Magnetic Circuits

Outline

- Ampere’s Law Revisited
- Review of Last Time: Magnetic Materials
- Magnetic Circuits
- Examples

Electric Fields

GAUSS

$$\oint_S \epsilon_o \vec{E} \cdot d\vec{A} = \int_V \rho dV$$

$$= Q_{enclosed}$$

FARADAY

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\left(\int_S \vec{B} \cdot d\vec{A}\right)$$

$$emf = v = \frac{d\lambda}{dt}$$

Magnetic Fields

GAUSS

$$\oint_S \vec{B} \cdot d\vec{A} = 0$$

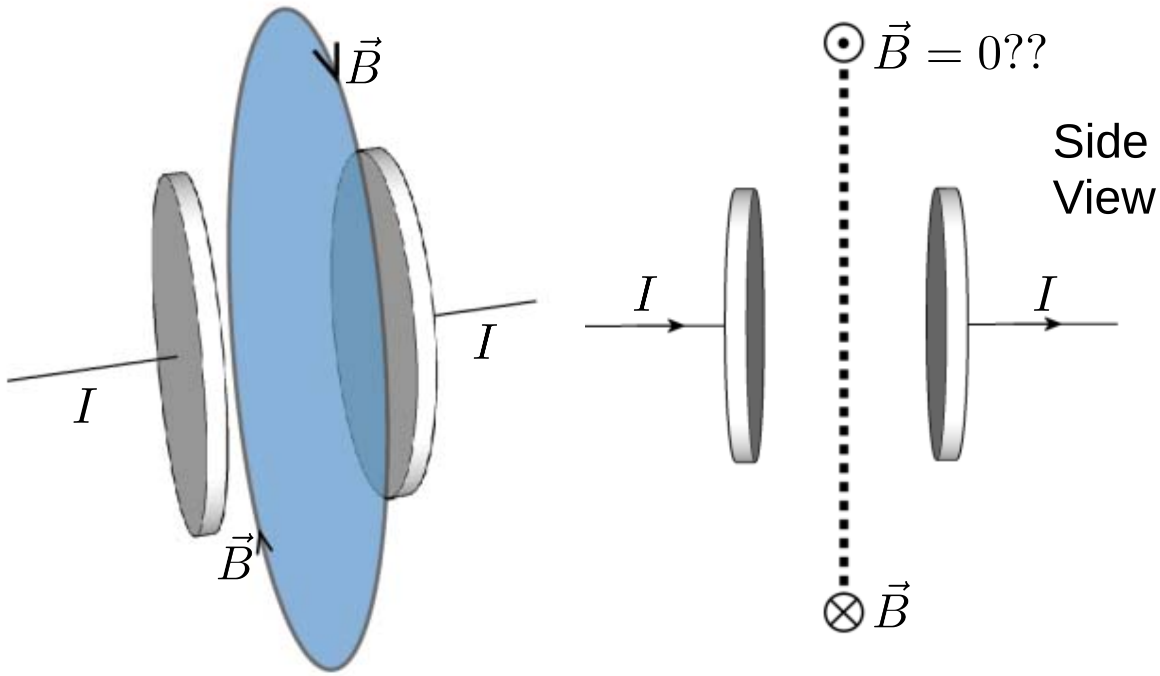
AMPERE

$$\oint_C \vec{H} \cdot d\vec{l}$$

$$= \int_S \vec{J} \cdot d\vec{A} + \frac{d}{dt}\int_S \epsilon\vec{E} \cdot d\vec{A}$$

Ampere's Law Revisited

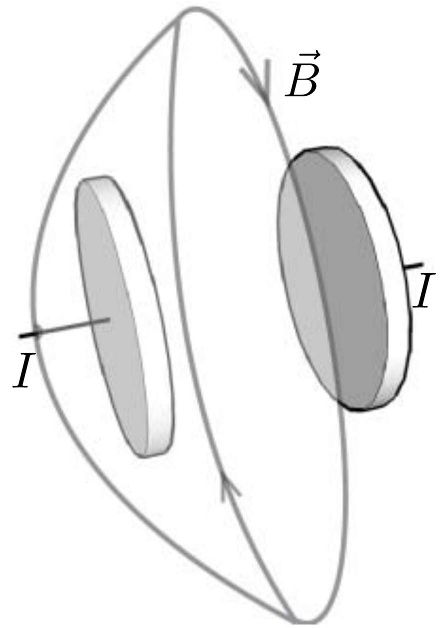
In the case of the magnetic field we can see that 'our old' Ampere's law can not be the whole story. Here is an example in which current does not gives rise to the magnetic field:

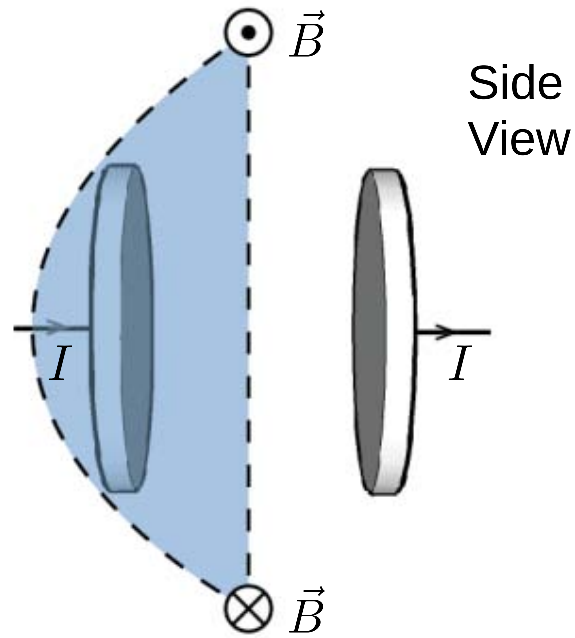


Consider the case of charging up a capacitor C which is connected to very long wires. The charging current is *I*. From the symmetry it is easy to see that an application of Ampere's law will produce *B* fields which go in circles around the wire and whose magnitude is $B(r) = \mu_o I/(2\pi r)$. But there is no charge flow in the gap across the capacitor plates and according to Ampere's law the *B* field in the plane parallel to the capacitor plates and going through the capacitor gap should be zero!
This seems unphysical.

Ampere's Law Revisited (cont.)

If instead we drew the Amperian surface as sketched below,
we would have concluded that *B* in non-zero !





Maxwell resolved this problem by adding a term to the Ampere's Law.
In equivalence to Faraday's Law,
the changing electric field can generate the magnetic field:

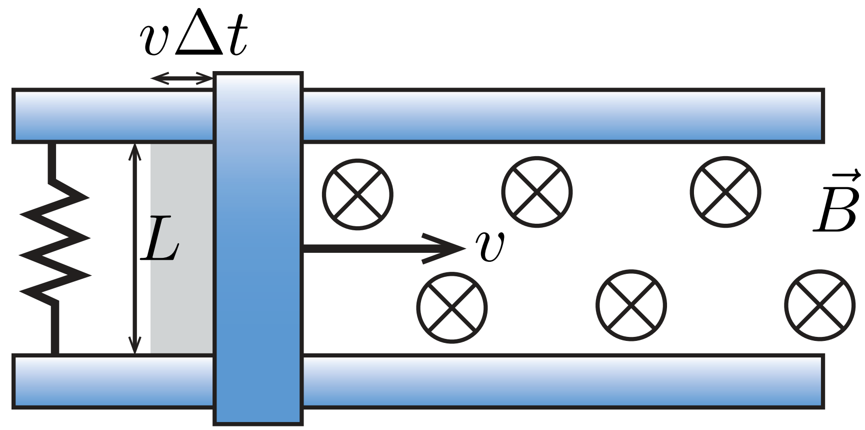
$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{A} + \frac{d}{dt} \int_S \epsilon \vec{E} \cdot d\vec{A}$$

COMPLETE AMPERE'S LAW

Faraday's Law and Motional emf

What is the emf over the resistor ?

$$emf = -\frac{d\Phi_{mag}}{dt}$$



In a short time Δt the bar moves a distance Δx = v*Δt, and the flux increases by $\Delta\Phi_{mag}$ = B (L v*Δt)

$$emf = \frac{\Delta\Phi_{mag}}{\Delta t} = BLv$$

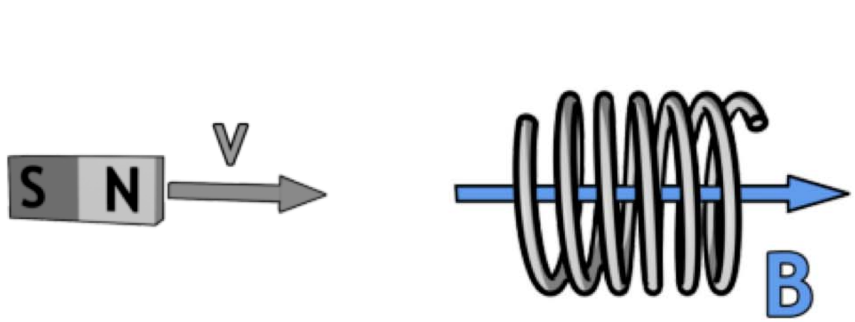
There is an increase in flux through the circuit as the bar of length *L* moves to the right (orthogonal to magnetic field H) at velocity, *v*.

from Chabay and Sherwood, Ch 22

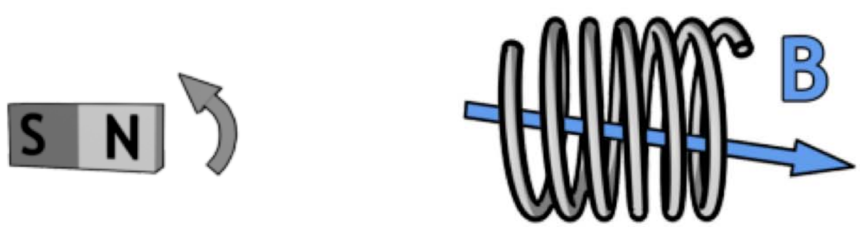
Faraday's Law for a Coil

The induced emf in a coil of N turns is equal to
N times the rate of change of the magnetic flux on one loop of the coil.

