Dielectrics and Dipoles

Reading – Shen and Kong – Ch. 10

Outline

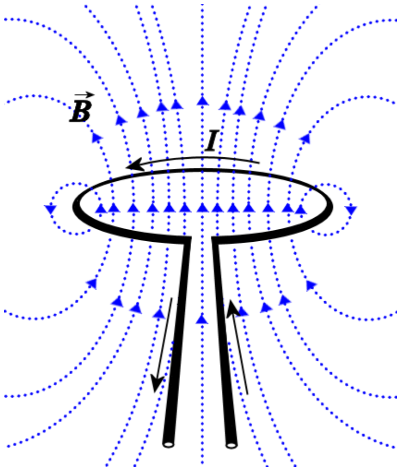
- Polarization and Dipole Density
- Dielectric Constant
- Microphones
- More Dielectric Actuators

True or False?

1. The magnetic moment is defined as

$$m = i\,\mathbf{a}$$

and has units of Amp-m^2



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2. In a linear magnetic material, the magnetization is given by $\vec{M} = \chi_m \vec{H}$ where χ_m is the magnetic susceptibility with units of m^3

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3. The energy stored in the magnetic field is ($\frac{1}{2}\ \mu_o H^2$)·(Volume) and has units of Joules.

______

Electric Fields

$$\oint_S \epsilon_o \vec{E} \cdot d\vec{A} = \int_V \rho dV$$

$$= Q_{enclosed}$$

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\left(\int_S \vec{B} \cdot d\vec{A}\right)$$

