14.662 Recitation 10

Goldberger (1984): Reverse Regression and Salary Discrimination

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Regressing Discrimination

- Often (both in academia and the "real world") discrimination is diagnosed by regressions of the form

$$y = x'a + bz + e \tag{1}$$

  where z indicates a sex/race and x are other relevant "qualifications"

- Another approach is the "reverse" regression of, for $q \equiv x'a$:

$$q = cy + dz + u \tag{2}$$

- A naïf might expect $d < 0$ if $a > 0$ ("if men earn more than equally-qualified women, they're less qualified than equally-paid women")
  - But that's only true for deterministic relationships
- We might think (2)$\succ$(1) if qualifications are measured with error (suppose, for some reason, we're not worried about OVB)
  - Goldberger (1984) shows this preference may be ill-founded

Forward and Reverse Regressions

A. Median Income by Schooling

	Schooling (years)							
					High School		College	
	None	1–4	5–7	8	1–3	4	1–3	4
	Dollars							
White	1569	1962	3240	3981	5013	5529	6104	7779
Nonwhite	1042	1565	2353	2900	3253	3735	4029	4840

B. Median Schooling by Income

	Income ($1000s)									
	None	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–9	10+
	Years									
White	8.4	8.0	8.4	8.7	9.5	10.5	11.4	12.1	12.4	14.0
Nonwhite	6.9	5.1	6.5	7.8	8.7	9.3	10.4	11.2	12.1	12.8

Source: Hashimoto and Kochin [20].

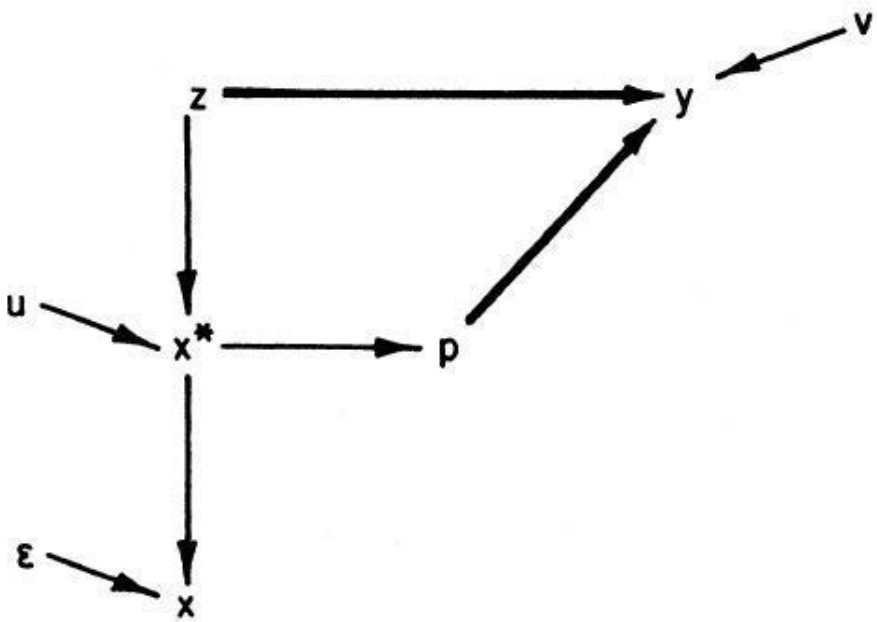
- Both $a > 0$ and $d > 0$ (Hashimoto and Kochin call this a "riddle")

Reverse $\succ$ Forward?

- Reverse regression often suggests less discrimination (in favor of men, whites, etc.), and sometimes even reverse discrimination
  - Conway and Roberts (1983): $a = 0.15$, $d = -0.01$ in a sex regression for 274 bank employees, education/experience/age controls
  - Abowd, Abowd, and Killingsworth (1983): $a, d > 0$ in a race regression from the 1976 Survey of Income and Education
- Conway and Roberts (1983): *"The problem of omitted job qualifications points to the weakness of a direct-regression-adjusted income differential [relative to reverse regression]"*
- Goldberger shows this is true only in very special case where salary is a deterministic function of productivity and gender
  - In a more general EIV model, forward reg. will be upward-biased and reverse reg. will be downward-biased
  - ...but in another "proxy variable" model forward can be unbiased while reverse is still downward-biased

Multivariate EIV

Data-generating process:



$$y = \alpha z + p + \nu$$

$$p = \beta x^*,\ x^* = \mu z + u,\ x = x^* + \varepsilon$$

where ν, u, and ε are all white noise terms

Forward Regression

- Estimate $E[y|x,z] = az + bx$ (we normalize everything to be mean-zero for women)

$$b = \frac{Cov(y,\tilde{x})}{Var(\tilde{x})} = \frac{Cov(\alpha z + \beta x^* + \nu, x^* + \varepsilon - \mu z)}{Var(x^* + \varepsilon - \mu z)}$$
$$= \frac{Cov((\alpha + \beta\mu)z + \beta u + \nu, u + \varepsilon)}{Var(u+e)} = \beta \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}$$

- So

$$\begin{aligned} a &= E[y|z=1] - bE[x|z=1] \\ &= \alpha + \beta E[x^*|z=1] - bE[x|z=1] \\ &= \alpha + (\beta - b)\mu \\ &= \alpha + \beta\mu \frac{\sigma_e^2}{\sigma_u^2 + \sigma_e^2} \end{aligned}$$