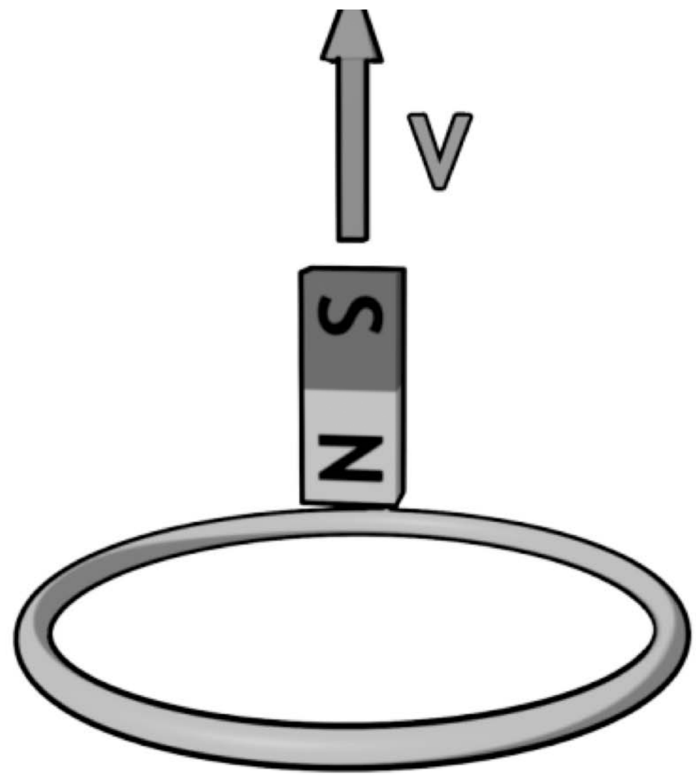
$$emf = -N\frac{d\Phi_{mag}}{dt}$$



Moving a magnet towards a coil produces a time-varying magnetic field inside the coil



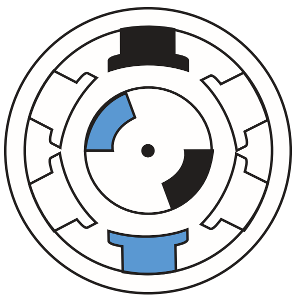
Rotating a bar of magnet (or the coil) produces a time-varying magnetic field inside the coil



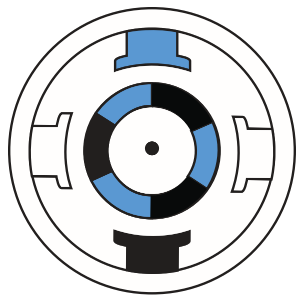
Will the current run
CLOCKWISE or ANTICLOCKWISE ?

<u>*Complex Magnetic Systems*</u>

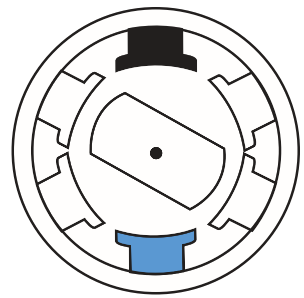
DC Brushless



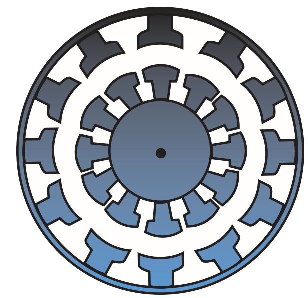
Stepper Motor



Reluctance Motor



Induction Motor



$$\int_C \vec{H} \cdot d\vec{l} = I_{enclosed}$$

$$\int_S \vec{B} \cdot d\vec{A} = 0$$

$$\vec{f} = q\left(\vec{v} \times \vec{B}\right)$$

We need better (more powerful) tools...

Magnetic Circuits: *Reduce Maxwell to (scalar) circuit problem*

Energy Method: *Look at change in stored energy to calculate force*

Magnetic Flux Φ [Wb] (Webers)

Magnetic Flux Density B [Wb/m^2] = T (Teslas)

Magnetic Field Intensity H [Amp-turn/m]

due to macroscopic & microscopic (→ B)

due to macroscopic currents (→ H)

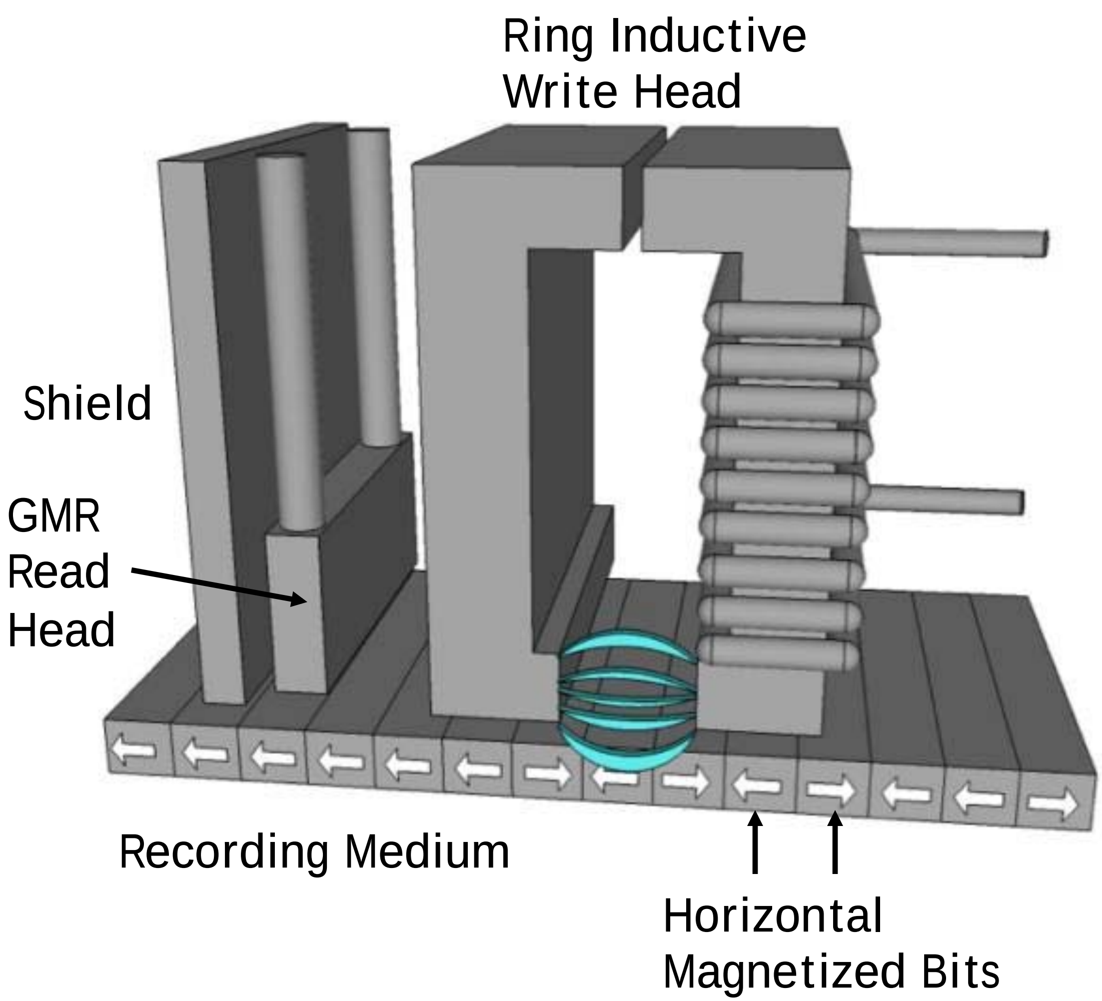
$$\vec{B} = \mu_o \left(\vec{H} + \vec{M} \right) = \mu_o \left(\vec{H} + \chi_m \vec{H} \right) = \mu_o \mu_r \vec{H}$$

