MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.007 – Electromagnetic Energy: From Motors to Lasers
Spring 2011

Pre-Lab 1: DC Motors
Due: Start of Lab 1 (7:00PM on Tuesday, Feb 8 / Wednesday, Feb 9, 2011)

Introduction

Before coming to the lab:

- Read this handout

- Complete the lab kit

- Answer the pre-lab questions at the end of this handout

- Print out and read the Lab 1 document

- Bring the Lab 1 document to lab with you, along with the completed lab kit and pre-lab questions answered. You will be checked off for your completion of the pre-lab. For Lab 1, the pre-lab is worth 30% of your total lab score.

1 Theory

This theory section attempts to explain everything about DC motors that you need to know for this lab. We start by reviewing the rotational dynamics from 8.01, specifically the relationship between speed, torque, and power. Next we look at a simple design for a DC motor and find expressions for the forces acting on the shaft. Finally we tie the two sections together and explain how a DC motor behaves in a circuit.

1.1 Rotational Dynamics Review

Remember back to 8.01 when you learned about rotation dynamics. Instead of balancing forces, you balanced torque. Torque is a force acting at a distance. You know torque from seesaws. If you are on the 10 m end of the seesaw in Figure 1, you need to weight three times as much as the person on the 30 m end to balance him out.
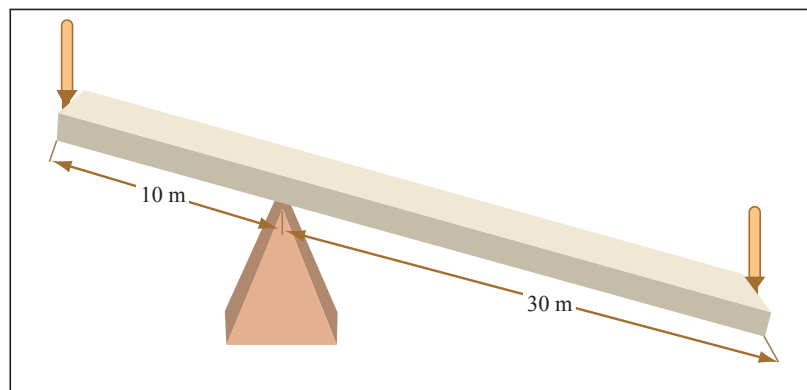
Figure 1: Seesaws demonstrate the need to balance torque, not force.

We can express this observation mathematically. Call your mass m_1 and your friend's mass m_2. The gravitational force is $m_1 g$, but your torque (acting counterclockwise at fulcrum) is $|\boldsymbol{\tau}| = |\mathbf{r} \times \mathbf{F}| = 10 \cdot m_1 g$ (we'll ignore the direction of the cross product). Likewise your partner's torque is $30 \cdot m_2 g$, but his acts clockwise. To have a fun time seesawing, you need the torques to balance. So $10 \cdot m_1 g = 30 \cdot m_2 g$ or $m_1 = 3m_2$. We're assuming that the board is massless.

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$: the farther out you can apply the force, the more torque you exert. This is your leverage. If you weighed less than three times your partner, he or she should sit closer in on the board to reduce his torque. For more about torque, see the Wikipedia entry.

What does this mean for motors? In the next section we break apart a DC motor and show that the rotational force on the shaft acts at a distance (r), causing the shaft to rotate. The strength of the motor is quoted as the torque forcing the shaft to turn.

Now let's talk about friction. Imagine pushing a block across a plane when there is friction between the block and the plane. You push this block at a constant speed, so its kinetic energy does not change. But you are definitely doing work; your power is $\mathbf{P} = \mathbf{F} \cdot \mathbf{v}$. The energy that you put into the system is dissipated as heat through the frictional force. The block moves at a constant speed because the net force is zero: $\mathbf{F}_{\text{push}} = -\mathbf{F}_{\text{fric}}$.