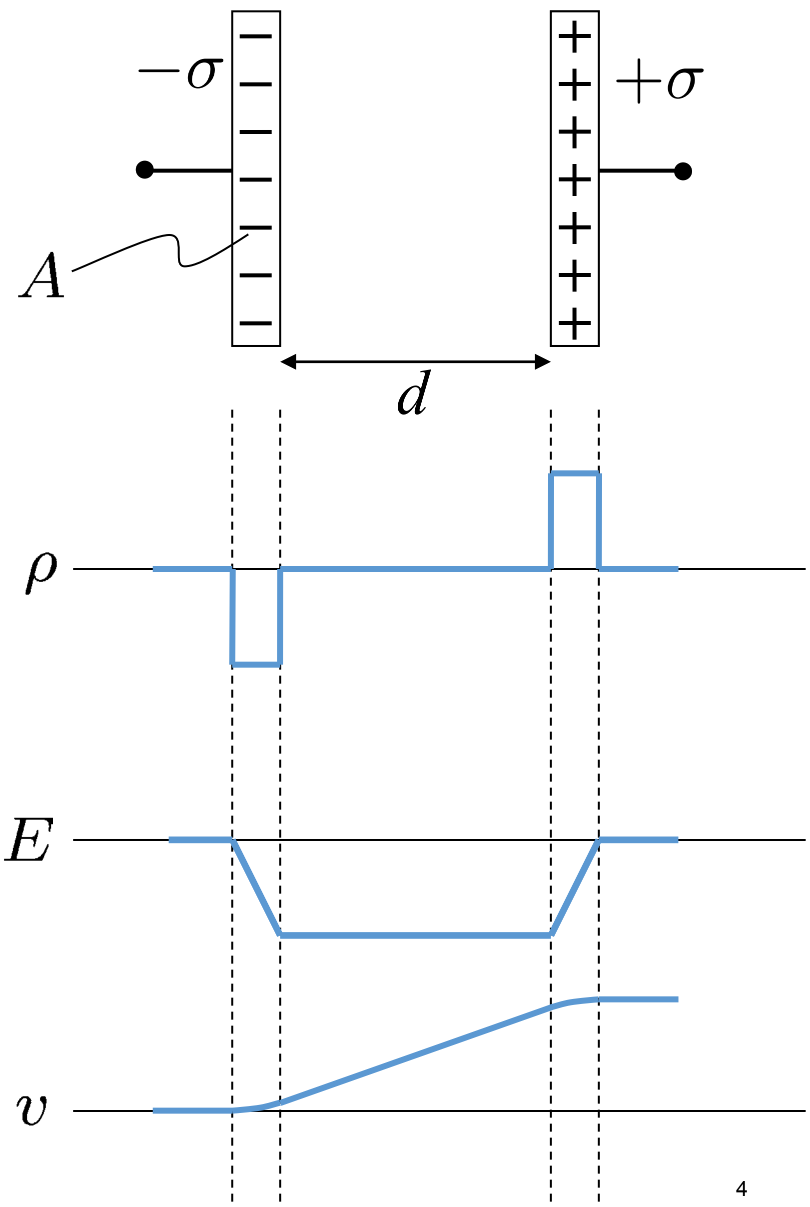
Magnetic Fields

$$\oint_S \vec{B} \cdot d\vec{A} = 0$$

$$\oint_C \vec{H} \cdot d\vec{l}$$

$$= \int_S \vec{J} \cdot d\vec{A} + \frac{d}{dt}\int_S \epsilon \vec{E} \cdot d\vec{A}$$

