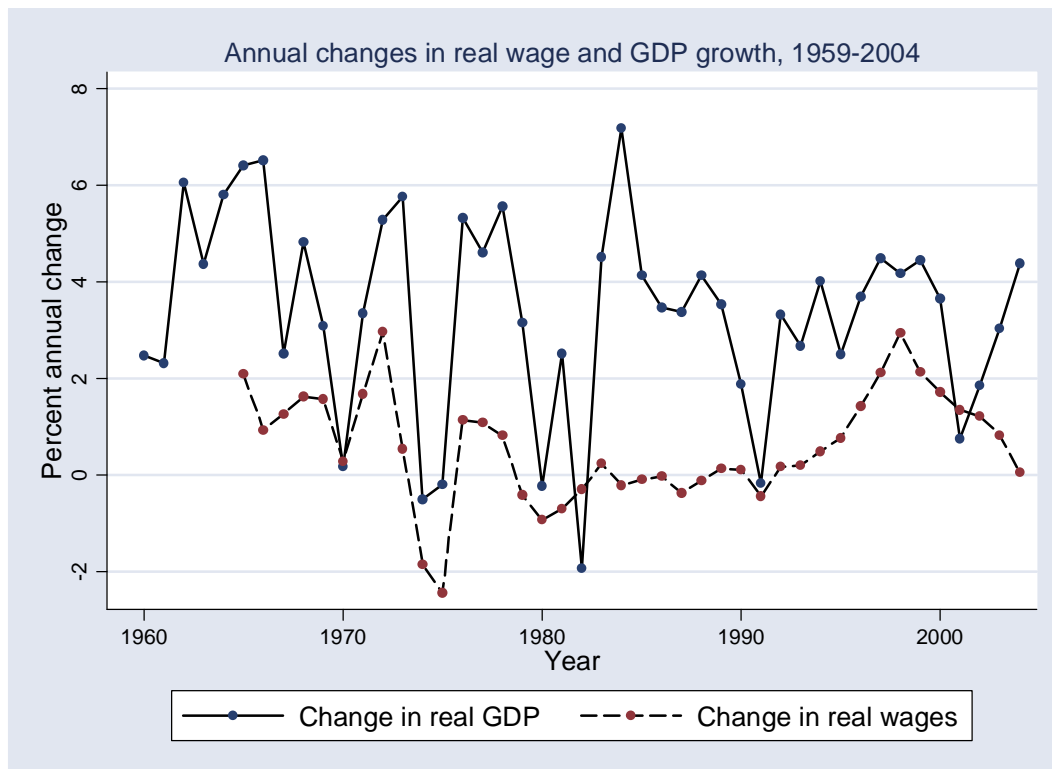
Source	SS	df	MS		Number of obs =	40
Model	16.8770354	3	5.62567846		F(3, 36) =	5.80
Residual	34.9164937	36	.969902602		Prob > F =	0.0024
					R-squared =	0.3259
					Adj R-squared =	0.2697
Total	51.793529	39	1.32803921		Root MSE =	.98484

ch_wages_r~1	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
ch_realGDP	.1477099	.1063342	1.39	0.173	-.0679459 .3633656
ch_realGD~80	.2831432	.1541685	1.84	0.075	-.029525 .5958114
yr1980	-.9166867	.6161876	-1.49	0.146	-2.166373 .3329997
_cons	.0633763	.3766141	0.17	0.867	-.7004325 .8271852

Note that the coefficient on the interaction term (i.e. β_2) is .283, exactly our difference in coefficients from running the regressions on the two periods separately. We can also see that this coefficient is marginally significant (the t-stat is slightly below 2, the p-stat is slightly above .05) – but given that our sample size is so small, this might be enough for

us to declare that the coefficients over the two periods are different. (Again, it's not clear that we actually have to do this, because the regressions we're running are on the actual populations – not samples – and so we aren't trying to infer anything about population coefficients from our regressions. Our regressions give us the actual population coefficients. Nonetheless, it's important to understand how one could use the regression described above to get the difference in coefficients from two separate regressions in one single regression).