The same effect occurs in rotational systems. Imagine a silly scheme where you twist a paper clip around a pencil and apply a constant *torque* to the paper clip. (This means that the *force* is rotating with the clip.) In the absence of friction, the paper clip will accelerate indefinitely. In reality, there is friction between the paper clip and the pencil, and the clip will only accelerate until $\tau_{\text{push}} = -\tau_{\text{fric}}$.

A short digression about the strength of the frictional force. In 8.01 we generally assume that the frictional force is independent of velocity. This approximation is useful when air resistance is negligible. Motor rotors can encounter significant air resistance, and we generally assume that the frictional force grows in strength proportional to the speed. As the motor spins faster, the frictional force will grow quite large.

When the frictional torque equals the torque that you are able to exert with your finger, the paper clip will cease to accelerate. Instead, it will turn at a constant rotational speed that we will measure in radians per second, ω. You are still doing work. Like the linear case in pushing the block, the power is $\tau\omega$ and all of the energy is dissipated as heat.
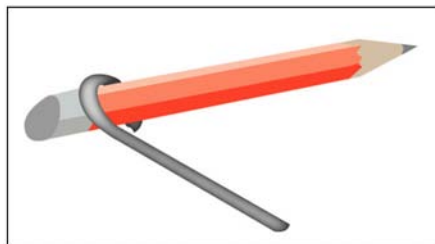

Image by MIT OpenCourseWare.

We can quantify the frictional torque. The force acts at a small radius but grows with increasing speed. So torque looks something like $\tau_{\text{f}} = rF_{\text{f}}(\omega)$. As we stated, the frictional force grows proportional to speed: $F_{\text{f}}(\omega) = c\omega$. Generally we roll the two constants (r and c) together into β: $\tau_{\text{f}}(\omega) = \beta\omega$. Here β looks like innocuous constant; however, it changes when you put the motor in a different environment. For instance, a heavier load will increase the friction on the motor shaft. β also changes when we use different gears.

When we turn the motor on, the shaft will accelerate to a maximum speed. This equilibrium occurs when the net torque is zero – when the forward torque from the Lorentz force is equally opposed by the frictional torque acting about the shaft.

1.2 Electromagnetics in a DC Motor

Every DC motor has a rotor and a stator, each with its own magnetic field.[1] The rotor rotates relative to the fixed stator because of the attraction between the two magnetic dipoles. Different motors have different methods of generating the magnetic fields; it is those differences that give us a variety of DC motor configurations.

The pedagogical DC motor (depicted in Figure 2) is made from two permanent magnets, a wire loop, a commutator, and a voltage source. The current carrying loop in the magnetic field experiences a Lorentz force, which acts to rotate the coil clockwise. You may want to take a moment to figure out which forces are causing the loop to rotate. (Recall that $\hat{\mathbf{F}} = \hat{\mathbf{I}} \times \hat{\mathbf{B}}$.) For example, when the plane of the loop is perpendicular to the B-field (which points from N to S), there is no net force on the loop. However, its momentum carries it past this equilibrium point. Furthermore, the current reverses direction through the coil because of the commutator, the metal contacts shown in silver. Because of this current reversal, the coil continues to feel clockwise torque and rotates further.
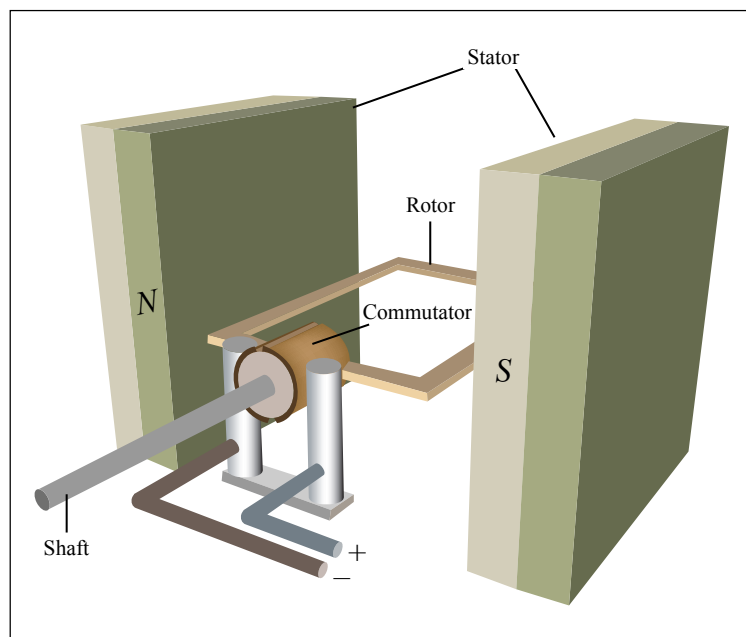

