Faraday's Law (Induced emf)

Reading - Shen and Kong – Ch. 16

<u>Outline</u>

- Magnetic Flux and Flux Linkage
- Inductance
- Stored Energy in the Magnetic Fields of an Inductor
- Faraday's Law and Induced Electromotive Force (*emf*)
- Examples of Faraday's Law

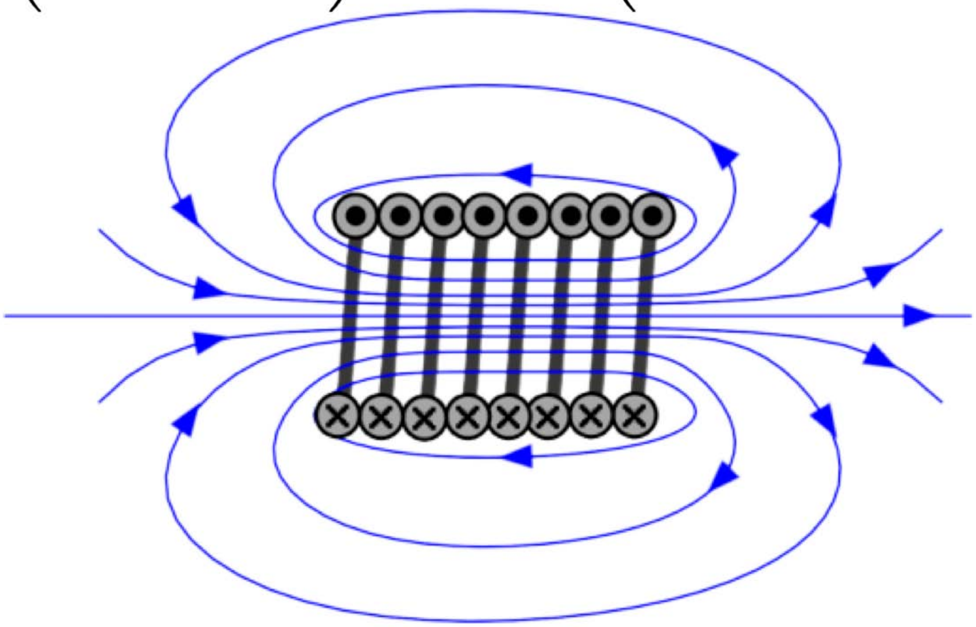
Magnetic Flux Φ [Wb] (Webers)

Magnetic Flux Density B [Wb/m^2] = T (Teslas) ← due to macroscopic & microscopic

Magnetic Field Intensity H [Amp-turn/m] ← due to macroscopic currents

$$\Phi = \int \overline{B} \cdot d\overline{A}$$

$$\overline{B} = \mu_o\left(\overline{H} + \overline{M}\right) = \mu_o\left(\overline{H} + \chi_m \overline{H}\right) = \mu_o \mu_r \overline{H}$$



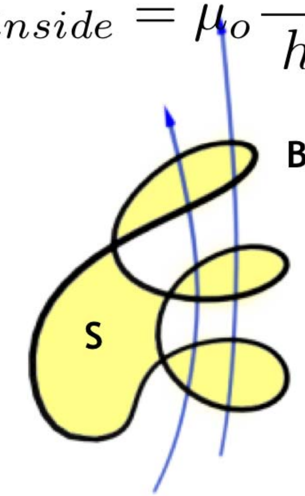
Flux Linkage of a Solenoids

FOR A SUFFICIENTLY LONG SOLENOID…

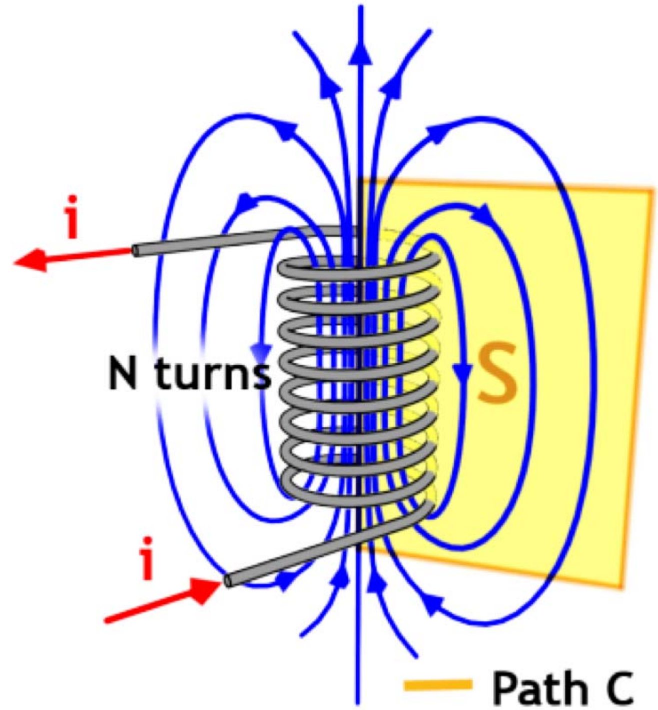
$$H_{inside} = \frac{Ni}{h}$$

$$B_{inside} = \mu_o \frac{Ni}{h}$$

$$\Phi_{inside} = \mu_o \frac{Ni}{h} A$$



In the solenoid the individual flux lines pass through the integrating surface S more than once



B = Magnetic flux density inside solenoid
A = Solenoid cross sectional area
N = Number of turns around solenoid

$$\lambda = N\Phi = NBA = \mu_o \frac{N^2 i}{h} A$$

… flux linked by solenoid

Inductors

… is a passive electrical component that stores energy in a magnetic field created by the electric current passing through it. (This is in equivalence to the energy stored in the electric field of capacitors.)

IN GENERAL: $$\lambda = Li$$

FOR A LINEAR COIL: $$\lambda = N\Phi = \mu_o \frac{N^2 i}{h} A = Li \quad \Rightarrow \quad L = \mu_o \frac{A}{h} N^2$$

The magnetic permeability of the vacuum: $\mu_o = 4\pi \times 10^{-7}\,\mathrm{H/m}$ [henry per meter]

An inductor's ability to store magnetic energy is measured by its inductance, in units of henries. The henry (symbol: H) is named after Joseph Henry (1797–1878), the American scientist who discovered electromagnetic induction independently of and at about the same time as Michael Faraday (1791–1867) in England.

EQUIVALENCE OF UNITS:

$$\lambda = N\Phi = NBA = \mu_o \frac{N^2 i}{h} A$$

$$\mathrm{H} = \frac{\mathrm{Wb}}{\mathrm{A}} = \frac{\mathrm{V\cdot s}}{\mathrm{A}} = \Omega\cdot\mathrm{s} = \frac{\mathrm{J}}{\mathrm{A}^2} = \frac{\mathrm{J/C\cdot s}}{\mathrm{C/s}} = \frac{\mathrm{J\cdot s^2}}{\mathrm{C}^2} = \frac{\mathrm{m^2\cdot kg}}{\mathrm{C}^2} = \frac{\mathrm{m^2\cdot kg}}{\mathrm{s^2\cdot A^2}}$$

Stored Energy in an Inductor

FROM 8.02: $v(t) = L\frac{di(t)}{dt}$ ← voltage over an inductor

$$P_{elec} = v \cdot i = L\frac{di}{dt} \cdot i = \frac{1}{2}L\frac{d}{dt}i^2$$

If L is not a function of time ...

$$P_{elec} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = \frac{dW_s}{dt}$$

... where E is energy stored in the field of the inductor any instant in time

$$W_s(i, r) = \frac{1}{2}Li^2$$

FOR A LINEAR COIL: $L = \mu_o \frac{A}{h} N^2$

Calculation of energy stored in the inductor

$$W_s = \int vi\,dt = \int i\frac{d\lambda}{dt}dt = \int i\,d\lambda = \frac{1}{2}\frac{\lambda_0^2}{L}$$

Note that flux is $\lambda = N\phi = NA\mu_0 H_z$

And that inductance is $L = \mu_0 \frac{A}{h} N^2$

Energy stored is $$\mathrm{W_s} = \frac{1}{2}\frac{\left(NA\mu_0 H_z\right)^2}{\mu_0 \frac{A}{h} N^2} = \frac{1}{2}\mu_0 H_z^2 Ah$$