Faraday's Law

$$emf = -\frac{d\Phi_{mag}}{dt}$$

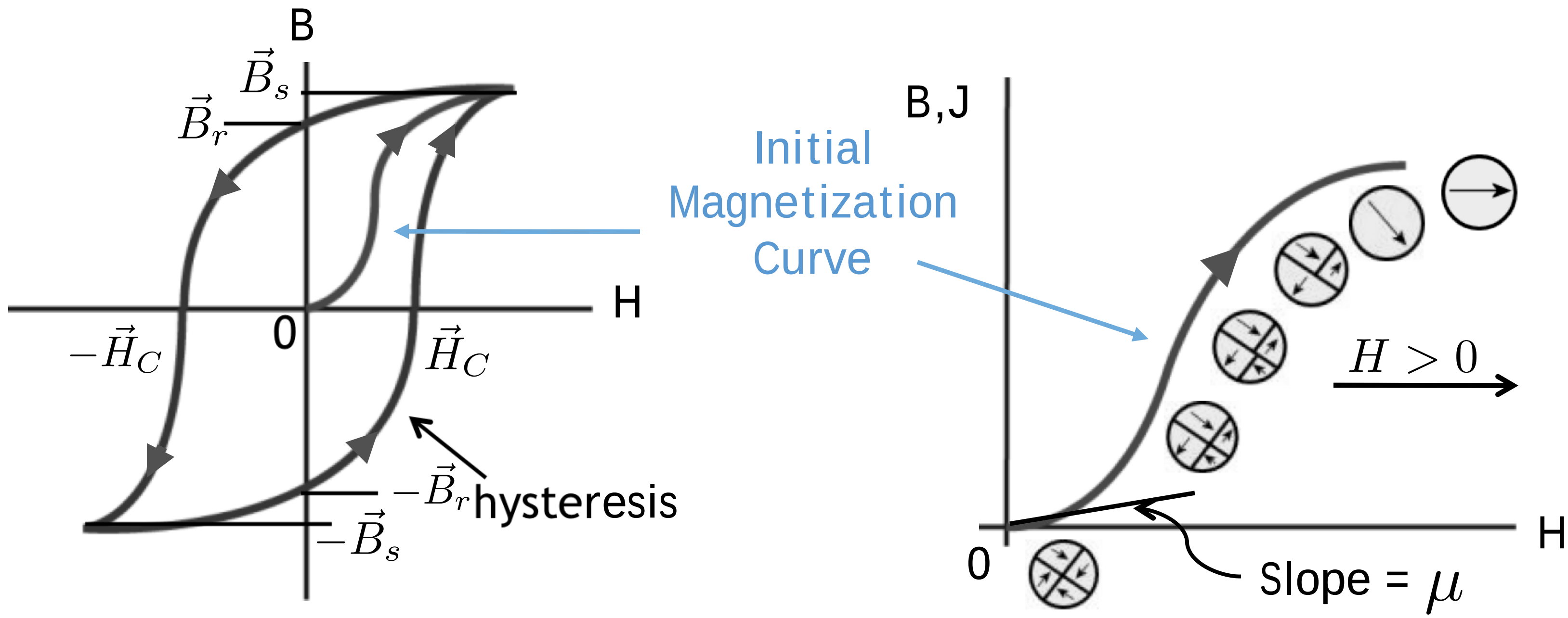
$$emf = \oint \vec{E}_{NC} \cdot d\vec{l} \quad \text{and} \quad \Phi_{mag} = \int \vec{B} \cdot \hat{n} d\vec{A}$$

Example: Magnetic Write Head



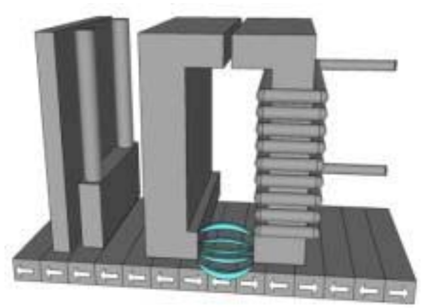
Bit density is limited by how well the field can be localized in write head

Review: Ferromagnetic Materials



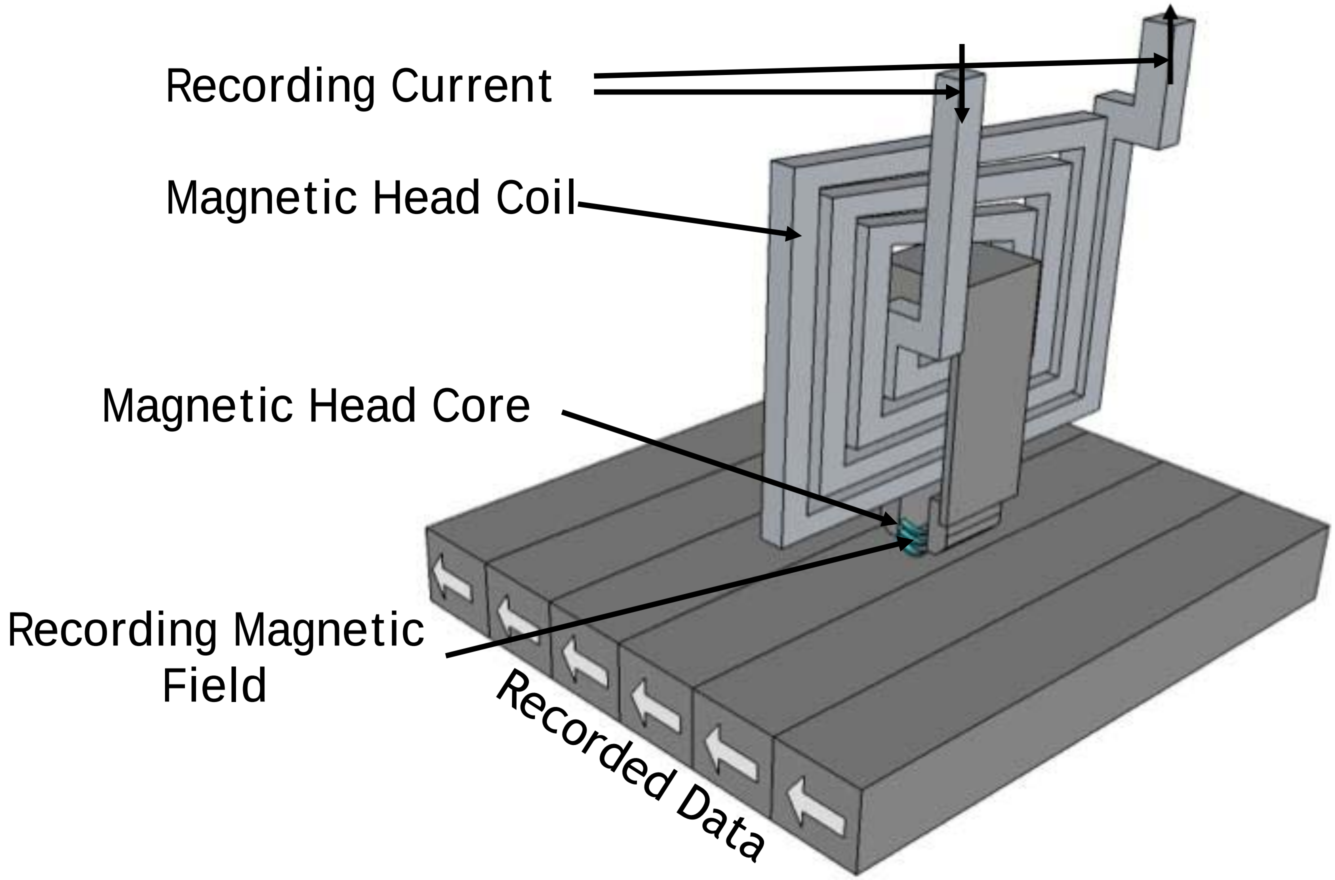
c
Hr : coercive magnetic field strength
Bs : remanence flux density
B : saturation flux density

Behavior of an initially unmagnetized material.
Domain configuration during several stages of magnetization.



$$H \sim i$$

Thin Film Write Head

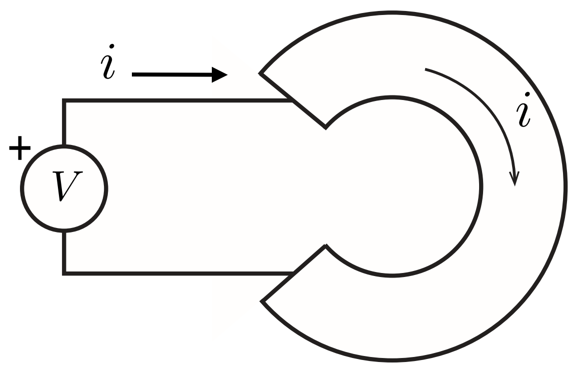


How do we apply Ampere's Law to this geometry (low symmetry) ?

$$\int_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{A}$$

Electrical Circuit Analogy

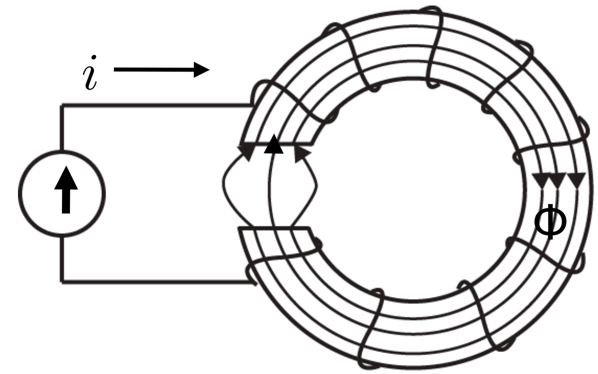
Charge is conserved...