Image by MIT OpenCourseWare.

Figure 2: **Basic Motor Operation.** Copper coil turns clockwise due to current through the coil. Torque is maximum when coil is positioned as pictured. The torque diminishes to zero as the coil rotates 90 degrees, but the coil's inertia carries it through this point. Also at 90 degrees, the current through the coil reverses, and it continues to feel a clockwise torque. The torque grows to a second maximum at 180 degrees, and then diminishes to zero at 270 degrees.

It should be obvious that the force on the wire loop is directly proportional to the current. Lorentz tells us $F = BIl$, where l is the length of the wire. In the above configuration, B and l are constant. To increase F, we must increase I. Furthermore, the force exerts a torque about the shaft dictated by the radius of the current loop. We encapsulate these constants (B, l, and r) into the *motor constant*, K, so that motor torque is $\tau_{\mathrm{m}} = KI$. The motor constant describes how well the motor converts current through the armature into torque.

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[1]A little vocabulary. There are two motor parts: the *stator* and the *rotor*. As you might have guessed, the rotor rotates inside a stationary stator. Another term you might encounter is the *armature*, which is sometimes incorrectly used as a synonym for the rotor. The armature refers to the coil in which current changes directions. In the motor below, the rotor contains the oscillating current and is the armature. In DC motors without commutators (brushless DC motors), the stator contains the oscillating current and functions as the armature.

An engineer wouldn't attempt to calculate K from first principles – instead she would measure it directly after the motor was built. If she then wanted a larger K, she could choose from a variety of options: increase the strength of the magnets, make the motor longer, or add more loops. It is the first option that we will explore in the lab.

Many DC motors use permanent magnets like the one above. The other way for the stator to generate a magnetic field is to replace the permanent magnets with solenoids. The advantage of solenoids is that we can easily increase B, a parameter that we said was fixed in the permanent magnet design. In the solenoid stator configuration, K is a function of the current through the solenoids. In Section 1.3.3, it will become evident why tuning K might be desirable.

There's one other electromagnetic effect to consider. As the loop rotates, the magnetic flux through it changes sinusoidally. From Faraday's law, we know that this changing flux will produce a back electromotive force (EMF) on the red and black wires that opposes the forward voltage. Although this voltage, like the torque, changes sinusoidally in time, we can approximate it as an average constant voltage. The value of this voltage is proportional to the speed of rotation and the motor constant K. In fact, $V_{\text{Bemf}} = K\omega$. It might surprise you that the same motor constant K is used here in what looks like a completely different equation.