So the conclusion is that the relationship between wage growth and GDP growth is indeed different post-1980 than pre-1980. In particular, the magnitude of the relationship between the two is weaker post-1980 – that is, a change in GDP growth is associated with a smaller change in wage growth. Additionally, the strength of the relationship is weaker post-1980 – after 1980, it appears that variation in GDP growth explains a smaller fraction of the variation in wage growth. Of course this doesn't prove that the relationship changed around 1980 – but a graph might help convince us of this:



Indeed, prior to 1980 annual wage changes match annual GDP changes fairly well. Throughout the entire 1980 decade, this relationship is non-existent as real wages remain stagnant and GDP grows (although the growth rate is declining); post 1990, the relationship looks a bit stronger again.

2) We'd expect that if production is growing (i.e. GDP is increasing), then demand for labor should increase in response – so both employment and wages should increase, and we would observe a positive relationship between wage and GDP growth. On the other

hand, one could imagine a labor supply shock occurring (say, if tax rates change drastically, or if a large social program with negative employment effects is enacted) – if labor supply shifted backwards for some reason, then we would observe wages increase, and since employment falls, production would fall as well (at least in the short term). In this case, we would observe a *negative* relationship between wage and GDP growth. Hence, there is not a clear structural interpretation of the relationship between wage and GDP growth – one could observe a positive or negative relationship, depending on whether labor demand expands or labor supply contracts.

There are other considerations that might limit the extent of a relationship between GDP and wage growth. If labor is extremely expensive (perhaps due to institutional constraints such as a minimum wage, or due to strong unions demanding high wages), or if capital is relatively inexpensive or easy to implement for increasing production (if, for example, unions are weak and unable to resist the introduction of labor-replacing technologies), then even with a non-shifting supply curve, one might expect increased economic growth to have much smaller effects on wage growth (since increased production is derived from the use of capital rather than labor).

The fact that there is no clear theoretical relationship between economic growth and wage growth means that we cannot view our estimate of the relationship between the two as measuring any structural parameter in a supply and demand model. All that we can say is that for our sample, we found the above relationship between economic growth and wage growth.

C. Analytical Problem

We are interested in $\frac{d \ln(\text{Tax Revenue})}{d \ln(\text{Tax Rate})} = \frac{d(\text{Tax Revenue})/(\text{Tax Revenue})}{d(\text{Tax Rate})/(\text{Tax Rate})}$, the elasticity of tax revenue with respect to the tax rate.

Define the following variables

- τ = the tax rate
- w = the hourly wage rate
- s = take-home earnings = $(1 - \tau)w$
- $H(s)$ = individual labor supply
- R = tax revenue

Then, we know that

$$R = \tau w H(s)$$

$$\frac{dR}{d\tau} = \frac{\partial R}{\partial \tau} + \left(\frac{\partial R}{\partial H(s)} \right) \left(\frac{dH(s)}{ds} \right) \left(\frac{\partial s}{\partial \tau} \right)$$

$$\frac{dR}{d\tau} = wH(s) + (\tau w) \left(\frac{dH(s)}{ds} \right) (-w)$$

$$\frac{dR/R}{d\tau/\tau} = \frac{\tau}{R} \left[wH(s) + (\tau w) \left(\frac{dH(s)}{ds} \right) (-w) \right]$$

$$\frac{dR/R}{d\tau/\tau} = \frac{1}{wH(s)} \left[wH(s) - (\tau w^2) \left(\frac{dH(s)}{ds} \right) \right]$$

$$\frac{dR/R}{d\tau/\tau} = 1 - \left(\frac{\tau w^2}{wH(s)} \right) \left(\frac{dH(s)}{ds} \right)$$

$$\frac{dR/R}{d\tau/\tau} = 1 - (\tau w) \left(\frac{dH(s)/H(s)}{ds} \right)$$

$$\frac{dR/R}{d\tau/\tau} = 1 - (\tau w) \left(\frac{dH(s)/H(s)}{ds} \right)$$

$$\frac{dR/R}{d\tau/\tau} = 1 - \left(\frac{\tau w}{s} \right) \left(\frac{dH(s)/H(s)}{ds/s} \right)$$

$$\frac{dH(s)/H(s)}{ds/s} = \text{the elasticity of labor supply}$$

When this is large, since τ , w , and s are all positive, $\frac{dR/R}{d\tau/\tau}$ will be negative. Then, when there is a positive (percentage) change in the tax rate, there will be a negative (percentage) change in tax revenue.