Energy stored per volume $$\frac{\mathrm{W_s}}{Volume} = \frac{1}{2}\frac{\left(NA\mu_0 H_z\right)^2}{\mu_0 \frac{A}{h} N^2} = \frac{1}{2}\mu_0 H_z^2$$

General: Stored Energy in the Coil

FROM 8.02: $v(t) = L\frac{di(t)}{dt}$ ← voltage over an inductor

Since $\lambda = Li$ then $v(t) = \frac{d\lambda(t)}{dt}$

→ Change in the magnetic flux within the inductor generates voltage

---

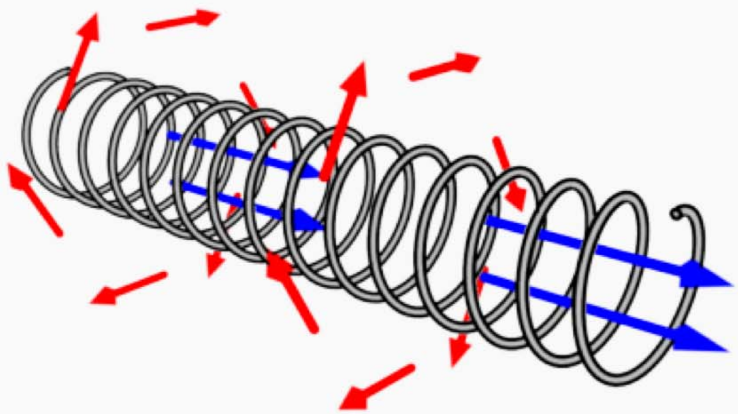
$$P_{elec} = v \cdot i = \frac{d\lambda}{dt} \cdot i$$

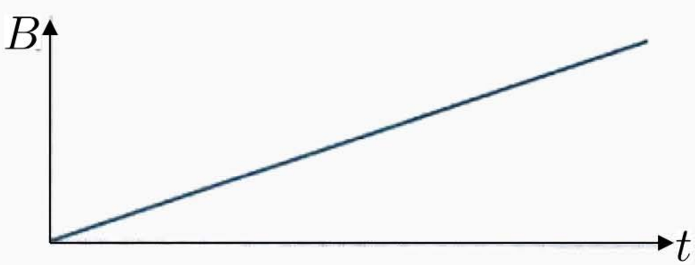
$$W_s = \int P_{elec}\, dt = \int i\frac{d\lambda}{dt}dt = \int i\, d\lambda$$

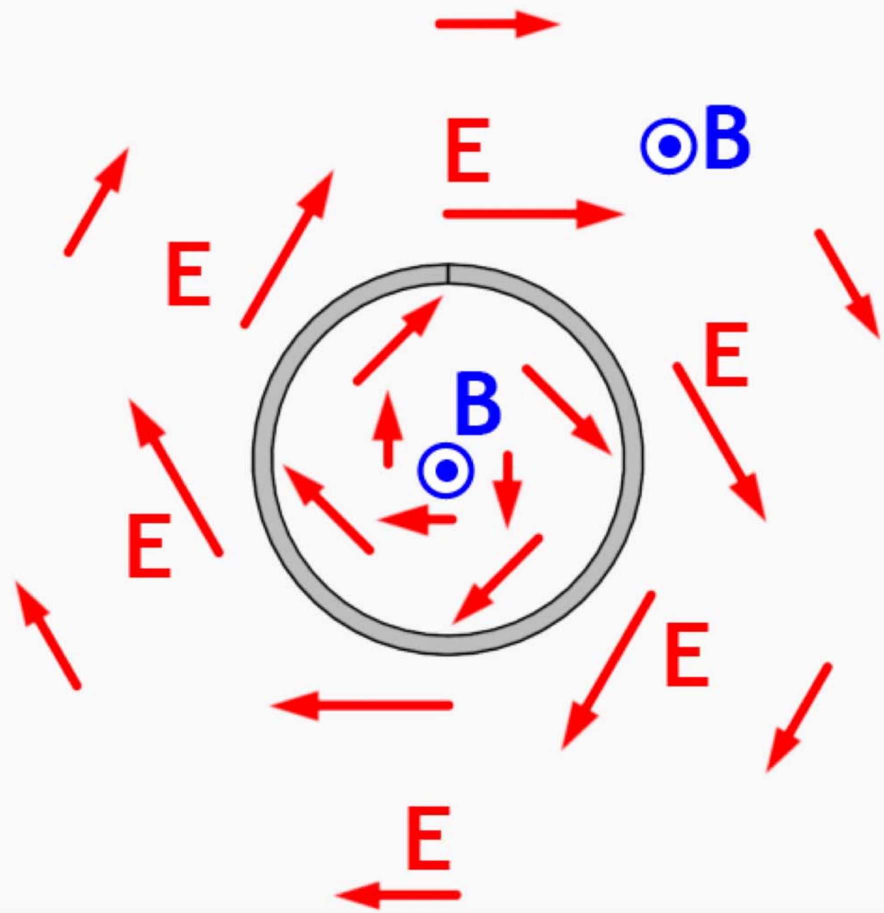
… where W_s is energy stored in the field of the inductor any instant in time

Induced electromotive force (emf)

Michael Faraday in 1831 noticed that time-varying magnetic field produces an emf in a solenoid







from Chabay and Sherwood, Ch 22

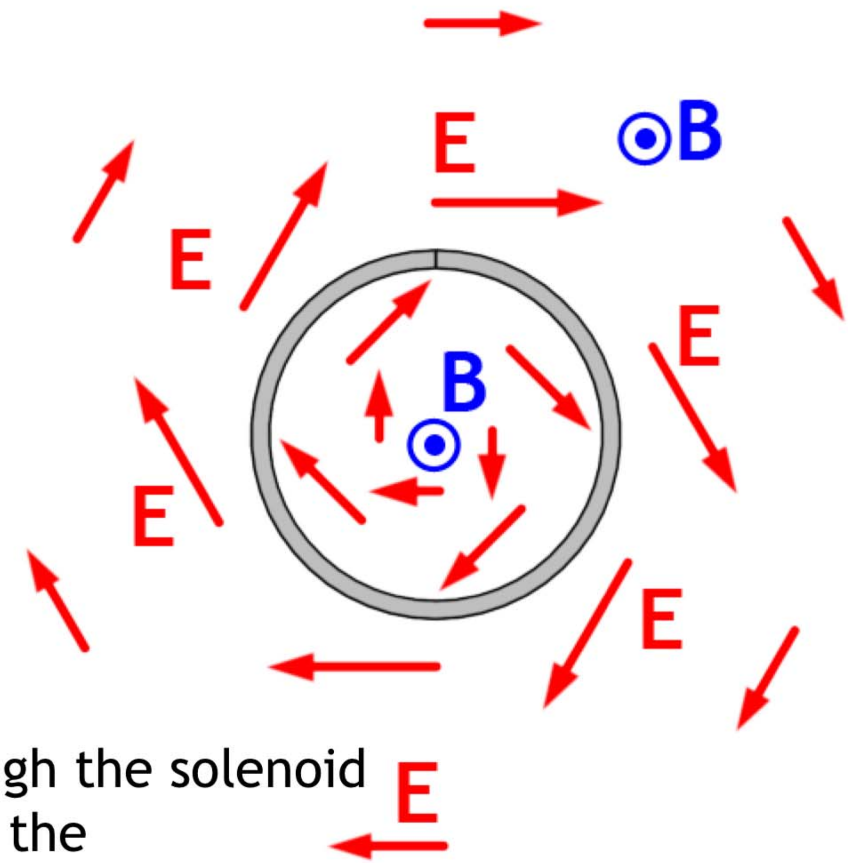
THEREFORE, THERE ARE
TWO WAYS TO PRODUCE ELECTRIC FIELD

(1) Coulomb electric field is produced by electric charges according to Coulomb's law:

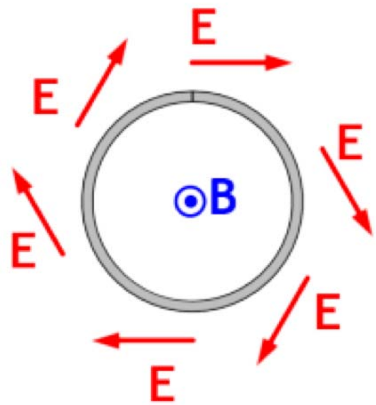
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

(2) Non-Coulomb electric field E_{NC} is associated with time-varying magnetic flux density dB/dt

For a solenoid, E_{NC}
- curls around a solenoid
- is proportional to *-dB/dt* through the solenoid
- decreases with *1/r*, where *r* is the radial distance from the solenoid axis

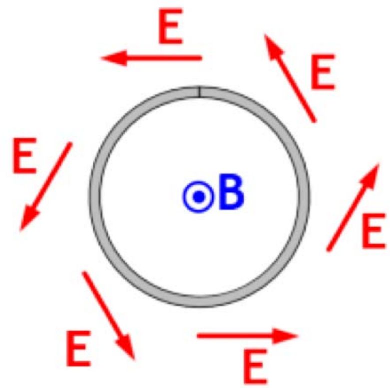


What is the Direction of Magnetically Induced (non-Coulomb) Field, E_{NC} ?



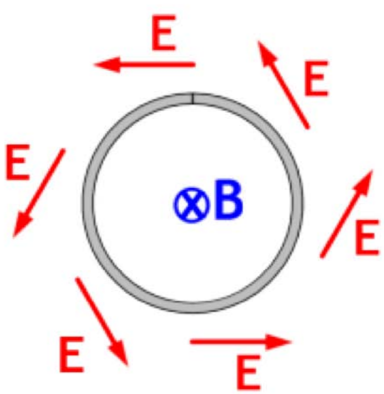
B out, *increasing*

$-\dfrac{dB}{dt}$ into page



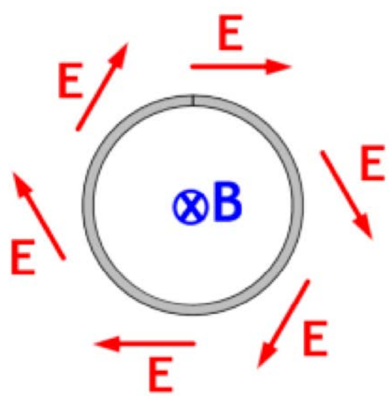
B out, *decreasing*

$-\dfrac{dB}{dt}$ out of page



B in, *increasing*

$-\dfrac{dB}{dt}$ out of page

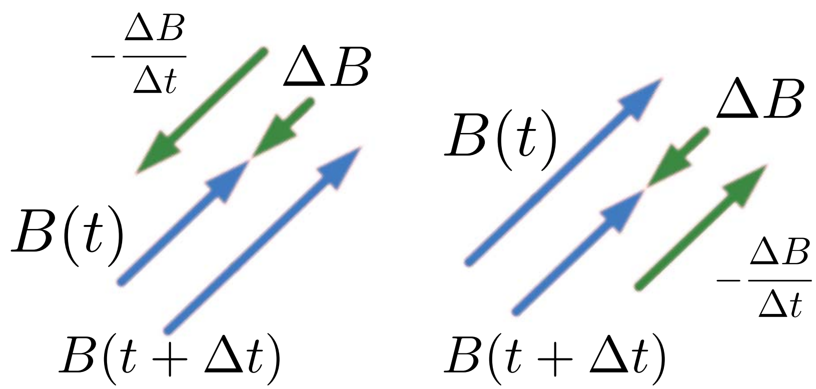


B in, *decreasing*

$-\dfrac{dB}{dt}$ into page

Find the change in the magnetic flux density as a basis for determining the direction of

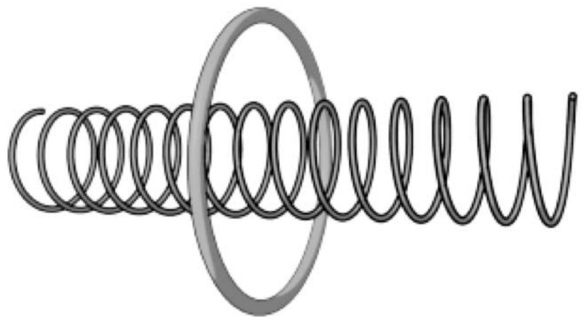
$$-dB_1/dt$$



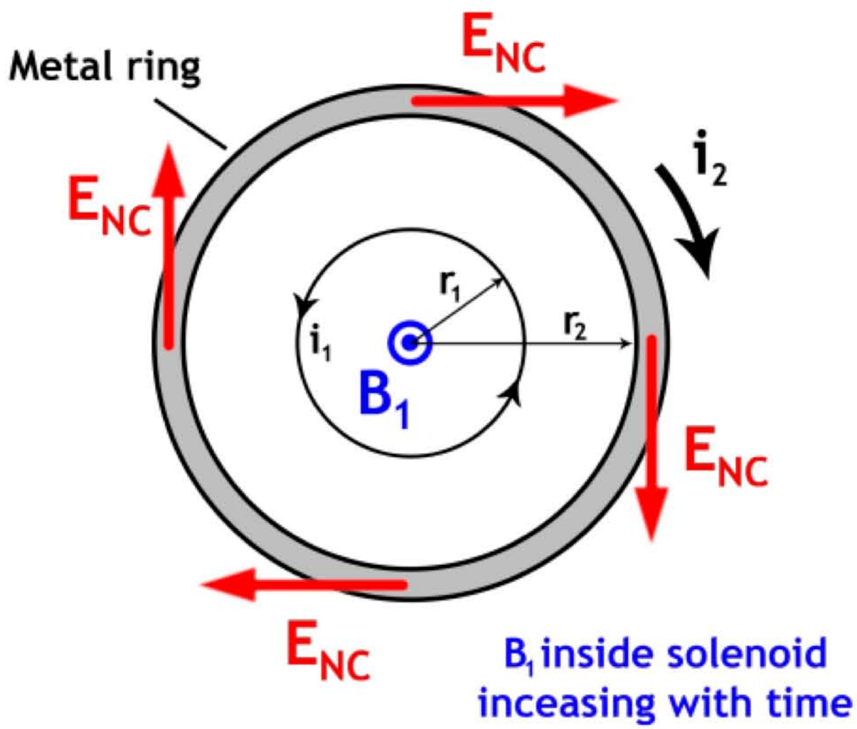
Lenz's Rule

The induced electric field would drive the current in the direction to make the magnetic field that attempts to keep the flux constant

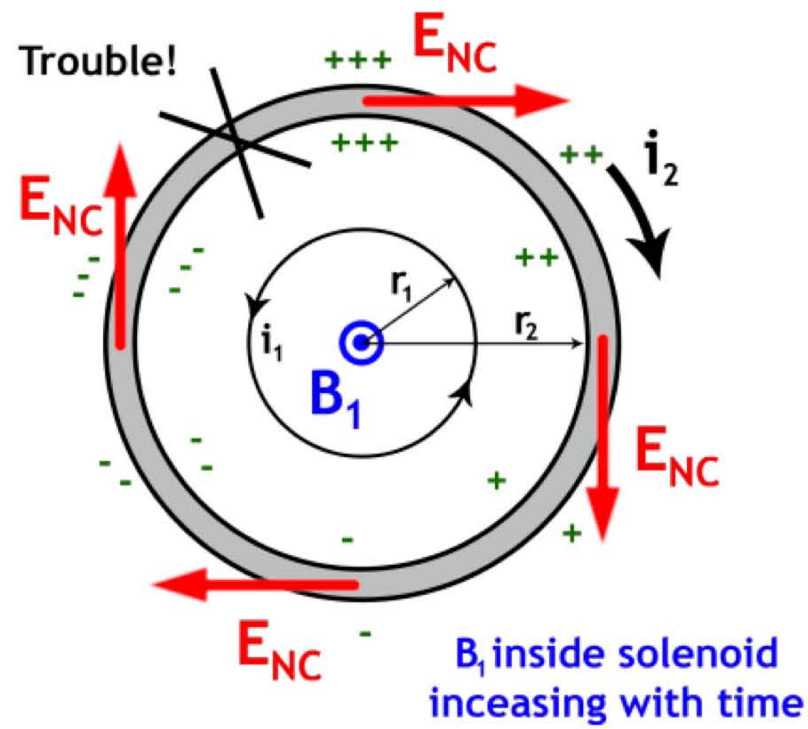
Magnetically Induced (non-Coulomb) E_{NC} Drives Current in a Loop Surrounding the Solenoid