1.3 Motors in Electrical Circuits

Now that we have our motor, let's put it into a circuit and predict its behavior. The equivalent circuit for a motor is a resistor in series with an inductor and a voltage source. The resistance comes from the wire loop of the rotor, and is typically a few ohms. The inductor also models the wire loop; however, it is only significant if there are many loops in the wire, or if the motor spins quickly relative to the R/L time constant. We will ignore it in this lab. Finally, we need a voltage source to model the back EMF.
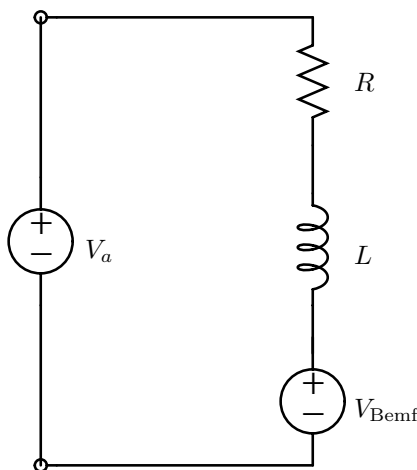

Figure 3: **Model DC Motor Circuit Diagram.**

We will drive our DC motor with a current source and a voltage source and compare the steady state speed and torque.

1.3.1 Current Source Operation

Given a current source, the torque is simply $\tau_{\text{m}} = KI$. The steady state speed is found from solving the equilibrium $\tau_{\text{m}} = \tau_{\text{f}}$, where τ_{f} is the frictional torque. $\tau_{\text{f}} = \beta\omega$; thus in steady state $KI = \beta\omega_{\text{ss}}$, and:

$$\omega_{\text{ss}} = \frac{K}{\beta}I \tag{1}$$

where ω_{ss} is the steady-state ω.

1.3.2 Voltage Source Operation

Given a voltage source, V_{a}, the current through the motor in steady state will be $(V_{\text{a}} - V_{\text{Bemf}})/R$. Hence the motor torque, $\tau_{\text{m}} = KI$, will be $K(V_{\text{a}} - V_{\text{Bemf}})/R$. Furthermore, the back EMF is $K\omega$. To find steady state quantities, we equate τ_{m} to the frictional torque, $\tau_{\text{f}} = \beta\omega$:

$$\frac{K(V_{\text{a}} - K\omega_{\text{ss}})}{R} = \beta\omega_{\text{ss}}$$

ω_{ss} happens to be linear in V_{a}:

$$\omega_{\text{ss}} = \frac{K}{K^2 + R\beta}V_{\text{a}} \tag{2}$$

and the motor torque is:

$$\tau_{\text{m}} = KI = K\frac{V_{\text{a}} - V_{\text{Bemf}}}{R} = \frac{K}{R}\left(1 - \frac{K^2}{K^2 + R\beta}\right)V_{\text{a}}$$

1.3.3 Optimal K

Let's do something clever and plot Equation 2 as a function of K (Figure 4). We discover that, for a given voltage, the motor reaches a maximum speed when:

$$K = \sqrt{R\beta} \tag{3}$$

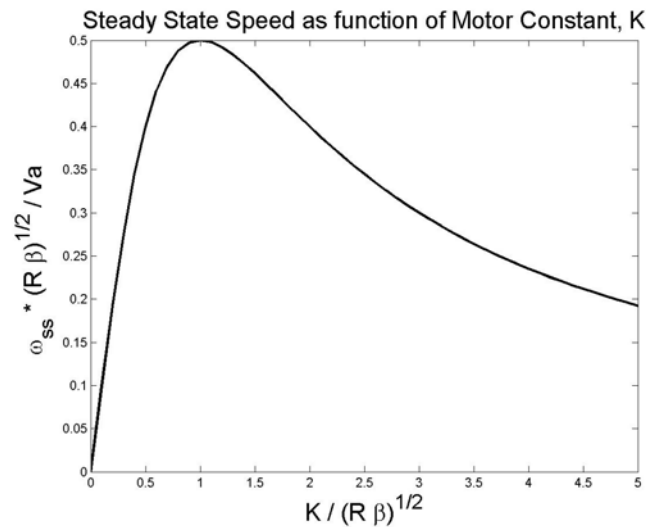**We want to adjust K to reach this optimality condition.**

Figure 4: Steady state speed, ω_{ss}, reaches a maximum when $K = \sqrt{R\beta}$.