Electrical

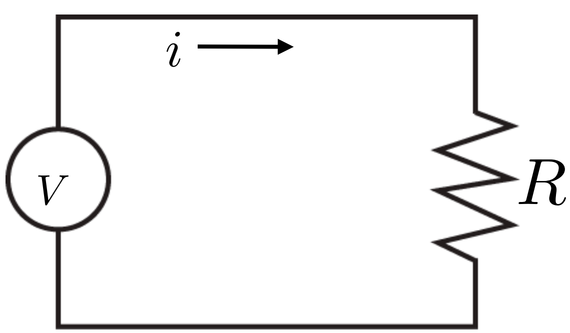
Flux is ‘conserved’...

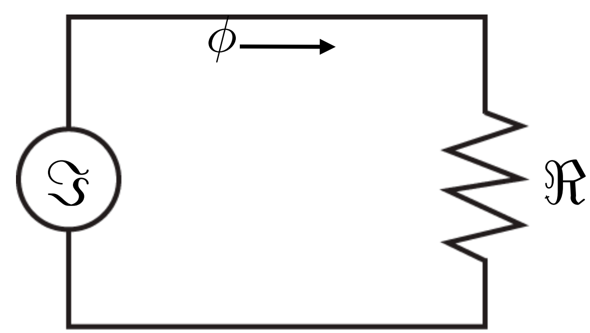
$$\int_S \vec{B} \cdot d\vec{A} = 0$$



Magnetic

EQUIVALENT
CIRCUITS

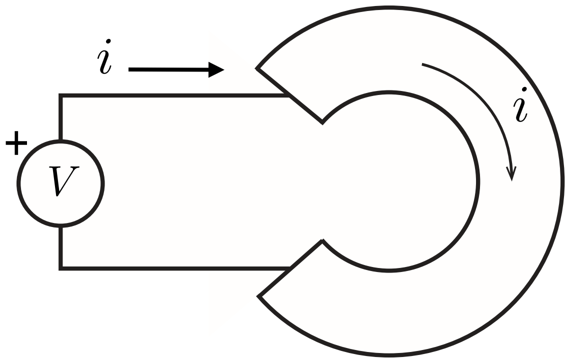




Electrical Circuit Analogy

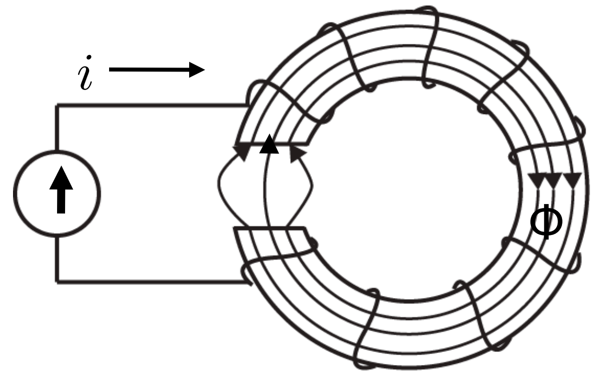
Electromotive force (charge push)=

$$v = \int \vec{E} \cdot d\vec{l}$$



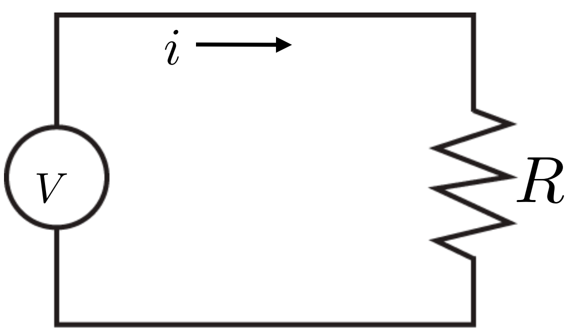
Magneto-motive force (flux push)=

$$\oint_C \vec{H} \cdot d\vec{l} = I_{enclosed}$$

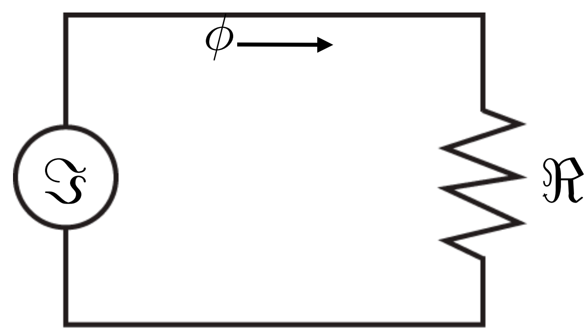


EQUIVALENT CIRCUITS

Electrical

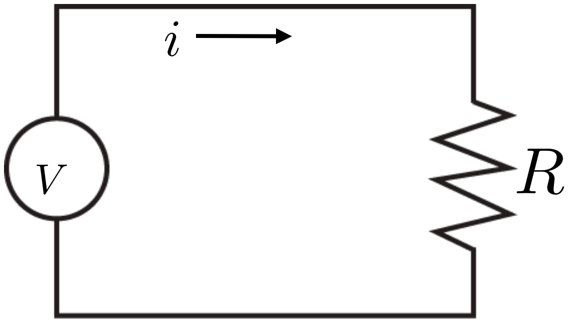


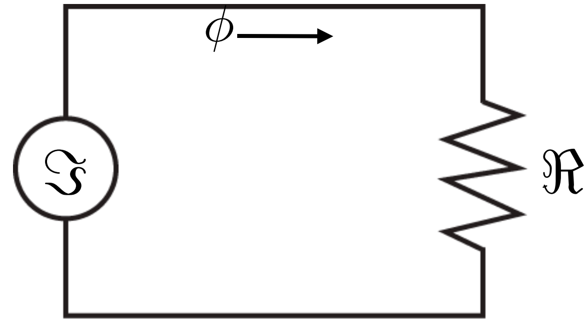
Magnetic



Electrical Circuit Analogy

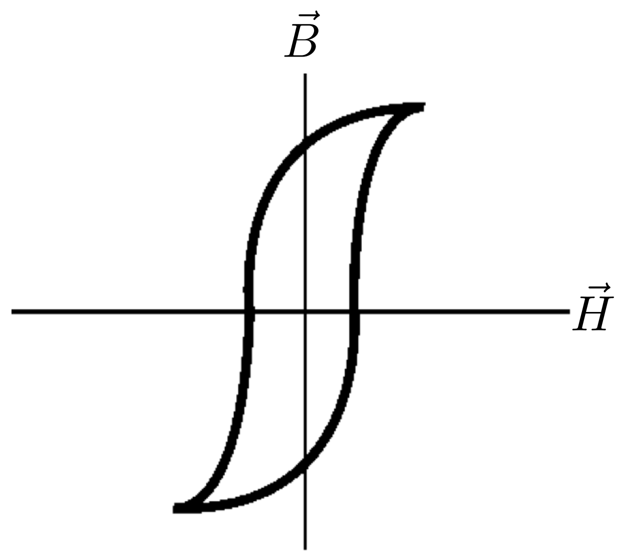
Material properties and geometry determine flow – push relationship