METAL RING IS PLACED AROUND A SOLENOID



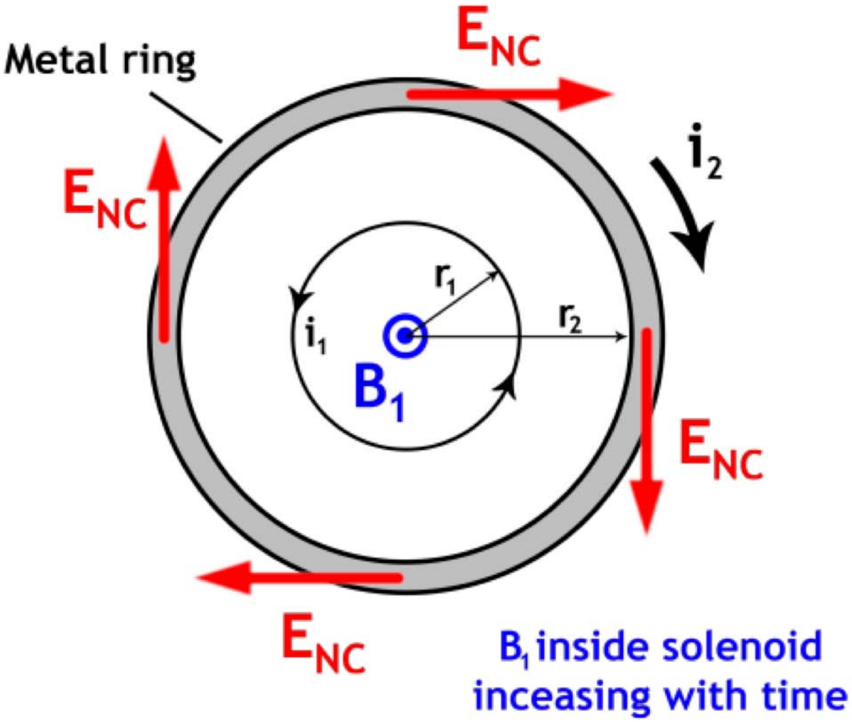
END VIEW: The non-Coulomb electric field drives a current I_2 in the ring

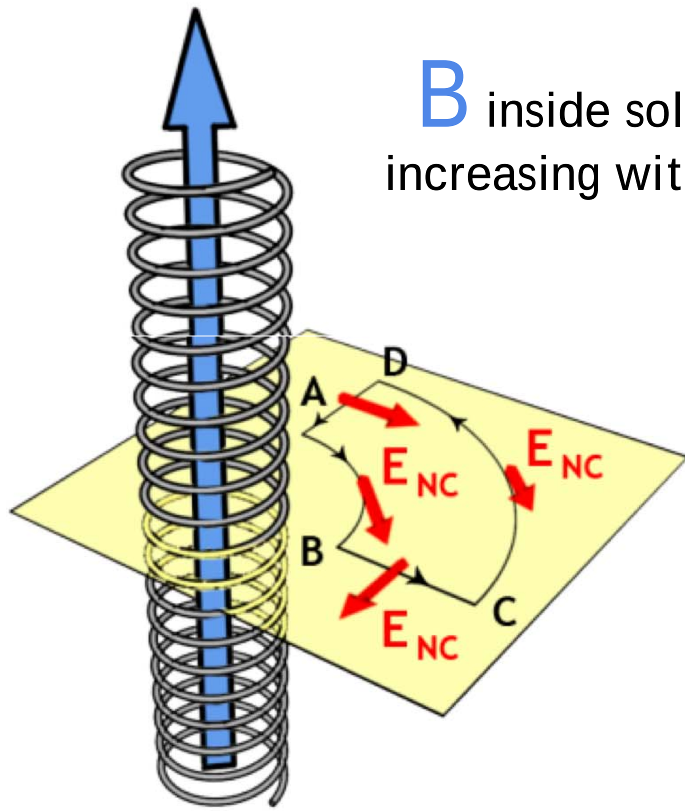


This pattern of surface charge is impossible, because it would imply a huge *E* at the marked location, and in the wrong direction !

$$emf = \oint \overline{E}_{NC} \cdot d\overline{l} = E_{NC}(2\pi r_2)$$

$$I_2 = emf / R$$ where R is the ring resistance





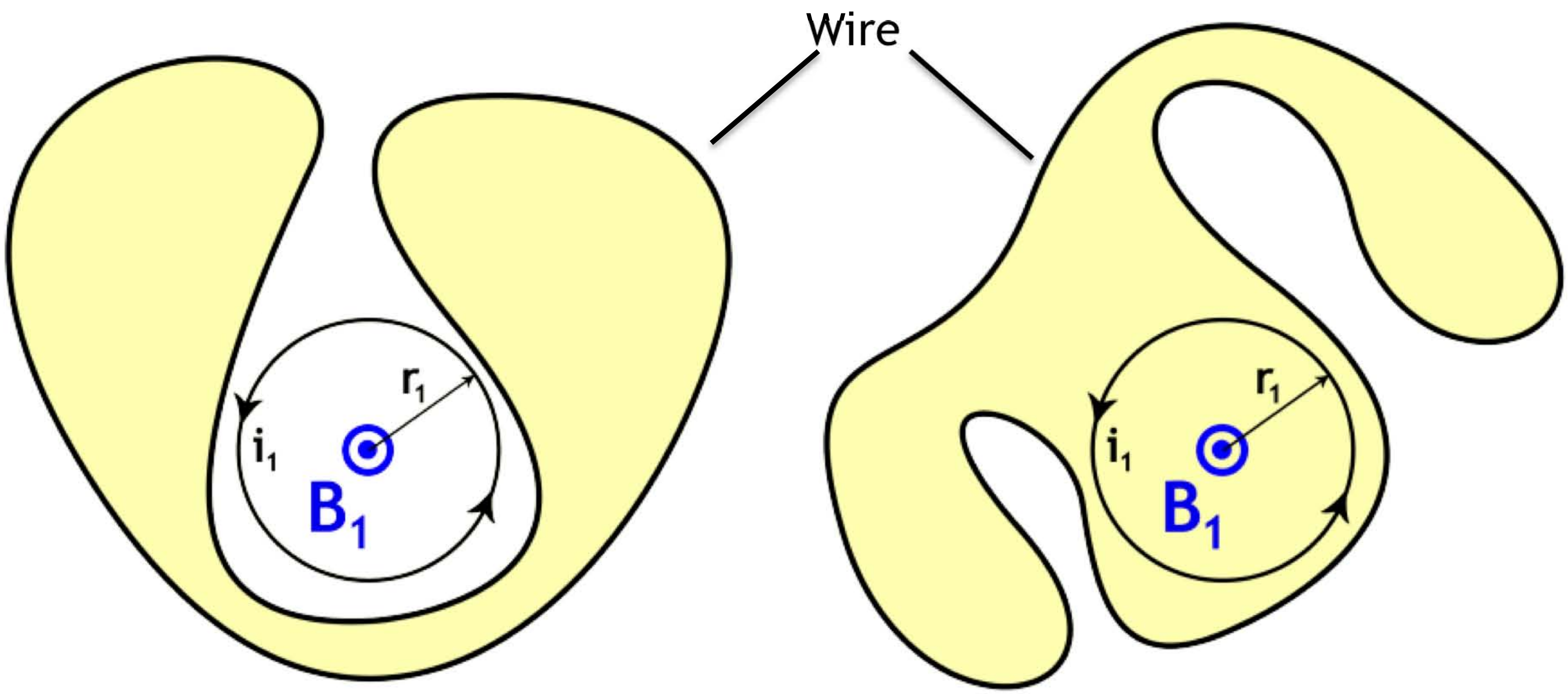
B inside solenoid increasing with time

Integral of E_{NC} along a path that does not encircle the solenoid is zero since $E_{NC} \sim 1/r$

What if we double the value of r_2 ? → Still get the same *emf* around the loop

emf in a ring encircling the solenoid is the same for any radius

Will Current Run in these Wires ?