2 Prelab

This prelab has two sections. First you will use the kit distributed in class to build your own motor. Second, there are some questions at the end of this handout for you to answer. **You need to bring your motor and the answers to the questions to the lab before starting the lab!** This pre-lab should take about 1 hour.

2.1   Build your own motor

You will need the following to build your motor (extras of some items have been included).

- (2) paper clips
- (1) paper cup
- (1) 1.5V size "D" battery
- (1) 1.5 meter of armature wire
- (1) bar magnets
- (1) piece of sandpaper to strip wire ends
- (1) wire cutter
- (1) battery strap
- (1) 9V battery

Please return the Scotch tape, 1.5V "D" battery and wire cutter when you come to the lab. The rest is for yours to keep!

2.2   Instruction

Figure 5: Place the cup upside down. Squeeze the magnet between the ledge. This should hold the magnet firmly in place.
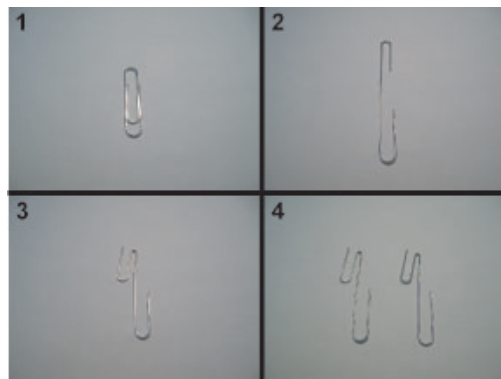

Figure 6: Bend the paper clips as shown in the figure

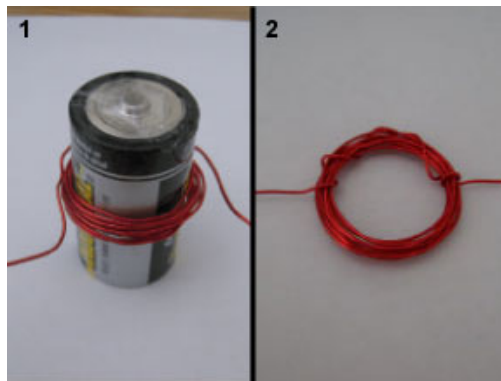Figure 7: Insert the paper clips between the ledge and the side of the magnet as shown in the figure.

Figure 8: It is easiest to coil the wire around the "D" battery to make the rotor. We have found 11 loops to be optimal.

Here's the part of the problem where you'll need to think. You will need to strip the ends of the rotor before placing it on the paper clips to achieve an electrical connection. The easiest way to strip the wire is to place it on a table and sand away the dark red insulation. One side you should strip the wire through all $360°$. The other side will be the commutator and you should only strip a $90°$ to $180°$ arc. Think about how current flows through the wire and the forces acting on the rotor. If you strip the full $360°$, the rotor will oscillate back and forth. It is important that current only pass through the coil when you want it to!

Figure 9: Hang your coil on the paper clips. Make sure your coil is sitting centered above the magnet. Attach the battery strap to the paper clips and you are done!

When you connect the battery, the motor should spin. You may have to give a little push to get it started, especially if you attempt to start it with the rotor in the position where the commutator prevents current from flowing.

2.3   Questions

Complete the following questions before coming to lab.

(1) You are the proud owner of a battery powered go-cart. The DC motor, powered by a voltage source, has been engineered to run optimally with a 70 kg driver. However, you use it to transport party supplies, and the total mass is 100 kg.

    (a) Which motor constant (K, β, or R) changes with the increased load? Does it increase or decrease?

    (b)   * Does the speed decrease? (Hint: Look at Figure 4. Changing the constant in Part **(a)** moved the operating point to a different point on this curve.)

           * How does the power supply compensate? (Hint: The increased load causes the motor to draw more power. Does the power supply provide constant current or constant voltage? How can the power supply provide more power?)