Air Capacitors



$$Cv = q$$

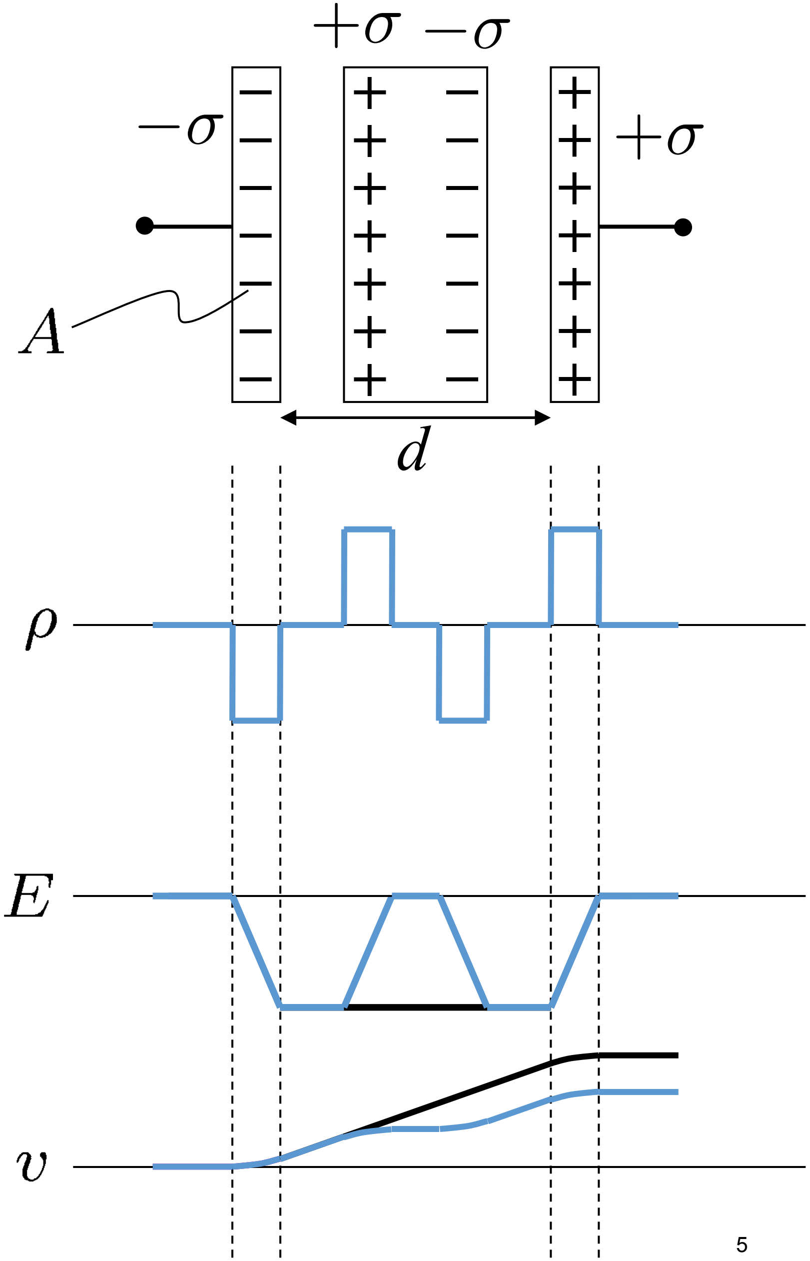
$$\epsilon_o \frac{A}{d} v = \sigma A$$

$$E = \frac{\sigma}{\epsilon_o}$$

$$\oint_S \epsilon_o \vec{E} \cdot d\vec{A} = \int_V \rho dV$$

$$v = -\int \vec{E} \cdot d\vec{l}$$

... let's insert a metal sheet between capacitor plates

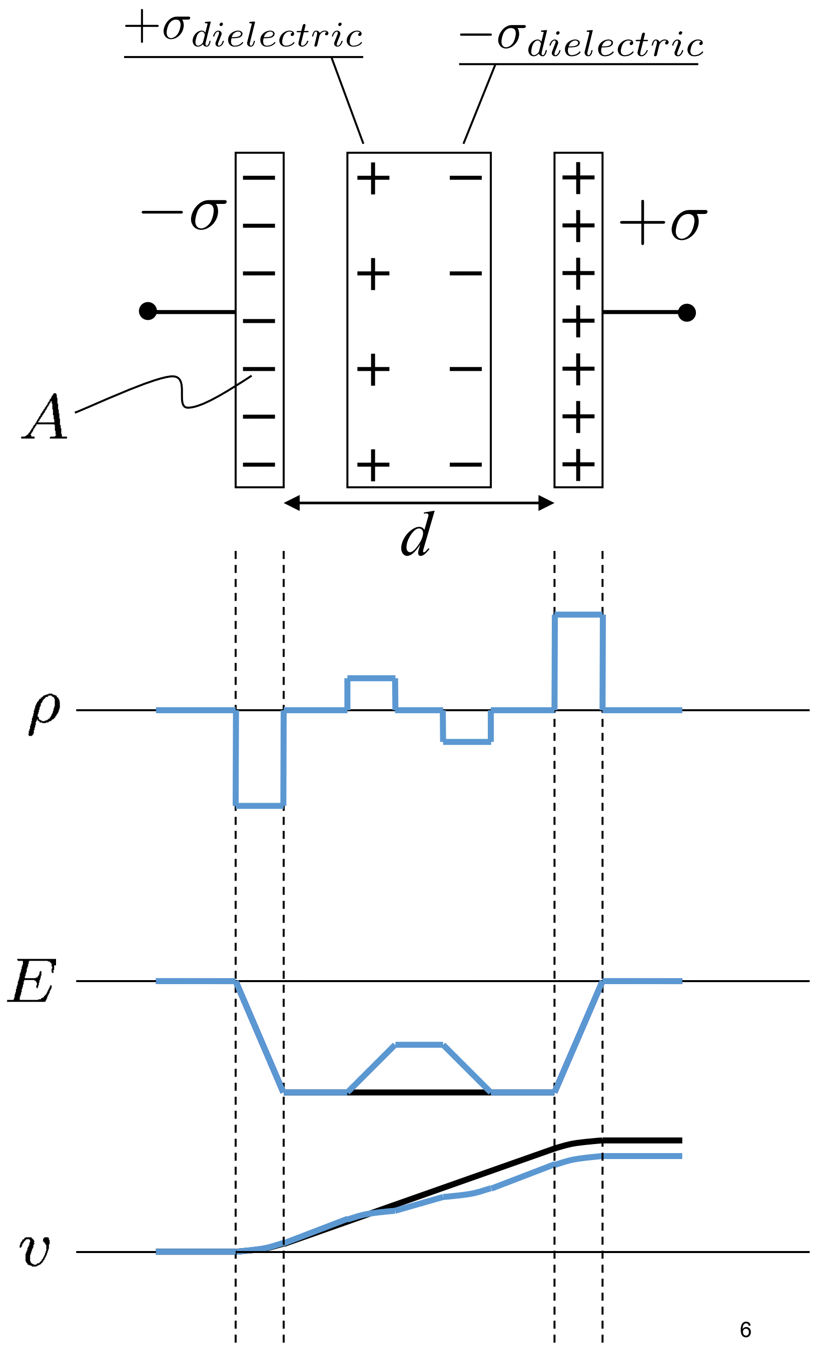


$$Cv = q$$

$$\epsilon_o \frac{A}{d} v = \sigma A$$

$$E = \frac{\sigma}{\epsilon_o}$$

- Capacitance increases
- Since σ remained the same then v decreases