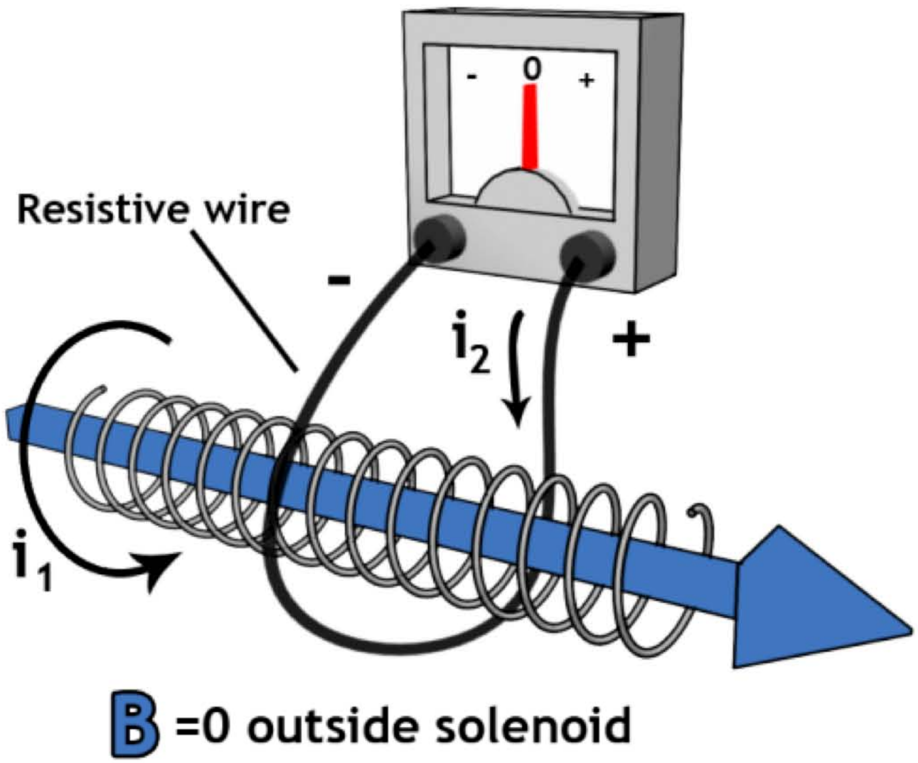


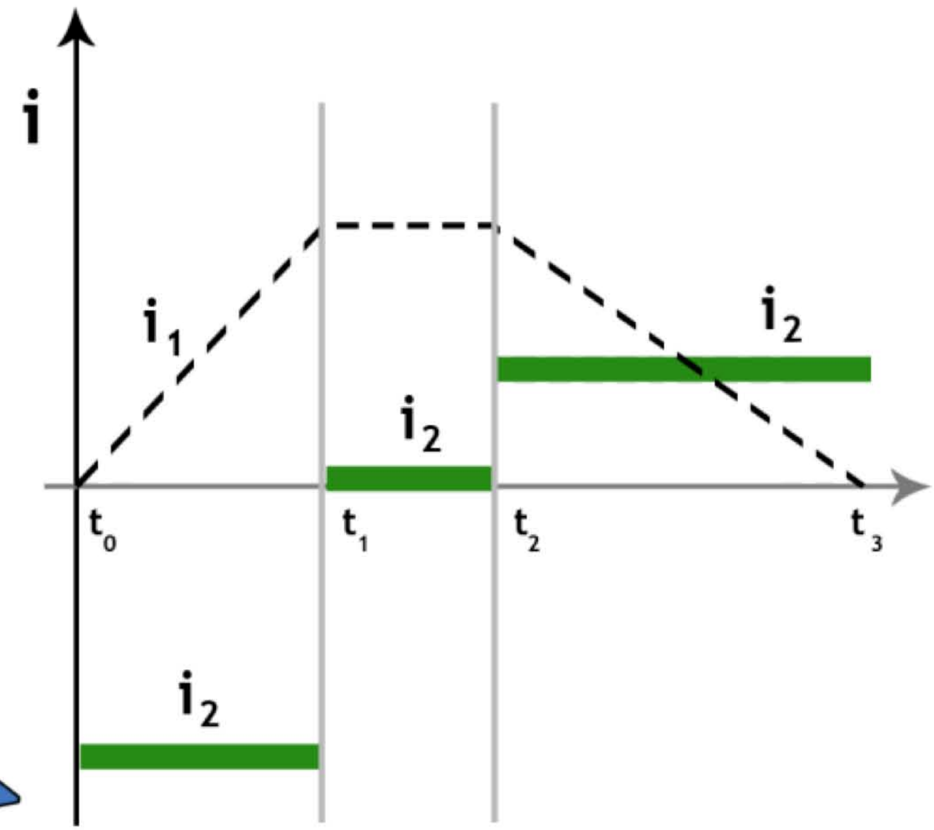
B_1 inside solenoid inceasing with time

B_1 inside solenoid inceasing with time

Example

An ammeter measures current in a loop surrounding the solenoid. Initially I_1 is constant, so B_1 is constant, and no current runs through the ammeter.