OHM's LAW $\quad \vec{J} = \sigma \vec{E}_{DC}$

$$\vec{B} = \mu_o \mu_r \vec{H}$$



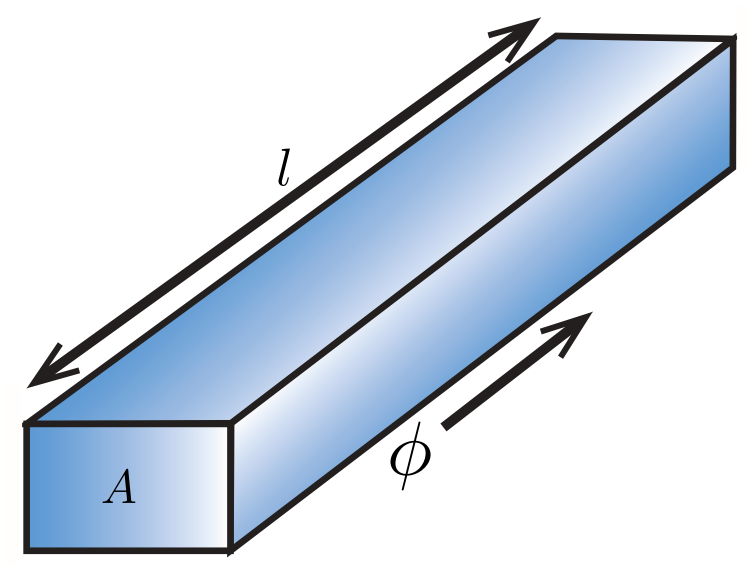
Recovering macroscopic variables:

$$I = \int \vec{J} \cdot d\vec{A} = \sigma \int \vec{E} \cdot d\vec{A} = \sigma \frac{V}{l} A$$

$$V = I \frac{l}{\sigma A} = I \frac{\rho l}{A} = IR$$

$$Ni = \Phi \Re$$

Reluctance of Magnetic Bar



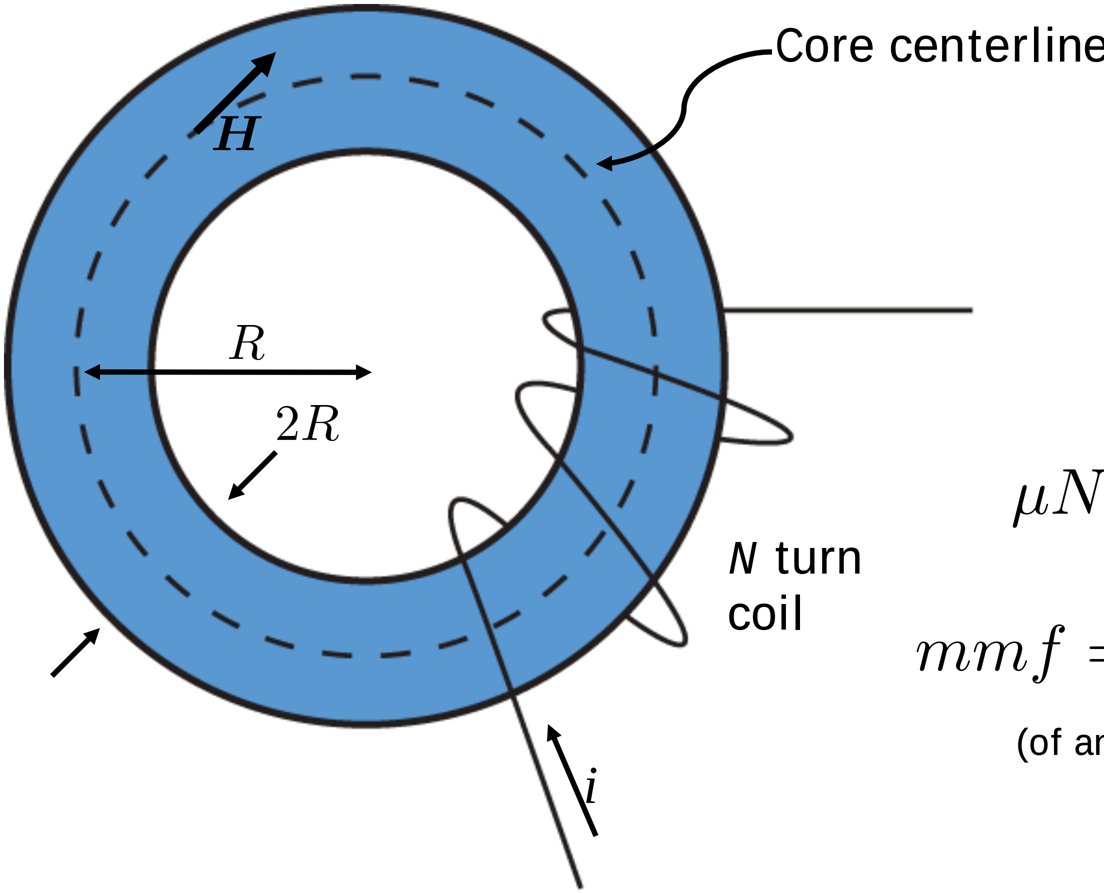
Magnetic "OHM's LAW"

$$Ni = \Phi \Re$$

$$\Re = \frac{l}{\mu A}$$

The reluctance $\mathcal{R}$ of a magnetic path depends on the mean length l, the area A, and the permeability μ of the material.

Flux Density in a Toroidal Core



$$B = \frac{\mu N i}{2\pi R}$$

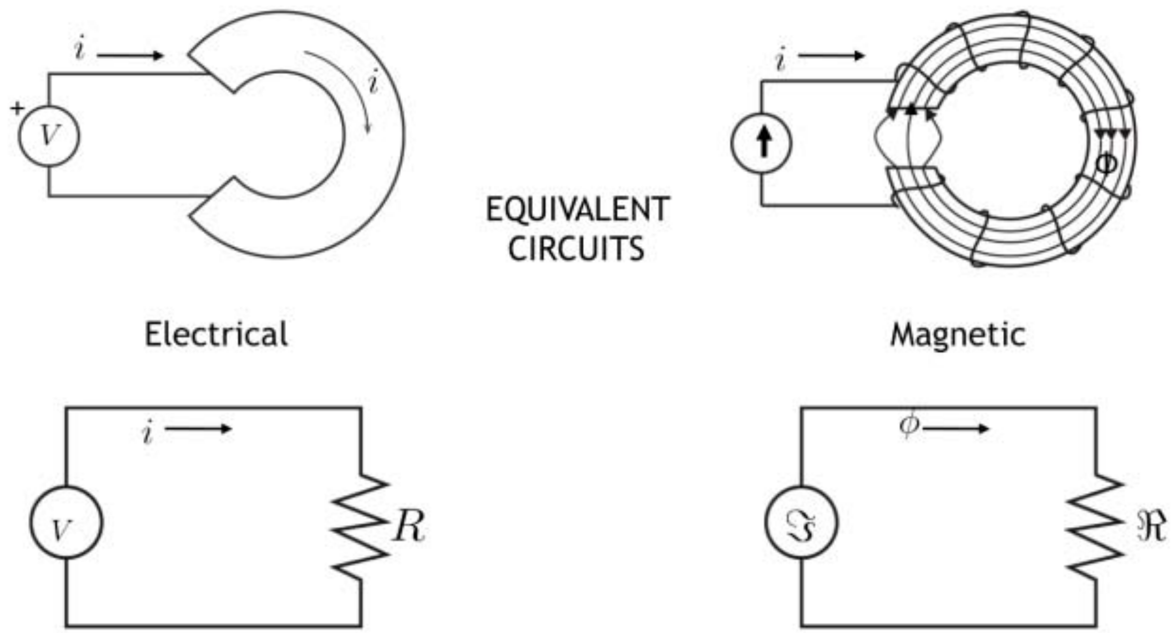
$$\mu N i = 2\pi R B = l B$$

$$mmf = Ni = \frac{lB}{\mu} = \boxed{\Phi \frac{l}{\mu A}}$$

(of an N-turn coil)

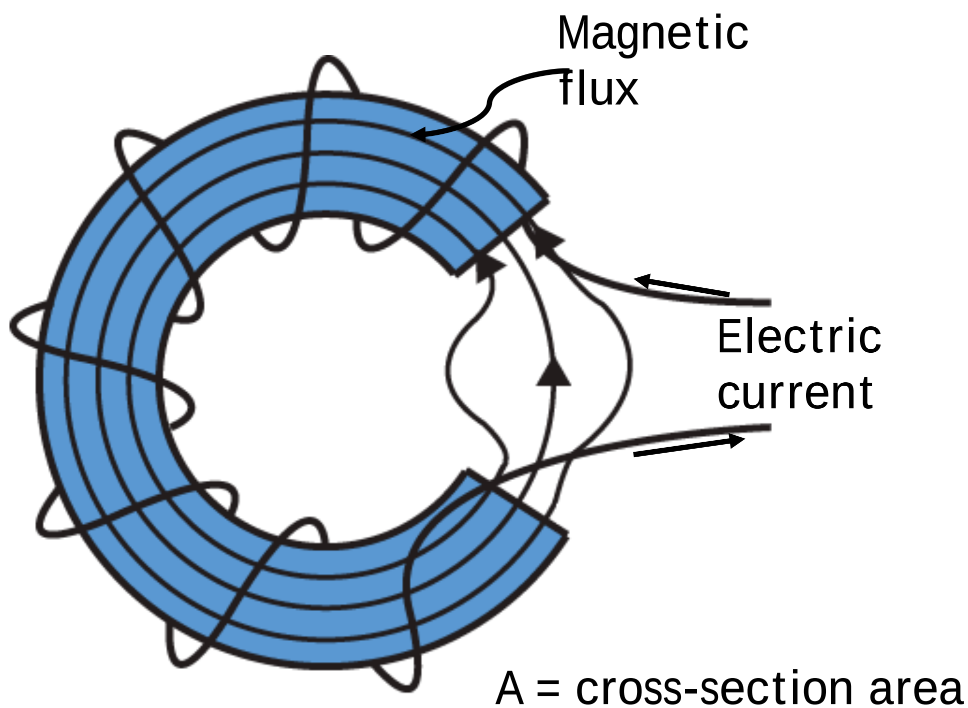
$$mmf = \Phi \Re$$

Electrical Circuit Analogy



Electrical	Magnetic
Voltage v	Magnetomotive Force $\Im = Ni$
Current i	Magnetic Flux ϕ
Resistance R	Reluctance $\Re$
Conductivity $1/\rho$	Permeability μ
Current Density J	Magnetic Flux Density B
Electric Field E	Magnetic Field Intensity H

Toroid with Air Gap



Why is the flux confined mainly to the core ?