… let's insert an insulator sheet between capacitor plates

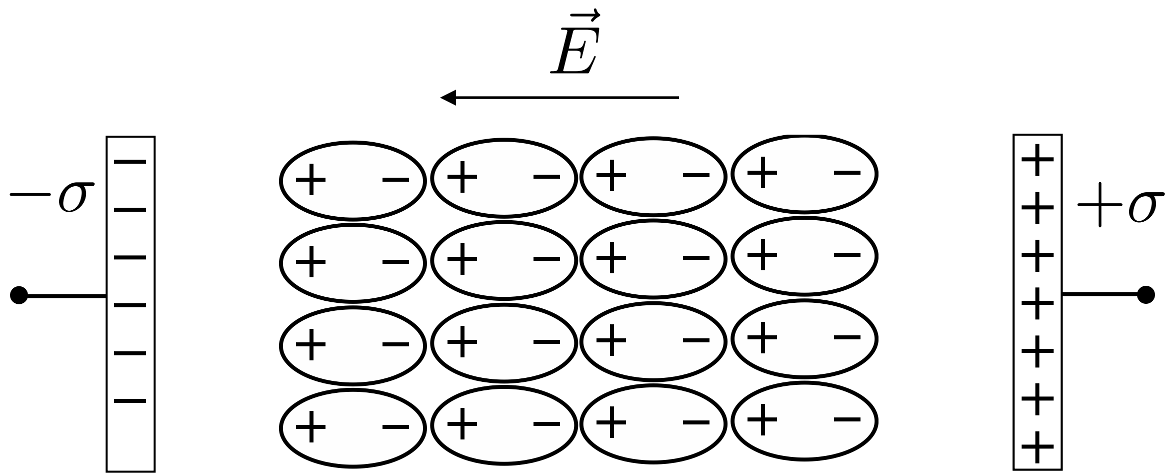
$$\sigma \geq \sigma_{dielectric}$$

- Capacitance increases
- Since σ remained the same then v decreases

Why is there $\sigma_{dielectric}$?

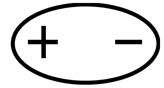
What is the magnitude of $\sigma_{dielectric}$?

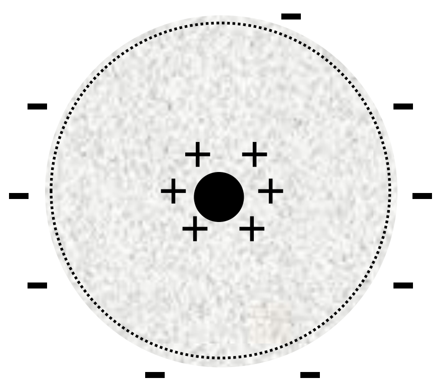
When molecules are in an electric field they get stretched



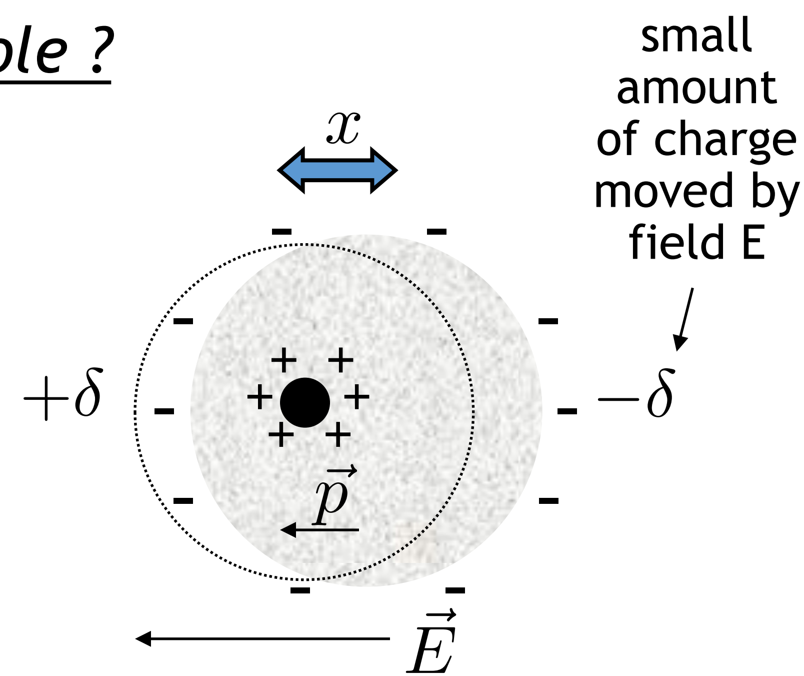
Electric field polarizes molecules …
… turns them into DIPOLES