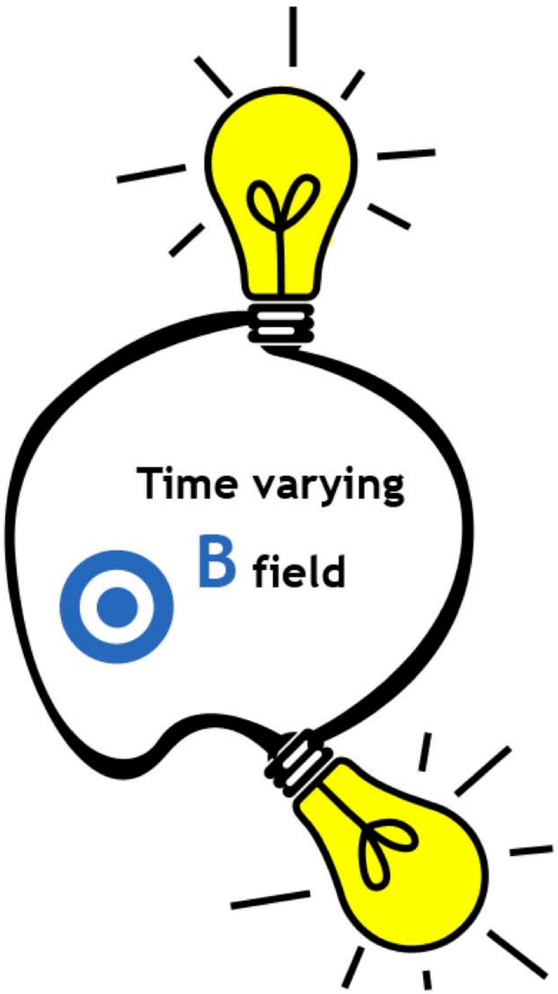




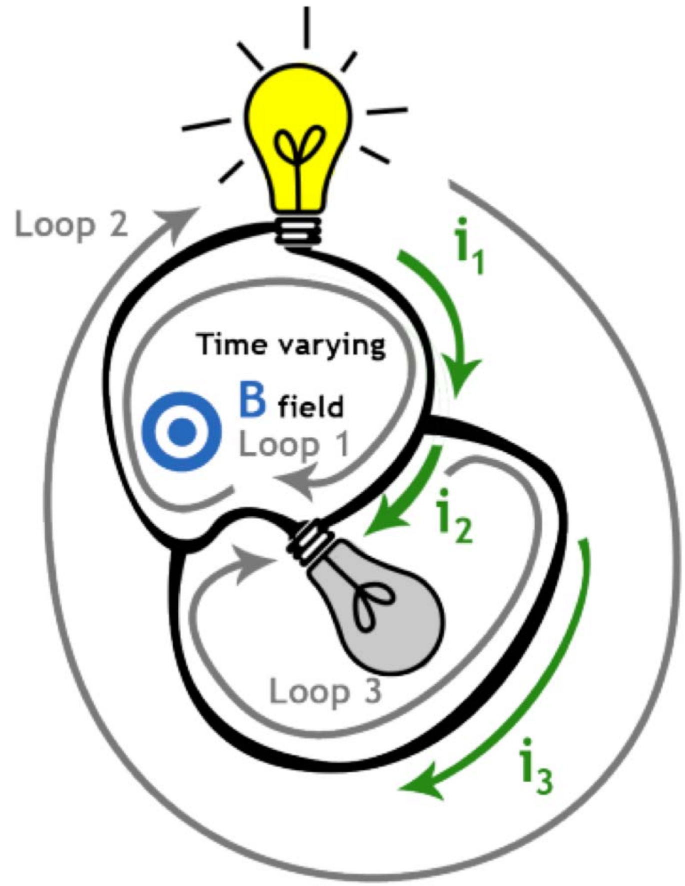
Vary the solenoid current I_1 and observe the current I_2 that runs in the outer wire, through the ammeter

from Chabay and Sherwood, Ch 22

Peculiar Circuit – Two Bulbs Near a Solenoid



Two light bulbs connected around a long solenoid with varying B.



… Add a thick copper wire.

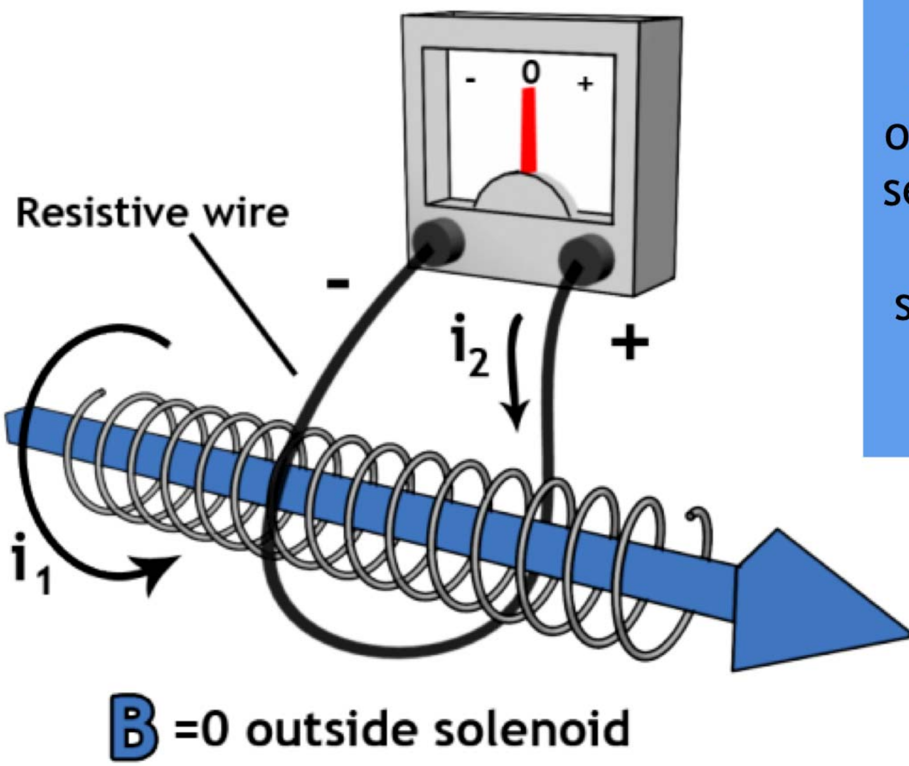
Loop 1: $emf - R_1 I_1 - R_2 I_2 = 0$

Loop 2: $emf - R_1 I_1 = 0$

Loop 3: $R_2 I_2 = 0$ (no flux enclosed)

Question:

If we use a solenoid with twice the cross-sectional area, but the same magnetic flux density (same magnitude of I_1), what is the magnitude of I_2?



We can build the solenoid with the larger cross-sectional area, *2*A*, out of two solenoids with the initial cross-sectional area, *A*. Each of the smaller solenoids would induce current I_2, so by superposition, for the twice-as-big solenoid the current would be twice-as-big !

$$|emf| = |\frac{d}{dt}(B_1\, A)|$$

$$|emf| = |\frac{d}{dt}(B_1\,A)| = |\frac{d}{dt}(\Phi_{mag})|$$

$$\Phi_{mag} = \int \overline{B} \cdot d\widehat{a} = \int B_\perp \, da$$

Faraday's Law

The induced *emf* along a round-trIp path is equal to the rate of change of the magnetic flux on the area encircled by the path.

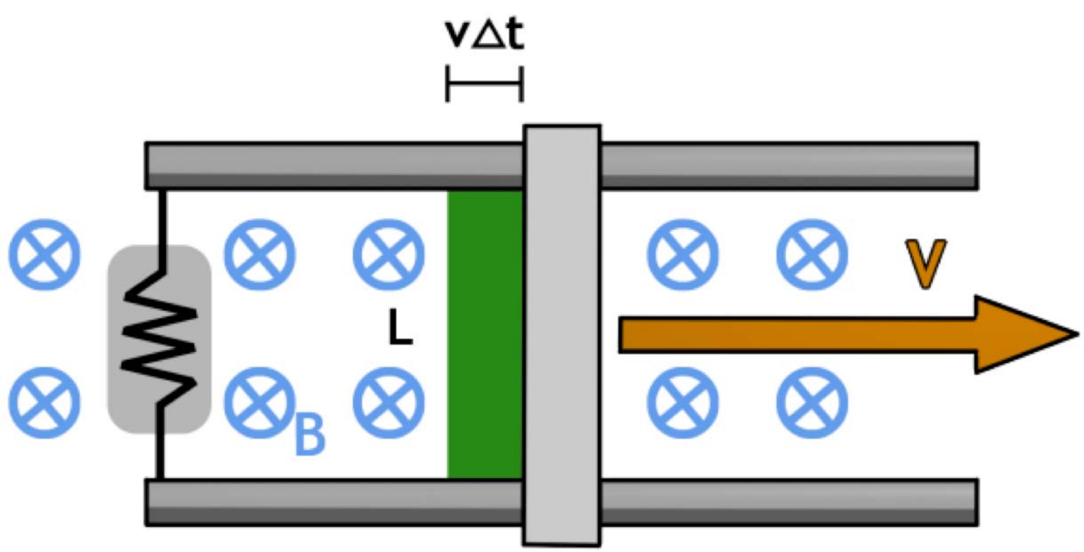
$$emf = -\frac{d\Phi_{mag}}{dt}$$

$$\oint \overline{E}_{NC} \cdot d\widehat{l} = -\frac{d}{dt} \int \overline{B} \cdot d\widehat{a}$$

Faraday's Law and Motional emf

What is the emf over the resistor ?

$$emf = -\frac{d\Phi_{mag}}{dt}$$



In a short time Δt the bar moves a distance $\Delta x = v^*\Delta t$, and the flux increases by $\Delta\Phi_{mag} = B\,(L\,v^*\Delta t)$