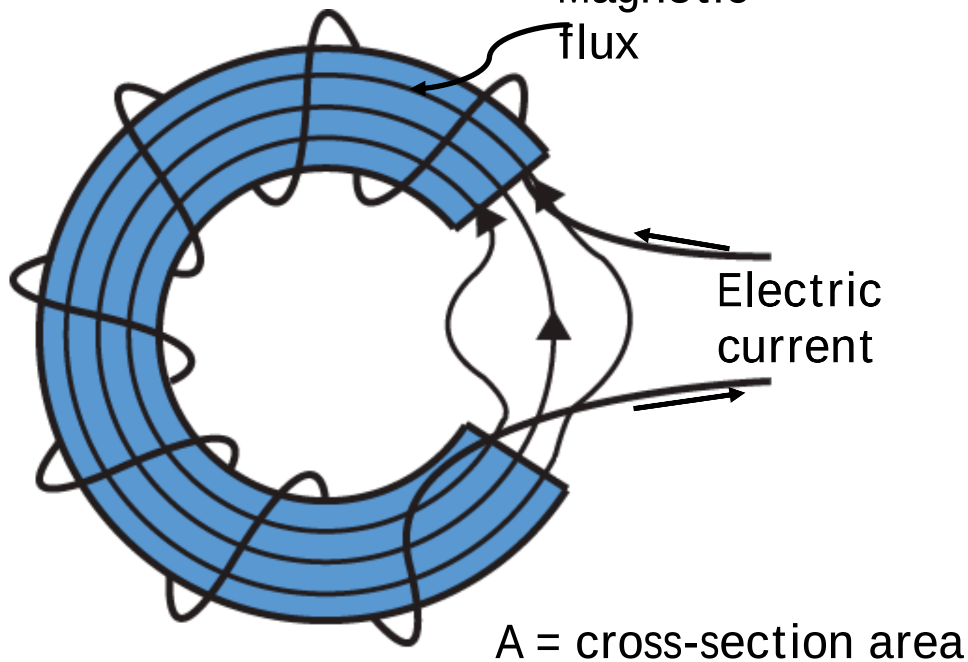
Can the reluctance ever be infinite (magnetic insulator) ?

Why does the flux not leak out further in the gap ?

Fields from a Toroid

$$\int_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{A}$$

$$= I_{enclosed}$$



$$H = \frac{Ni}{2\pi R}$$

$$\vec{B} = \mu_o \left(\vec{H} + \vec{M}\right)$$

$$B = \mu \frac{Ni}{2\pi R}$$

$$\Phi = BA = Ni \frac{\mu A}{2\pi R}$$

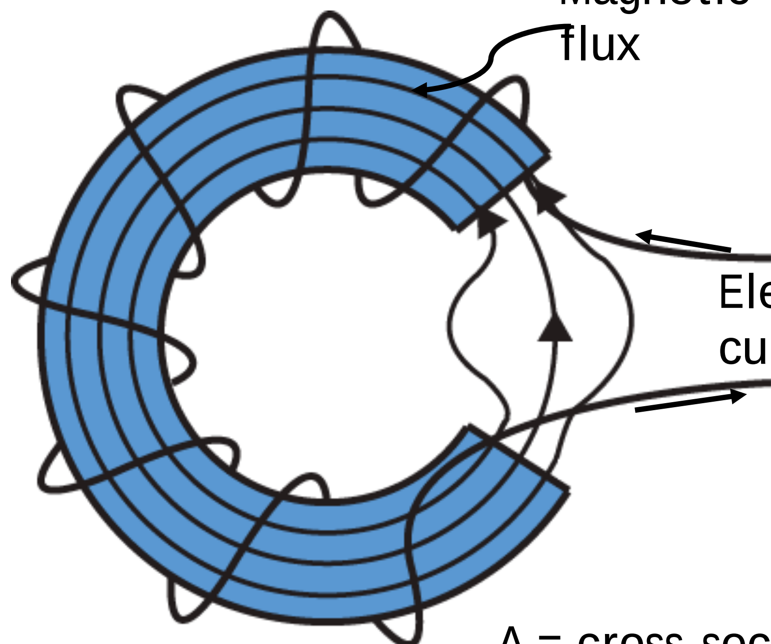
Scaling Magnetic Flux

$$Ni = \Phi \Re$$

&

$$\Re = \frac{l}{\mu A} \Rightarrow \Re = \frac{2\pi R}{\mu A}$$

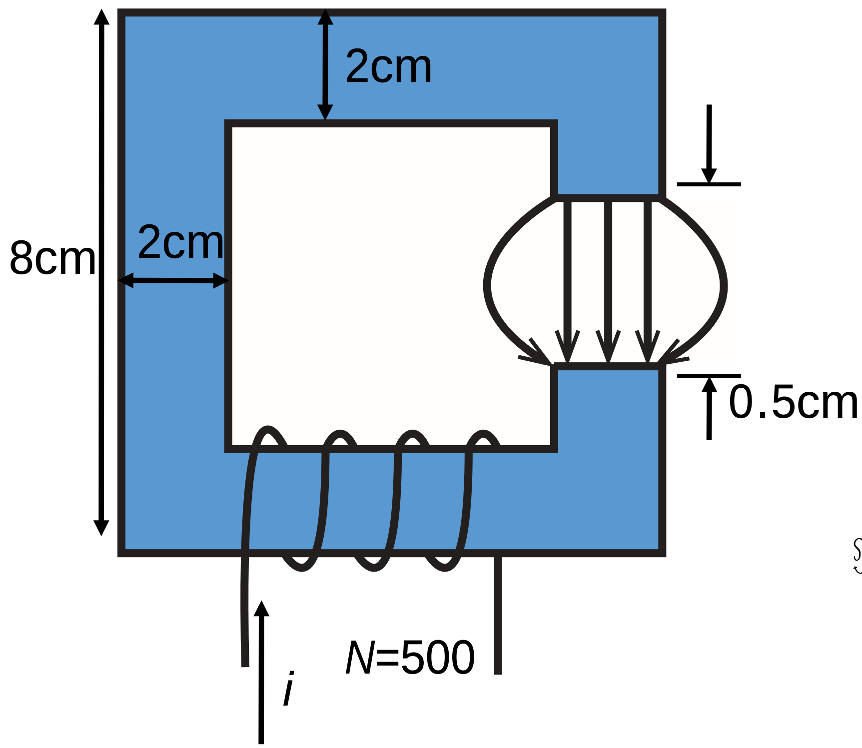
$$\Phi = BA = Ni \frac{\mu A}{2\pi R}$$



Same answer as Ampere's Law (slide 9)

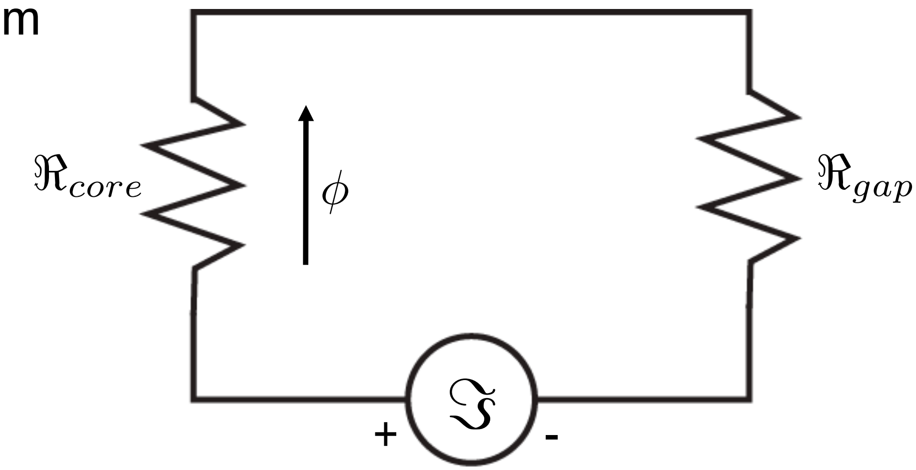
Magnetic Circuit for 'Write Head'