What is a Dipole ?





NO external E filed



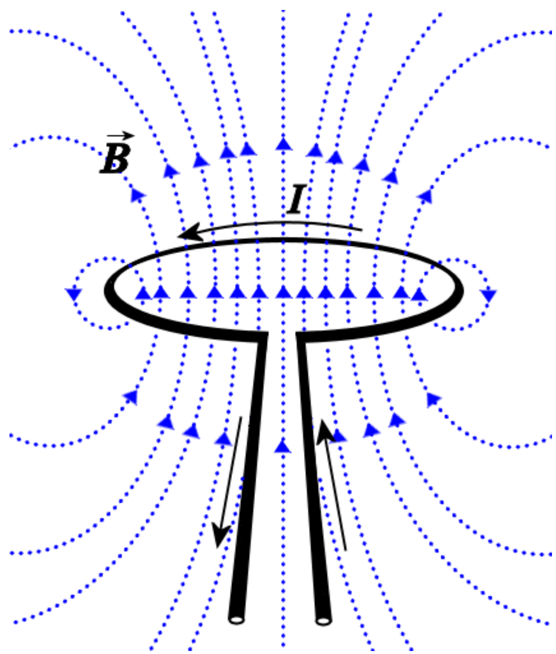
external E filed stretches the atom/molecule

Dipole Moment = $\vec{p} = \delta\vec{x}$

electric dipole moment, *p* (or **electric dipole** for short), is a measure of the polarity of a system of electric charges. Here ***x*** is the displacement vector pointing from the negative charge to the positive charge. This implies that the electric dipole moment vector points from the negative charge to the positive charge. Note that the electric field lines run away from the positive charge and toward the negative charge. There is no inconsistency here, because the electric dipole moment has to do with the positions of the charges, not the field lines.