- Regression puts more weight on a positive correlate to a noisy signal

Reverse Regression

- By substitution,

$$y = (\alpha + \beta\mu)z + \beta u + v$$

- Estimate $E[x|y,z] = cy + dz$

$$c = \frac{Cov(x,\tilde{y})}{Var(\tilde{y})} = \frac{Cov(\mu z + \varepsilon + u, \beta u + v)}{Var(\beta u + v)} = \frac{\beta\sigma_u^2}{\beta^2\sigma_u^2 + \sigma_v^2}$$

- So

$$\begin{aligned} d &= E[x|z=1] - cE[y|z=1] \\ &= \mu - c(\alpha + \beta\mu) \\ &= \frac{\sigma_v^2}{\beta^2\sigma_u^2 + \sigma_v^2}\mu - c\alpha \end{aligned}$$

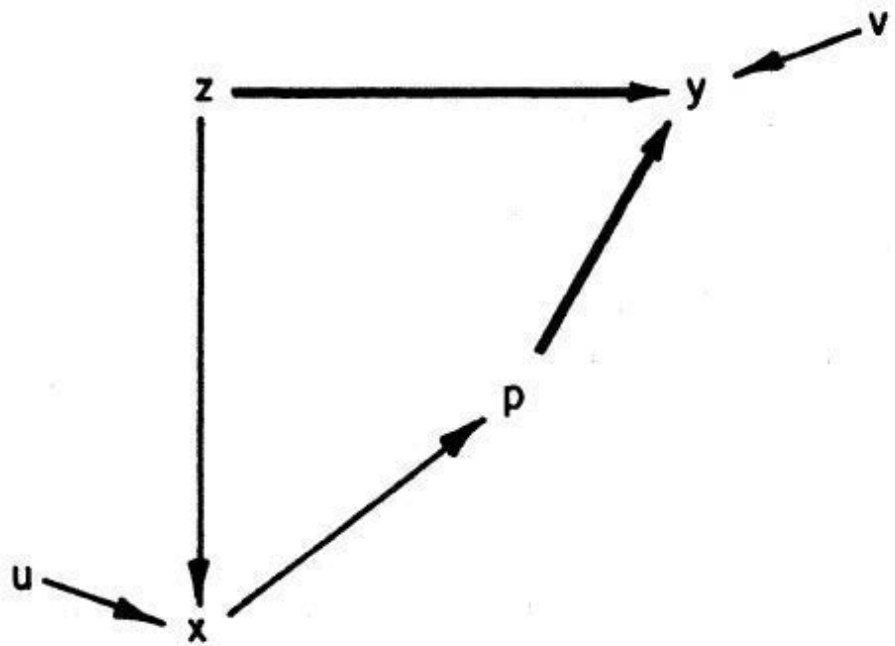
- Implied discrimination coefficient: $-d/c = \alpha - \mu\sigma_u^2/(\beta\sigma_v^2) < \alpha$

Comparing Forward and Reverse

- Forward regression gives an upper bound on α, while reverse regression gives a lower bound
  - Bounds are tighter when μ is smaller (so z and x^* are less correlated)
  - α is closer to a when β is smaller or σ_u^2 is larger
  - α is closer to $-d/c$ when β is larger or σ_u^2 is smaller
- If $\sigma_v^2 = 0$ (deterministic salary function), $d = -c\alpha$ and reverse regression is indeed unbiased (but not otherwise)
  - Dempster (1982): *"[we are] somewhat skeptical about the existence of a chance mechanism whereby the employer creates a random disturbance an adds it"*
  - Are mismeasured qualifications fallible measures of true productivity ($\sigma_v^2 = 0$) or of its determinants ($\sigma_v^2 > 0$)?

x as a "Proxy" for True Qualification

Data-generating process:



$$y = \alpha z + p + \nu$$

$$p = \beta x + \varepsilon,\ x = \mu z + u$$

where ν and ε are white noise terms

Forward and Reverse Regression

- Note that by substitution

$$y = \alpha z + \beta x + \varepsilon + \nu$$

since ε and ν are white noise, forward regression will be unbiased

- Reverse regression: for $E[x|y,z] = cy + dz$

$$c = \frac{Cov(x,\tilde{y})}{Var(\tilde{y})} = \frac{Cov(\mu z + u, \beta u + \varepsilon + \nu)}{Var(\beta u + \varepsilon + \nu)} = \frac{\beta \sigma_u^2}{\beta^2 \sigma_u^2 + \sigma_\varepsilon^2 + \sigma_\nu^2}$$

and $d = \frac{\sigma_\nu^2}{\beta^2 \sigma_u^2 + \sigma_\varepsilon^2 + \sigma_\nu^2} \mu - c\alpha$ as before; $-d/c < \alpha$

- Now reverse regression bias persists even if the salary function is deterministic: we have $\sigma_e^2 + \sigma_\nu^2 > 0$ even if $\sigma_\nu^2 = 0$

- Bias may be large enough that the reverse regression estimate may be of the wrong sign

Takeaways

- Goldberger (1984) is a nice illustration of how discrimination regressions may be hard to interpret
  - Whether forward or reverse is correct depends on assumed DGP
  - This kind of regression gymnastics builds character!
- Today we would likely care much more about OVB/misspecification
  - If the true wage CEF is nonlinear, forward regression may be sensible and reverse may not (Racine and Rilstone, 1994)
  - If men and women have unobservably different productivity, everything goes out the window
- Is it clear we want to control for productivity?
  - May capture a narrow definition of discrimination (Lundberg and Startz, 1983)

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