C. First, INELASTIC:

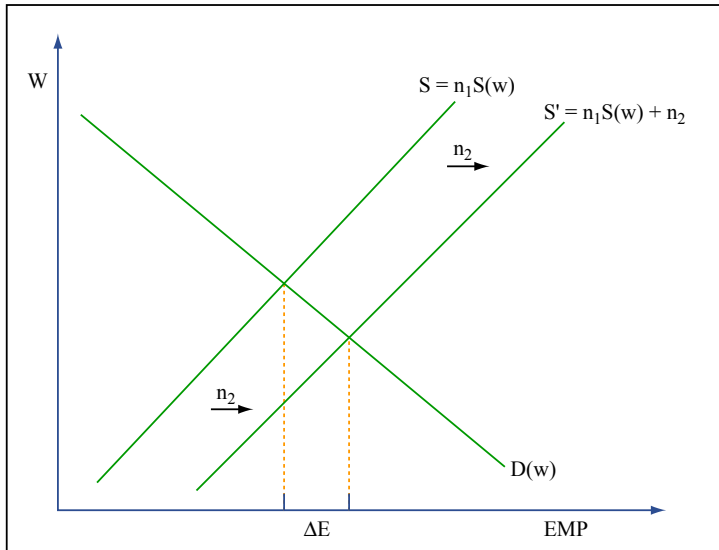


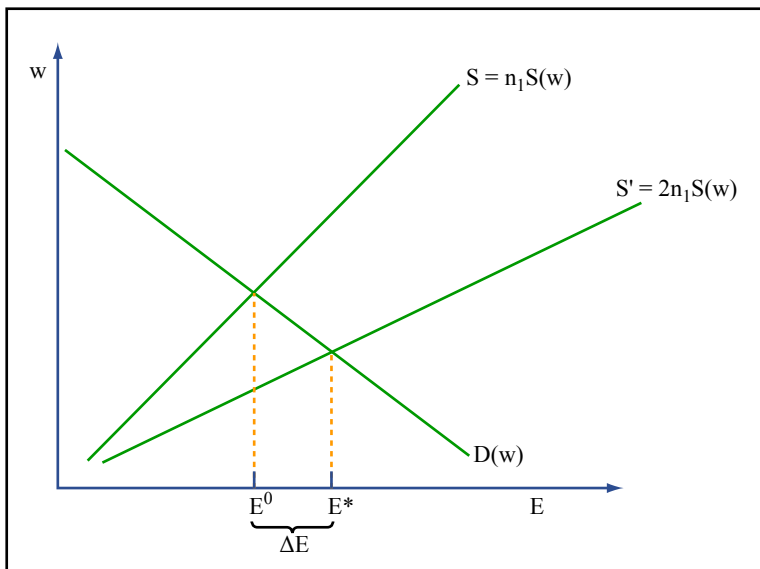
Figure by MIT OpenCourseWare.

Here, employment changes by $\Delta E < n_2$, because some natives reduce their LS in response to the immigration/wage reduction. Now immigrant LS is somewhat elastic. $LS = n_2S(w)$

Old $S = n_1S(w)$

New $S = n_1S(w) + n_2S(w) = (n_1 + n_2)S(w) = 2n_1S(w)$ (because $n_1 = n_2$)

Graphically, this makes the new supply curve flatter.



$$E^* = 2n_1S(w)$$

$$\frac{E^*}{2} = n_1S(w) = \text{new native employment}$$

$$\text{so } E^0 - \frac{E^*}{2} = \Delta \text{native employment}$$

Figure by MIT OpenCourseWare.

Note: why does the supply curve become flatter?

Simple example: Suppose $S(w)$ is linear (as we draw it for convenience), so $S(w) = \alpha w + \beta$.

Let $\alpha = 1, \beta = 0$ for simplicity. Then $\frac{S(w)}{\text{Employment}} = n_1 w$, so $w = \frac{E}{n_1} \Rightarrow$ supply curve. When

immigrants enter, $S(w) = 2n_1 w$, so $w = \frac{E}{2n_1} \Rightarrow$ new supply curve (which is flatter than the old curve).

$$D(w) = n_1 S(w) + n_2$$

Take total derivative (i.e. take the derivative with respect to n_2 on each side)

$$\frac{dD(w)}{dw} \frac{dw}{dn_2} = n_1 \frac{dS(w)}{dw} \frac{dw}{dn_2} + 1$$

Multiply by dn_2 , let $\frac{dD(w)}{dw} = D'$ and $\frac{dS(w)}{dw} = S'$

$$D' dw = n_1 S' dw + dn_2$$

From lecture (and section notes), $\frac{d \log w}{d \log n_2} = \frac{\phi}{n_D - \epsilon(1 - \phi)}$