Analogy Between Magnetic and Electric Dipoles

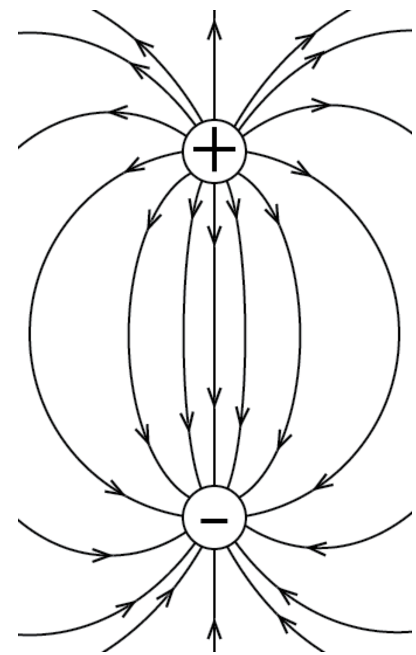
Magnetic Fields



Magnetic
Moment

$$m = i\,\boldsymbol{a}$$

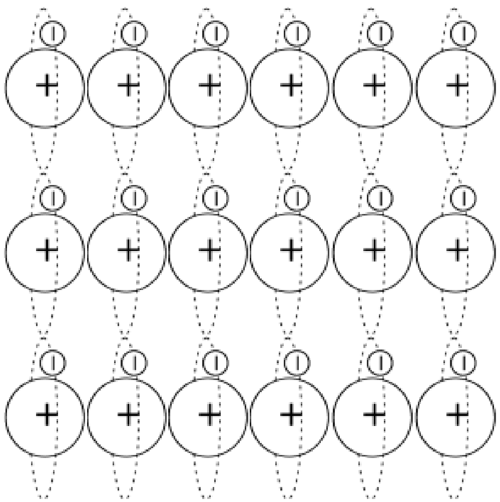
Electric Field

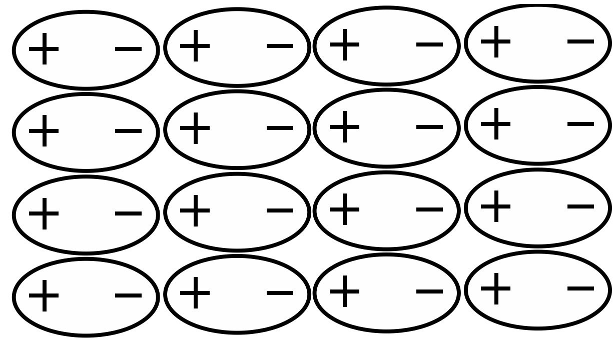


Electric Dipole
Moment

$$m = q\,\mathbf{d}$$

Superposition





The ***magnetization*** or *net magnetic dipole moment density* is given by

$$\vec{M} = N\vec{m}$$

[A/m]

Number of dipoles per unit volume [m^{-3}]

average magnetic dipole moment [A m^2]

The ***polarization*** or *net electric dipole moment density* is given by

$$\vec{P} = N\vec{p}$$

[C/m^2]

Number of dipoles per unit volume [m^{-3}]

average electric dipole moment [C m]

Induced Magnetization

For some materials, the *net magnetic dipole moment per unit volume* is proportional to the *H* field

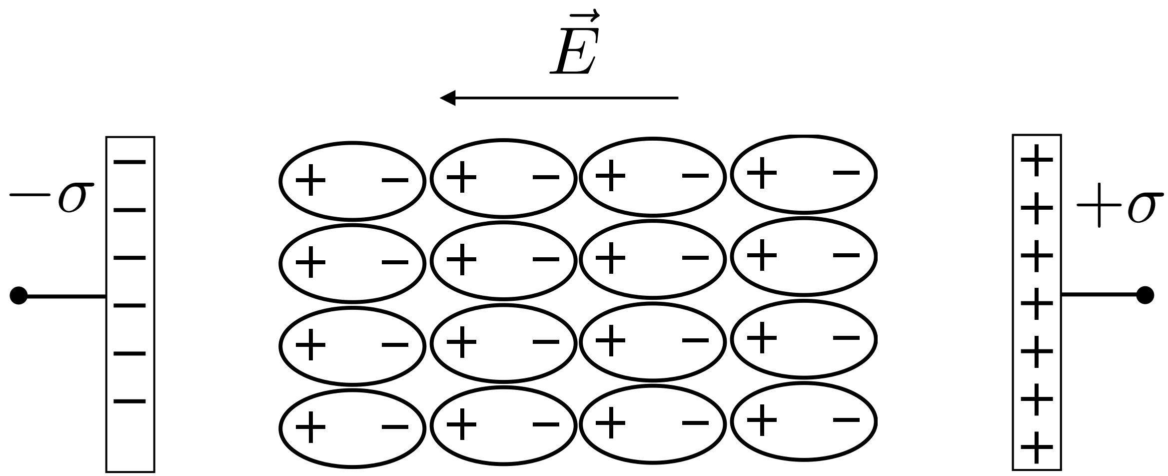
$$\overline{M} = \chi_m \, \overline{H}$$

MAGNETIC SUSCEPTIBILITY (dimensionless)

units of both ***M*** and ***H*** are A/m.

The effect of an applied magnetic field on a *magnetic* material is to create a net magnetic dipole moment per unit volume ***M***

Induced Polarization



Density of dipoles....

$$\vec{P} = N\vec{p}$$

$$= N\delta\vec{x} = N\alpha\vec{E}$$

polarization density, P (a.k.a **electric polarization**, or simply **polarization**) - density of permanent or induced electric dipole moments in a dielectric material. The SI unit of measure is coulombs per square meter.