    (c) The optimality condition $K = \sqrt{R\beta}$ no longer holds. Which constant should you change to bring the motor back to the optimal operating point? (There are three possible answers, but only one could be changed without re-engineering the entire cart.) What's an easy way to change this constant? (Hint: You'll be doing this in the lab.)

    (d) When you bring the car back the optimal operating point per Part **(c)**, does the speed recover? Does the motor continue to draw more power?

    (e) **Optional:** If the go-cart were put on an incline, the frictional torque would acquire a constant gravity term, L, so that $\tau_f = L + \beta\omega$. Balance τ_m and τ_f to find the effect of this additional term on ω_{ss} in Eqn (2).

(2) One of the goals of this lab is to introduce you to the oscilloscope and basic plotting functions in MATLAB. Find an Athena station and open MATLAB. (You can also download it from MIT at https://msca.mit.edu/cgi-bin/matlab. Follow the installation instructions carefully to get the license working correctly on your computer.) Try the following commands. If you become very frustrated, seek help from a friend or go through an online tutorial (a basic one can be found at http://www.cyclismo.org/tutorial/matlab).

    – Let's define a new variable, c. Type `c = 10` and press enter.

– Let's define a new array, x: `x = [1 2 3 4 5 6 7 8 9 10]`

– We can define arrays in a more compact manner: `y = 1:10`

– If we want non-integer spacing between the elements: `y = 1:.1:10`

– We can access different elements of the array. MATLAB indexing starts at 1: `y(2)`

– We can access a range of elements: `y(10:15)`

– We can scale an array: `c * x`

– MATLAB has many built in functions: `y = sin(x)`

– If we have two arrays of equal length, we can plot the corresponding pairs of points: `plot(x, y)`

– This sine curve looks awful. Try increasing the resolution (note the semicolon at the end): `x = (-3/2 * pi):.01:(3/2 * pi);`

– When we terminated the above statement with a semicolon, MATLAB suppressed the output. A semicolon also separates commands allowing you to enter two or more commands on one line.

– Now create the set of y coordinates: `y = sin(x); plot(x,y)`

– We can plot multiple lines on the same plot: `y2 = cos(x); plot(x, y, x, y2)`

– There are many ways to affect the look of the graphs: `plot(x, y, 'r.', x, y2, 'b--')`. Now each line is described by three variables: x coordinates, y coordinates, and the style in single quotes. `r.` indicates that the line is **r**ed and dotted (at this resolution the dots run together and appear like a solid bold line). `b--` indicates that the line is **b**lue and dashed. `k-` would be a solid blac**k.**

– We can add labels and title our graph: `xlabel('Phase')`, `ylabel('Magnitude')`, `title('Fundamental Trigonometric Relations')`

– You can label each line using the `legend` command: `legend('Sine', 'Cosine')`

– You can save your plot as a pdf. Do so, print it out, and tape a copy into your lab report.
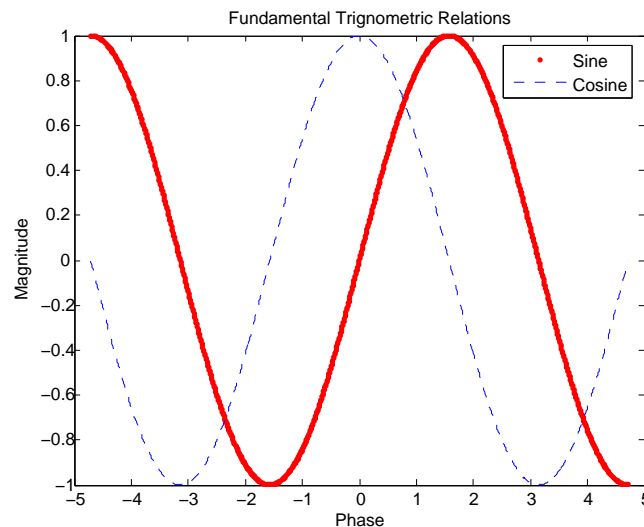

Figure 10: Graph obtained as a result of MATLAB walk through.

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