Core Thickness = 3cm



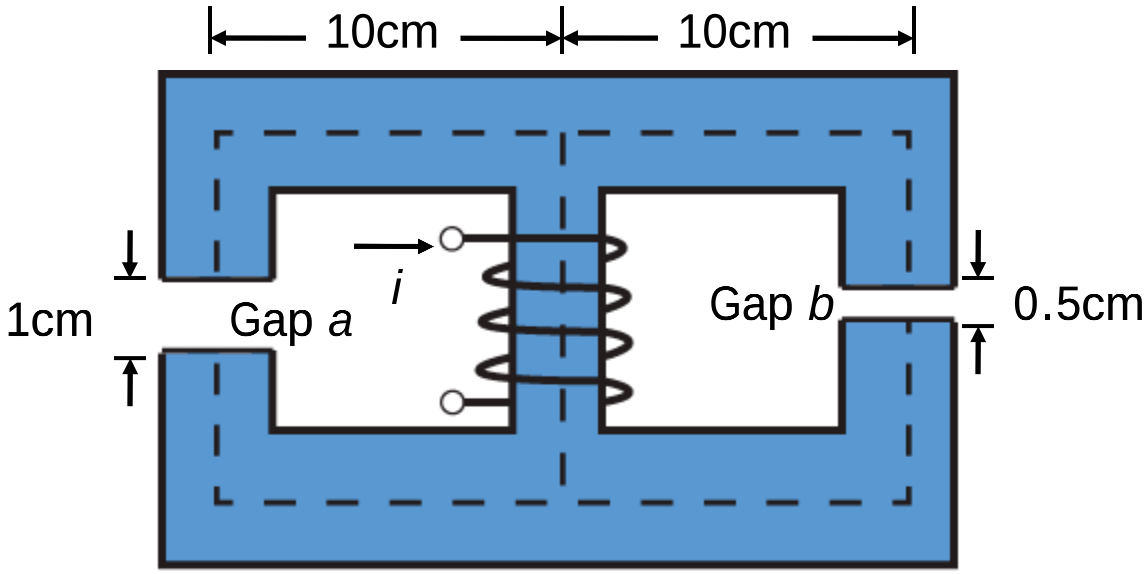
A = cross-section area

$$\Re = \frac{l}{\mu A}$$



$$\Phi \approx \boxed{}$$

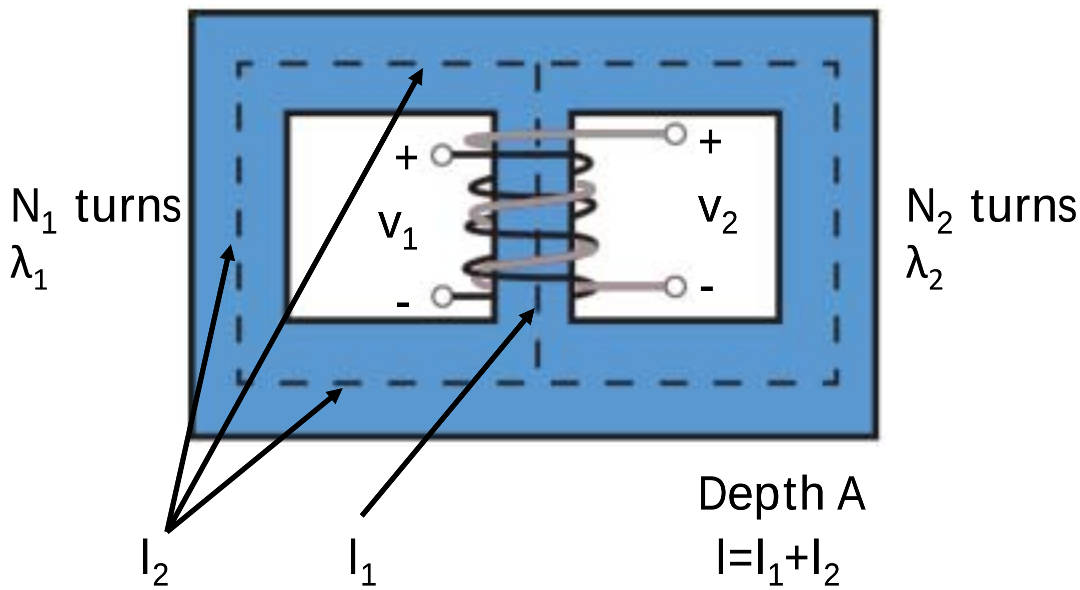
Parallel Magnetic Circuits



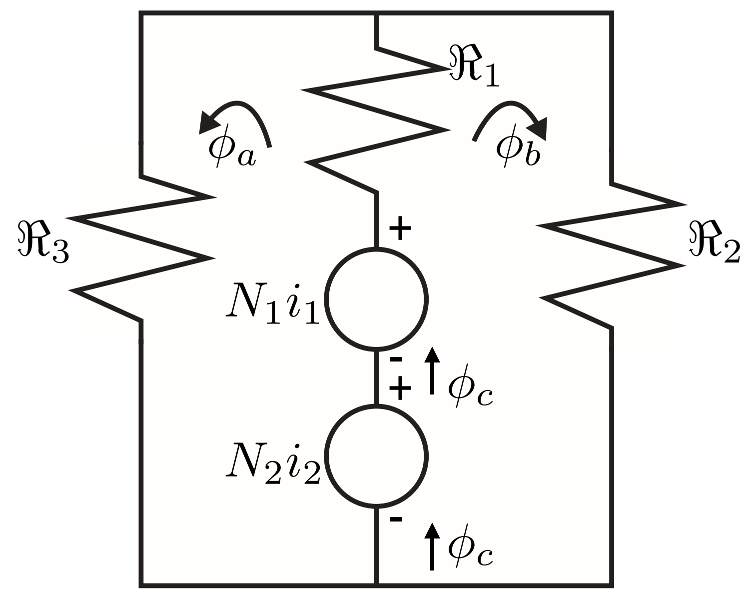
A = cross-section area

A Magnetic Circuit with Reluctances in Series and Parallel

"Shell Type" Transformer

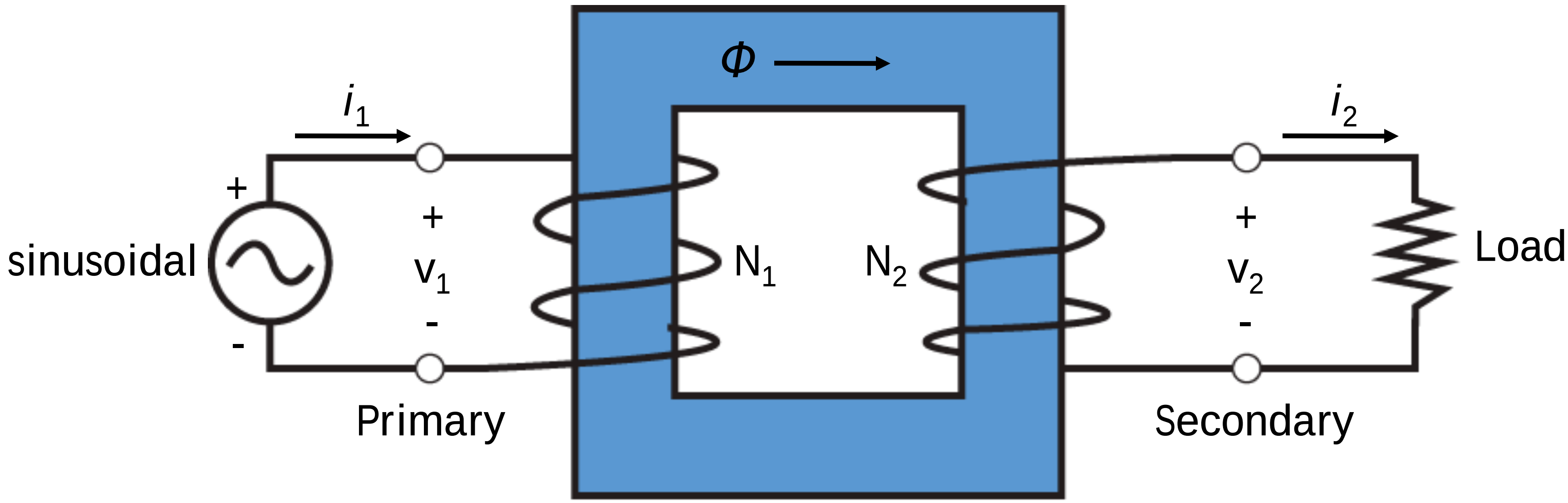


Magnetic Circuit



$$\Re_1 = \frac{l_1}{\mu A} \qquad \Re_2 = \frac{l_2}{\mu A}$$

Faraday Law and Magnetic Circuits



A = cross-section area

Flux linkage $\lambda = N\Phi$ $v = \frac{d\lambda}{dt}$

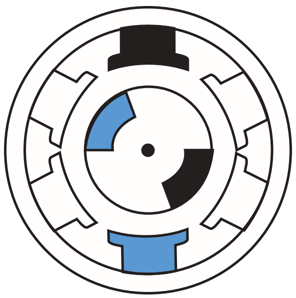
Step 1: Estimate voltage v_1 due to time-varying flux…

Step 2: Estimate voltage v_2 due to time-varying flux…

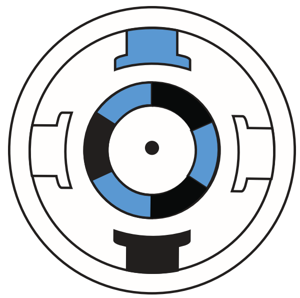
$$\frac{v_2}{v_1} = \boxed{}$$

Complex Magnetic Systems

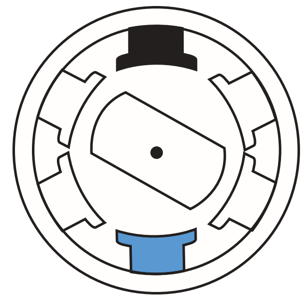
DC Brushless



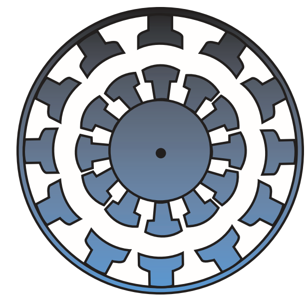
Stepper Motor



Reluctance Motor



Induction Motor



Powerful tools...