Electric field polarizes molecules...

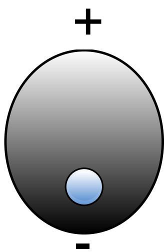
... equivalent to ... $\vec{P} = \epsilon_o \chi_e \vec{E}$

electric susceptibility of a dielectric material is a measure of how easily it polarizes in response to an electric field

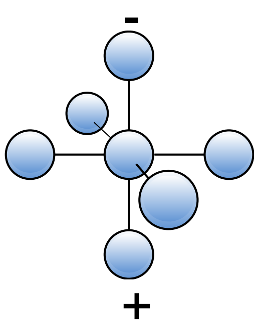
Origin of the Dielectric Response

$$\vec{P} = N\vec{p} = N\alpha\vec{E}$$

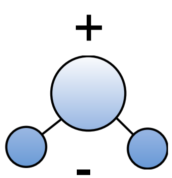
Polarizability, α, tells us how easy is to disturb charge distribution, like the electron cloud of an atom or molecule, from its normal shape by an external electric field.



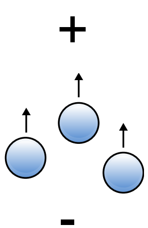
1. Electronic Polarizability (α_e)
Polarization of localized electrons



2. Ionic Polarizability (α_i)
Displacement of ions



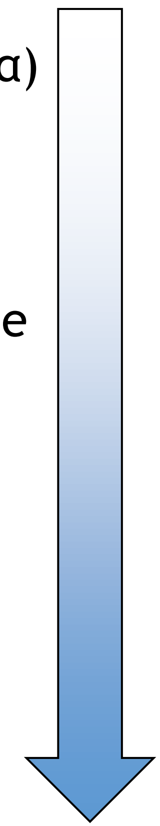
3. Dipolar Polarizability (α_d)
Reorientation of polar molecules



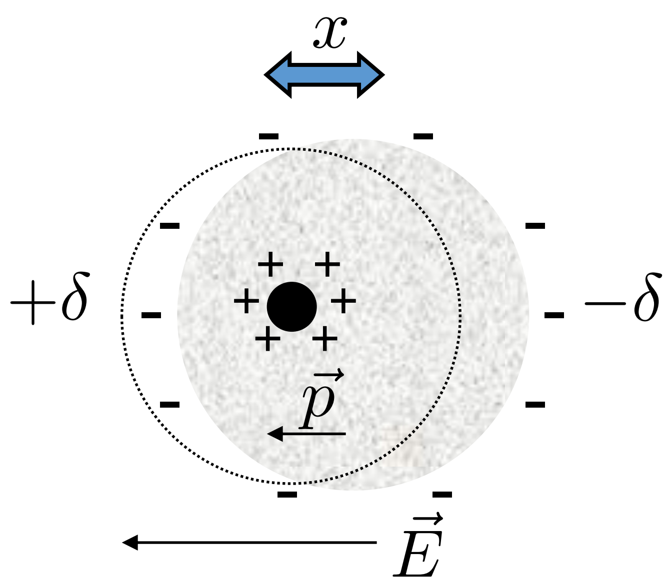
4. Space Charge Polarizability (α_s)
Long range charge migration