$L_1 = n_1 S(w) \leftarrow$ native LS

$$\frac{d \log L_1}{d \log n_2} = \frac{d \log L_1}{\underbrace{d \log w}_{\epsilon}} \frac{d \log w}{d \log n_2} = \frac{\epsilon \phi}{n_D - \epsilon(1 - \phi)}$$

We're interested in $\frac{dL_1}{dn_2}$. To get this, note that: $\frac{d \log L_1}{d \log n_2} = \frac{dL_1/L_1}{dn_2/n_2} = \frac{dL_1}{dn_2} \frac{n_2}{L_1}$, so multiplying

$\frac{d \log L_1}{d \log n_2}$ by $\frac{L_1}{n_2}$ should give us what we want:

$$\frac{L_1}{D} = 1 - \phi, \quad \frac{n_2}{D} = \phi, \quad \text{so } \frac{L_1}{n_2} = \frac{1 - \phi}{\phi}$$

and

$$\frac{dL_1}{dn_2} = \frac{\epsilon(1 - \phi)}{n_D - \epsilon(1 - \phi)}$$

ELASTIC

This isn't much harder theoretically, but the algebraic manipulation is somewhat confusing.

$$D(w) = n_1 S(w) + n_2 S(w)$$

Differentiate by dn_2 and multiply by dn_2 , noting that $\frac{dn_2 S(w)}{dn_2} = n_2 S'(w) \frac{dw}{dn_2} + S(w)$ by the chain rule:

$$D'(w)dw = n_1 S'(w)dw + n_2 S'(w)dw + S(w)dn_2$$

We want to get elasticities, so divide by D and multiply by $\frac{w}{w}$:

$$\frac{D'(w)}{D/w} \frac{dw}{w} = \frac{n_1 S'(w)}{D} w \frac{dw}{w} + \frac{n_2 S'(w)}{D} w \frac{dw}{w} + \frac{S(w)}{D} dn_2$$

Multiply right hand side by $\frac{S}{S}$:

$$n_D \frac{dw}{w} = \frac{n_1 S}{D} \left(\frac{S'}{S/w} \right) \frac{dw}{w} + \frac{n_2 S}{D} \left(\frac{S'}{S/w} \right) \frac{dw}{w} + \frac{S}{D} dn_2$$

Let $\epsilon = \frac{S'}{S/w}$ (supply elasticity); $\phi = \frac{n_2 S}{D}$ (share of immigrants);

$1 - \phi = \frac{n_1 S}{D}$ (share of natives) $\frac{S}{D} = \frac{\phi}{n_2}$

Substitute to obtain:

$$n_D \frac{dw}{w} = (1 - \phi)\epsilon \frac{dw}{w} + \phi\epsilon \frac{dw}{w} + \frac{dn_2}{n_2} \phi$$

$$n_D d \log w = \epsilon d \log w + \phi d \log n_2$$

$$d \log w (n_D - \epsilon) = \phi d \log n_2$$

$$\frac{d \log w}{d \log n_2} = \frac{\phi}{n_D - \epsilon}$$

$$\frac{d \log L_1}{d \log n_2} = \frac{\epsilon \phi}{n_D - \epsilon}, \quad \frac{d L_1}{d n_2} = \frac{\epsilon(1 - \phi)}{n_D - \epsilon}$$

So let's compare:

	$\frac{d \log w}{d \log n_2}$	$\frac{dL_1}{dn_2}$
INELASTIC	$\frac{\epsilon \phi}{n_D - \epsilon(1 - \phi)}$	$\frac{\epsilon(1 - \phi)}{n_D - \epsilon(1 - \phi)}$
ELASTIC	$\frac{\phi}{n_D - \epsilon}$	$\frac{\epsilon(1 - \phi)}{n_D - \epsilon}$

The denominator for both expressions is larger in the elastic case, so wages don't fall by as much (and native employment doesn't fall by as much) in response to immigration. Why? Because in the elastic case, an influx of *bodies* results in less LS than before, because immigrants themselves respond to a lower wage by reducing their own labor supply.

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