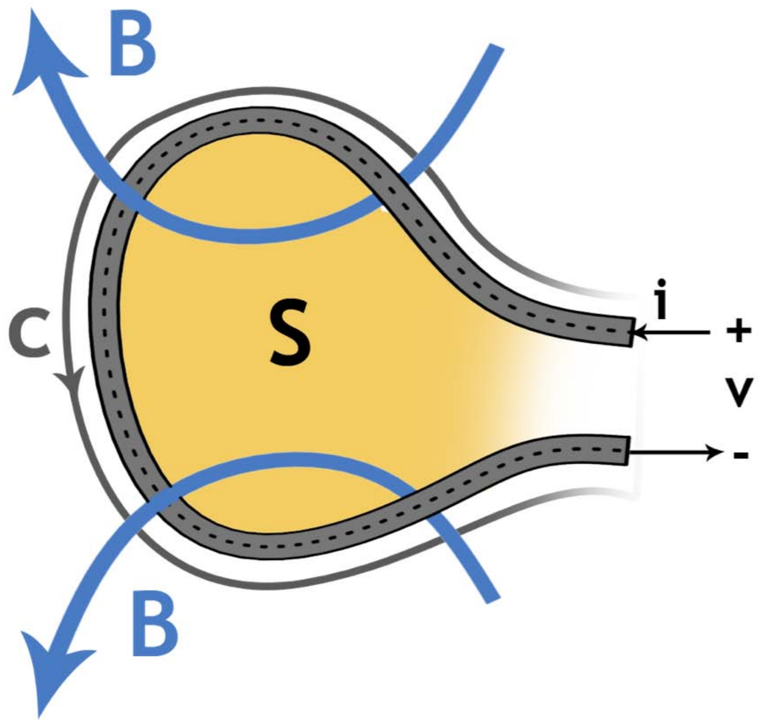
$$emf = \frac{\Delta\Phi_{mag}}{\Delta t} = BLv$$

There is an increase in flux through the circuit as the bar of length *L* moves to the right (orthogonal to magnetic field H) at velocity, *v*.

Terminal Voltages & Inductance

Assume:

- Perfectly conducting wire
- Stationary contour C
- Negligible magnetic flux at the terminals



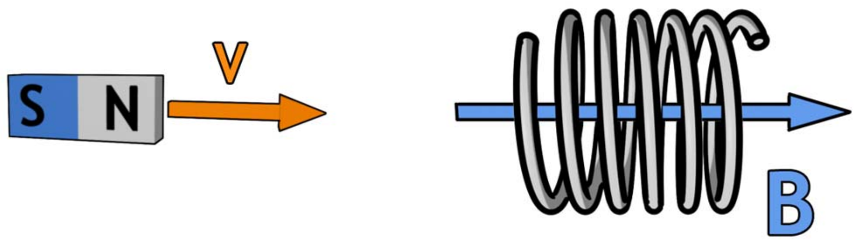
$$emf = d\lambda/dt$$

If the current i created the magnetic flux density B, then the flux linkage is given by $\lambda = Li$. In this case, $emf = L\, di/dt$. L is the self inductance of the coil.

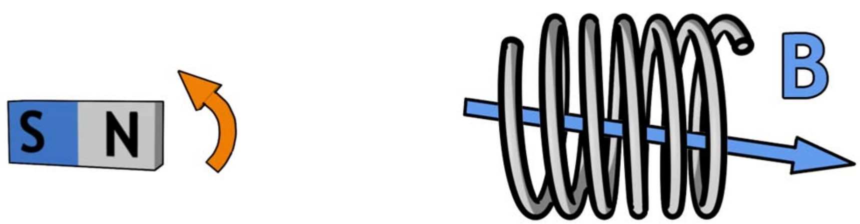
Faraday's Law for a Coil

The induced emf in a coil of N turns is equal to
N times the rate of change of the magnetic flux on one loop of the coil.

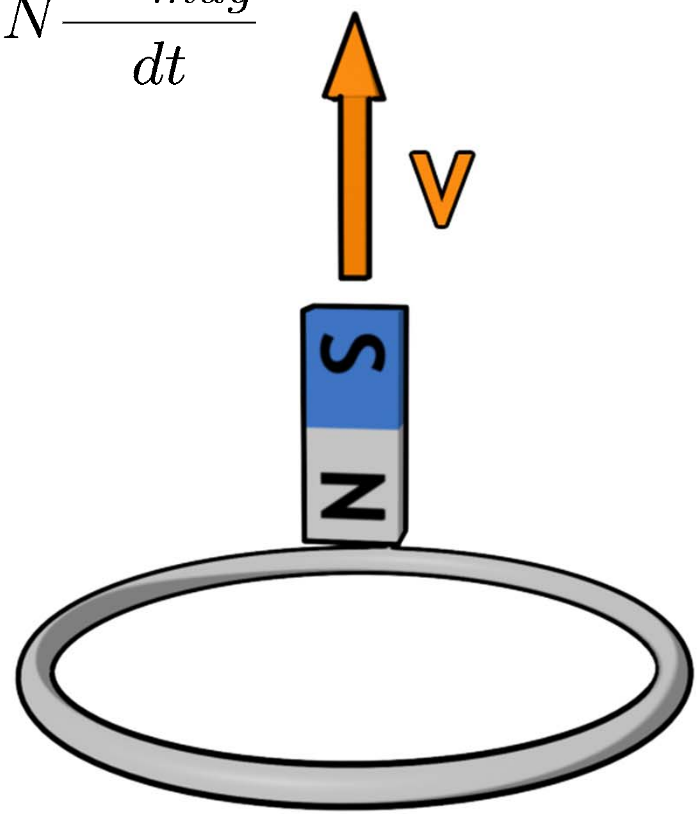
$$emf = -N\frac{d\Phi_{mag}}{dt}$$



Moving a magnet towards a coil produces a time-varying magnetic field inside the coil

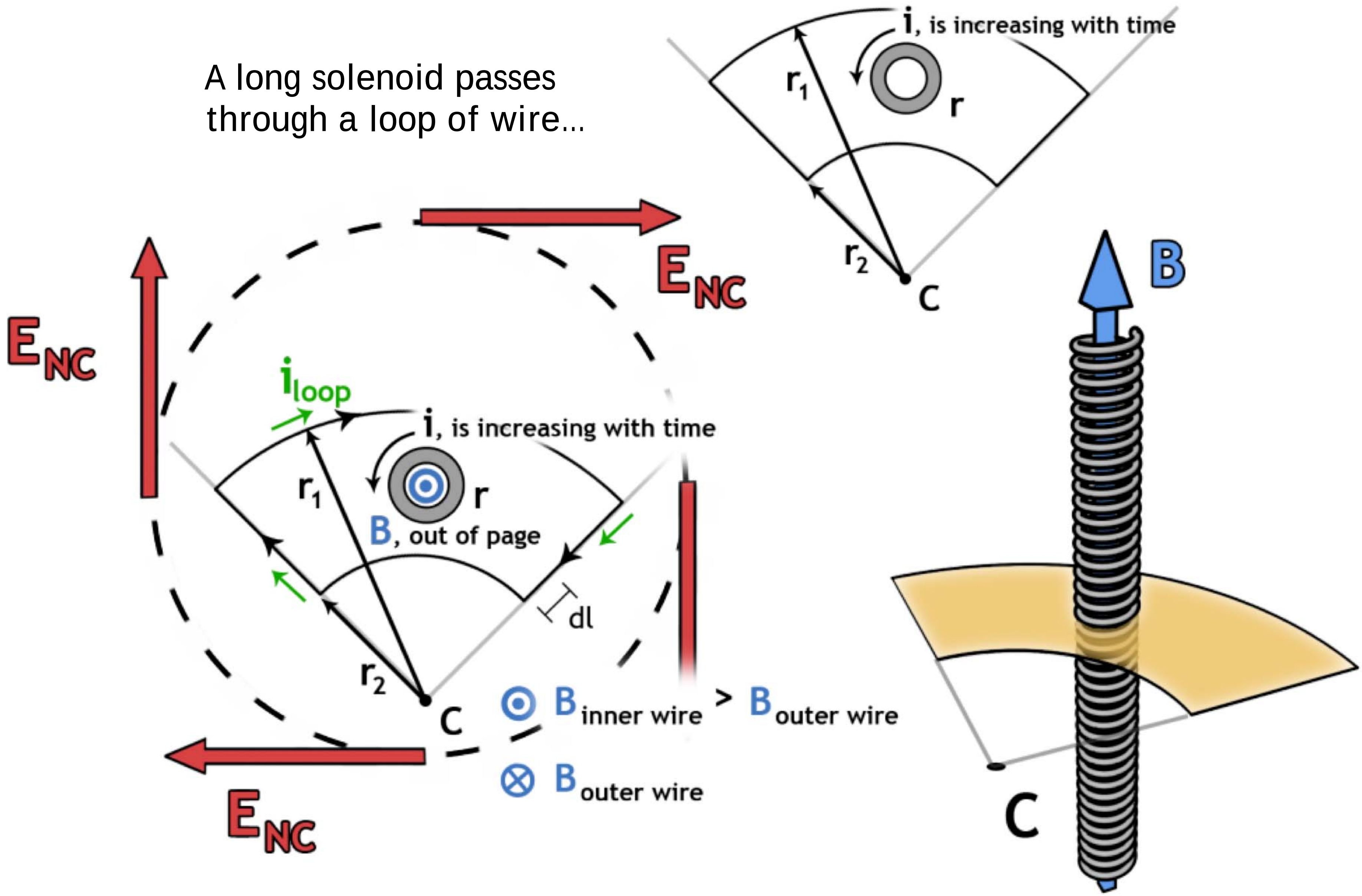


Rotating a bar of magnet (or the coil) produces a time-varying magnetic field inside the coil



Will the current run
CLOCKWISE or ANTICLOCKWISE ?