Magnetic Circuits: *Reduce Maxwell to (scalar) circuit problem*

Makes it easy to calculate B, H, λ

Energy Method: *Look at change in stored energy to calculate force*

Stored Energy in Inductors

In the absence of mechanical displacement...

$$W_S = \int P_{elec} dt = \int i v dt = \int i \frac{d\lambda}{dt} = \int i\left(\lambda\right) d\lambda$$

For a linear inductor:

$$i\left(\lambda\right) = \frac{\lambda}{L}$$

→ **Stored energy...**

$$W_S = \int_0^{\lambda} \frac{\lambda'}{L} d\lambda' = \frac{\lambda^2}{2L}$$

Relating Stored Energy to Force

Lets use chain rule ...

$$\frac{W_S\left(\Phi, r\right)}{dt} = \frac{\partial W_S}{\partial \Phi}\frac{d\Phi}{dt} + \frac{\partial W_S}{\partial r}\frac{dr}{dt}$$

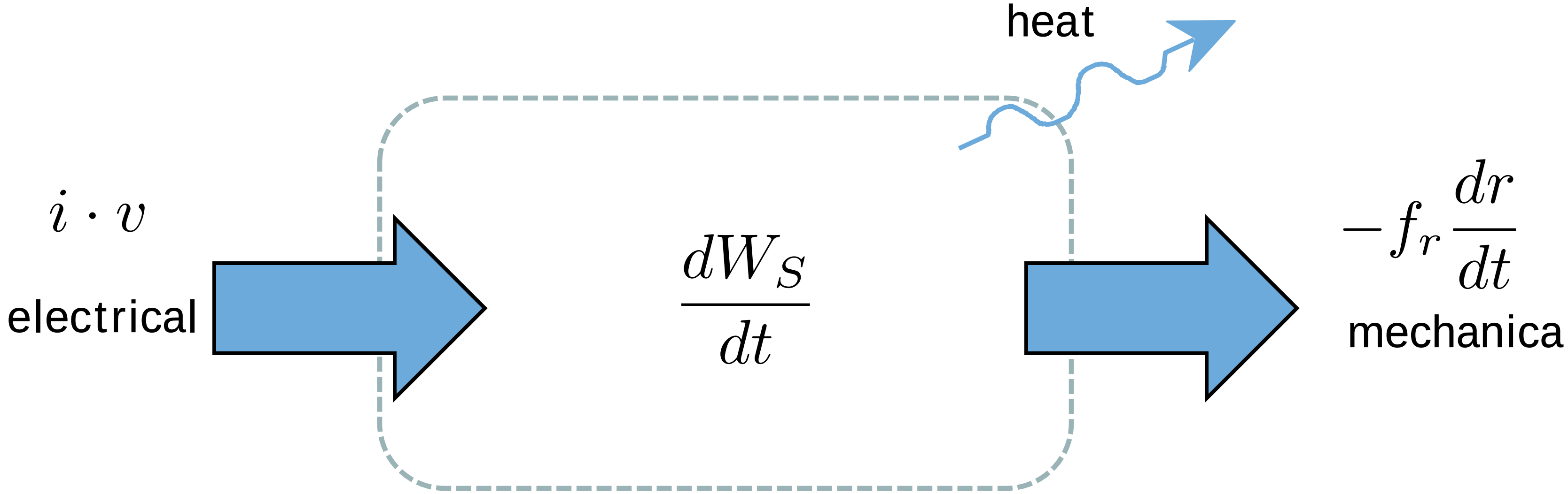
This looks familiar ...

$$\begin{aligned} \frac{dW_S}{dt} &= i \cdot v - f_r \frac{dr}{dt} \\ &= iL\frac{di}{dt} - f_r \frac{dr}{dt} \end{aligned}$$

Comparing similar terms suggests ...

$$f_r = -\frac{\partial W_S}{\partial r}$$

Energy Balance



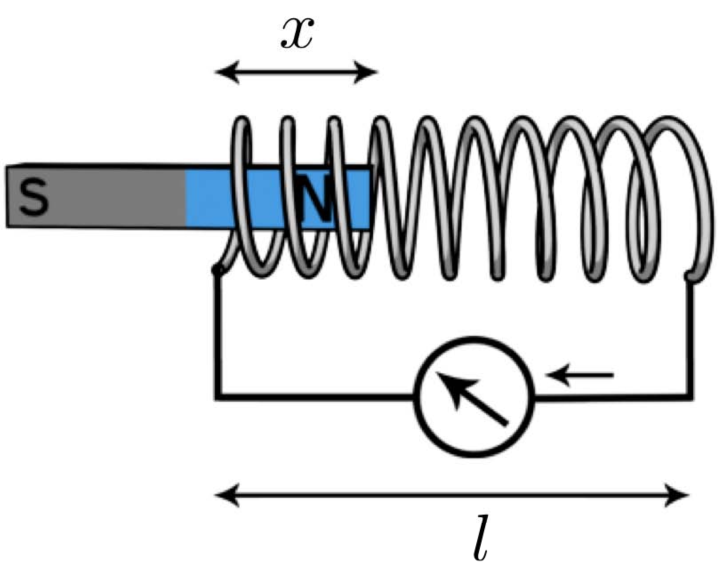
$$\frac{dW_S(\lambda, r)}{dt} = \frac{\partial W_S}{\partial \lambda}\frac{d\lambda}{dt} + \frac{\partial W_S}{\partial r}\frac{dr}{dt}$$

neglect heat

For magnetostatic system, dλ=0 **no electrical power** flow...

$$\frac{dW_S}{dt} = -f_r \frac{dr}{dt}$$

Linear Machines: Solenoid Actuator



Coil attached to cone

If we can find the stored energy, we can immediately compute the force...

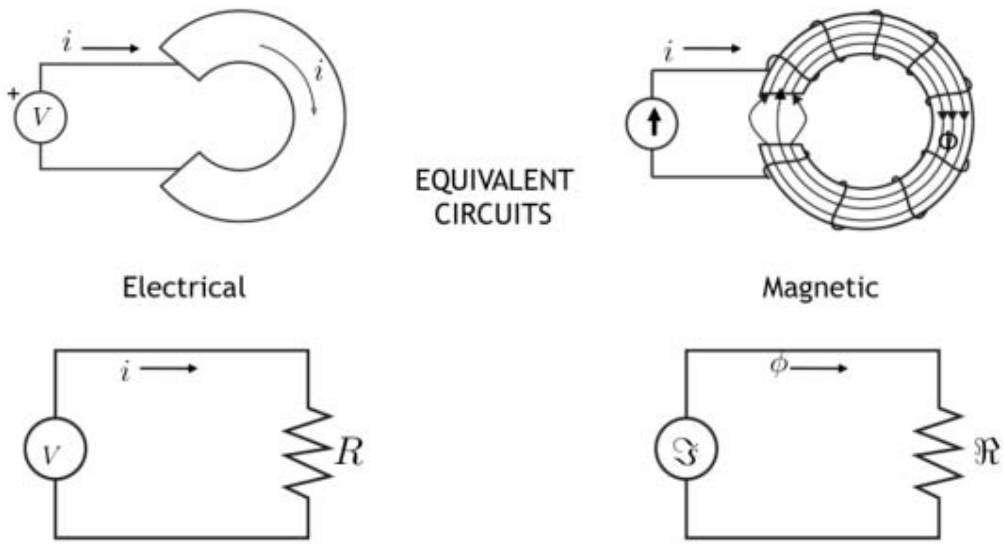
...lets take all the things we know to put this together...

$$f_r = -\frac{\partial W_S}{\partial r}|_\lambda \qquad W_S(\lambda, r) = \frac{1}{2}\frac{\lambda^2}{L}$$

KEY TAKEAWAYS

COMPLETE AMPERE'S LAW

$$\int_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{A} + \frac{d}{dt} \int_S \epsilon \vec{E} \cdot d\vec{A}$$



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Resistance R	Reluctance $\Re$
Conductivity $1/\rho$	Permeability μ
Current Density J	Magnetic Flux Density B
Electric Field E	Magnetic Field Intensity H

RELUCTANCE

$$\Re = \frac{l}{\mu A}$$

6.007 Electromagnetic Energy: From Motors to Lasers
Spring 2011