A long solenoid passes through a loop of wire…



Electric Fields

$$\oint_S \epsilon_o \overline{E} \cdot d\overline{A} = \int_V \rho dV$$

$$= Q_{enclosed}$$

GAUSS

FARADAY

$$\oint_C \overline{E} \cdot d\overline{l} = -\frac{d}{dt}\left(\int_S \overline{B} \cdot d\overline{A}\right)$$

$$emf = \frac{d\Phi}{dt}$$

Magnetic Fields

$$\oint_S \overline{B} \cdot d\overline{A} = 0$$

GAUSS

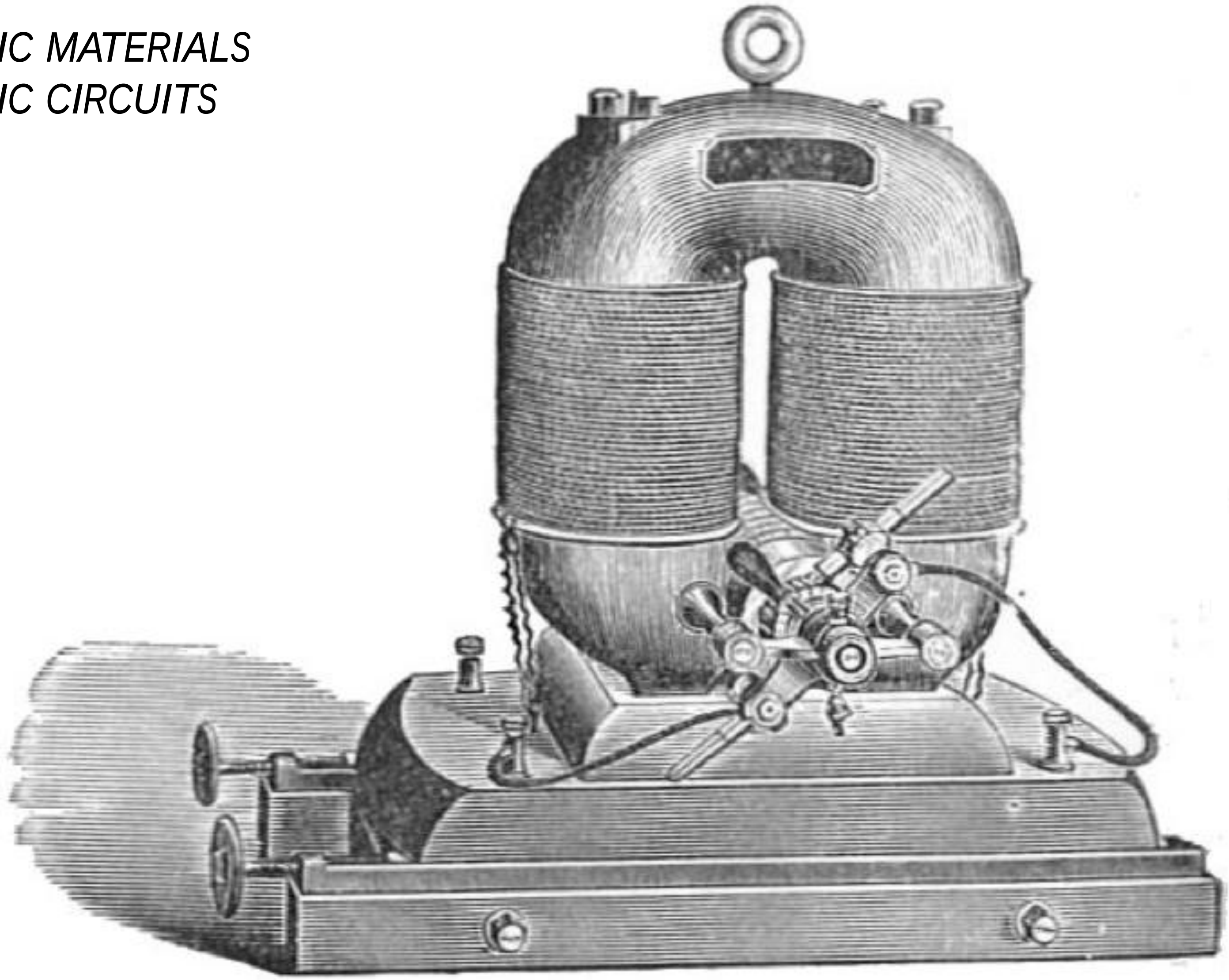
AMPERE

$$\oint_C \overline{H} \cdot d\overline{l}$$

$$= \int_S \overline{J} \cdot d\overline{A} + \frac{d}{dt} \int_S \epsilon E dA$$

Next ...

- *MAGNETIC MATERIALS*
- *MAGNETIC CIRCUITS*



KEY TAKEAWAYS

FOR A SUFFICIENTLY LONG SOLENOID...

$$\lambda = N\Phi = \mu_o \frac{N^2 i}{h} A = Li$$

INDUCTANCE:

$$\Rightarrow \quad L = \mu_o \frac{A}{h} N^2$$

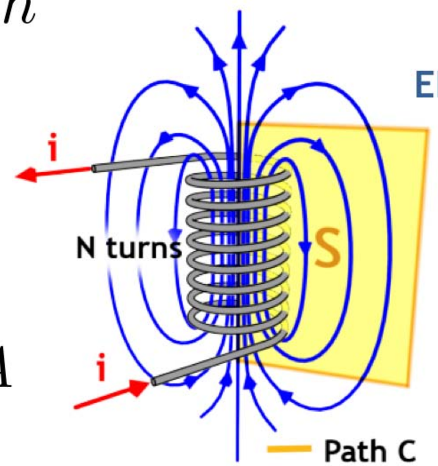
UNITS of INDUCTANCE:

$$\mathrm{H} = \frac{\mathrm{Wb}}{\mathrm{A}} = \frac{\mathrm{V \cdot s}}{\mathrm{A}}$$

$$H_{inside} = \frac{Ni}{h}$$

$$B_{inside} = \mu_o \frac{Ni}{h}$$

$$\Phi_{inside} = \mu_o \frac{Ni}{h} A$$



ENERGY STORED in an INDUCTOR:

$$W_s = \frac{1}{2} L i^2$$

$$\frac{W_s}{Volume} = \frac{1}{2} \mu_o H^2$$

THERE ARE TWO WAYS TO PRODUCE ELECTRIC FIELD

(1)Coulomb electric field is produced by electric charges according to Coulomb's law

(2)Non-Coulomb electric field E_{NC} is due to time-varying magnetic flux density dB/dt

Faraday's Law: The induced emf along a round-trIp path is equal to the rate of change of the magnetic flux on the area encircled by the path.

Lenz's Rule:
The induced electric field would drive the current in the direction to make the magnetic field that attempts to keep the flux constant

$$emf = -\frac{d\Phi_{mag}}{dt}$$

$$\oint_C \overline{E} \cdot d\bar{l} = -\frac{d}{dt} \left(\int_S \overline{B} \cdot d\overline{A} \right)$$

6.007 Electromagnetic Energy: From Motors to Lasers
Spring 2011