Polarizability (α) increases

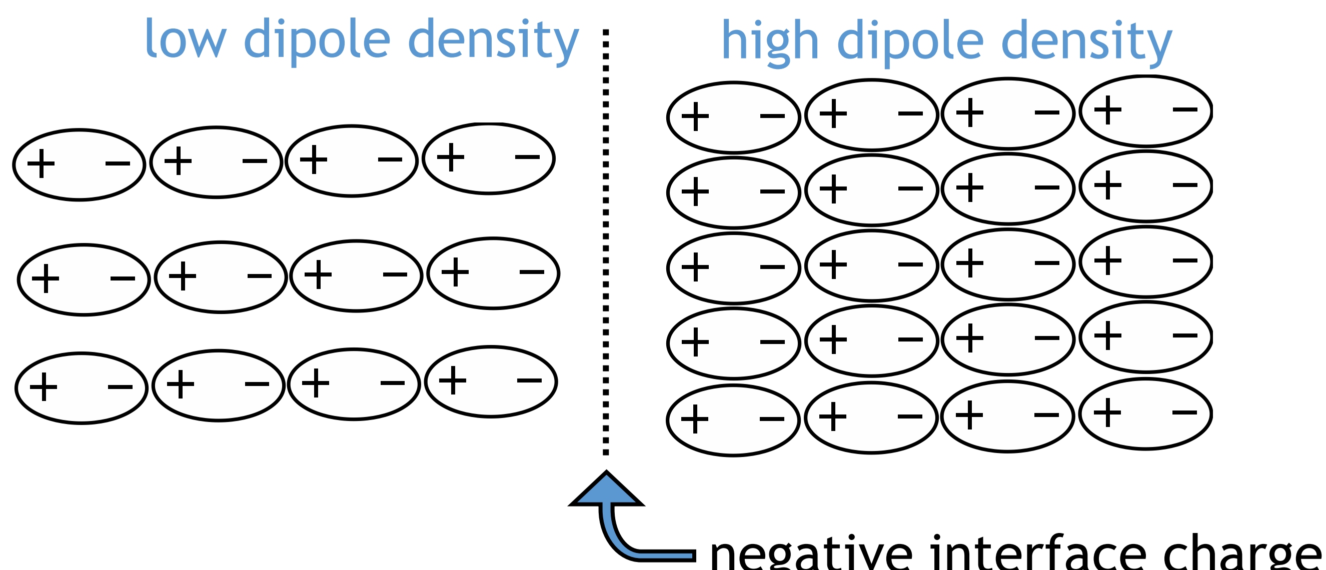
Response Time Increases (slower response)

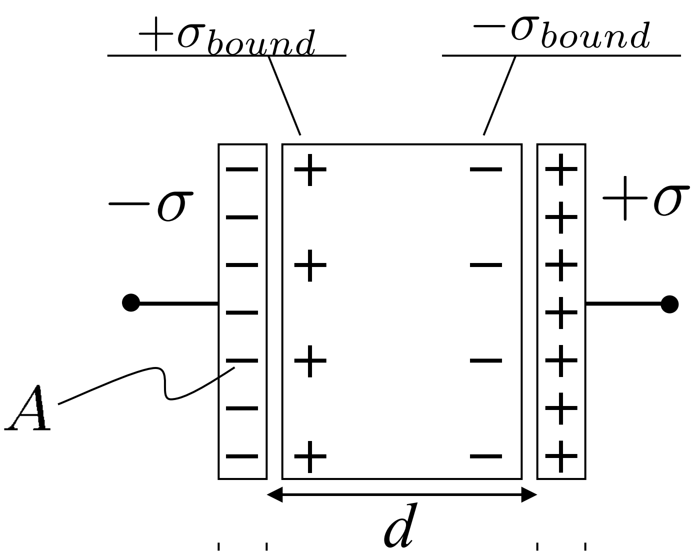


Bound Charges and Polarization

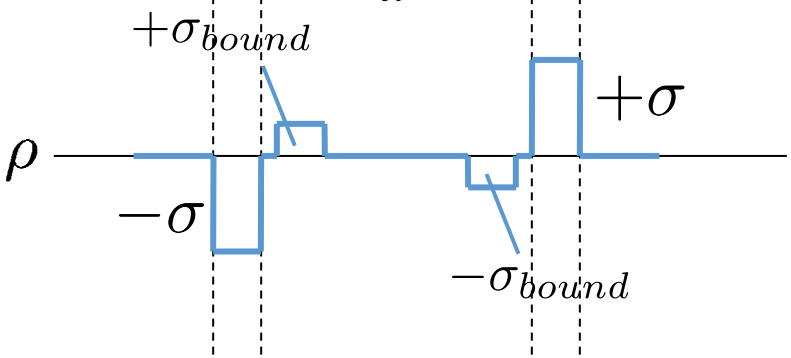


$$\rho_{bound} = -\vec{\nabla} \cdot \vec{P}$$





... let's insert an insulator sheet
between capacitor plates

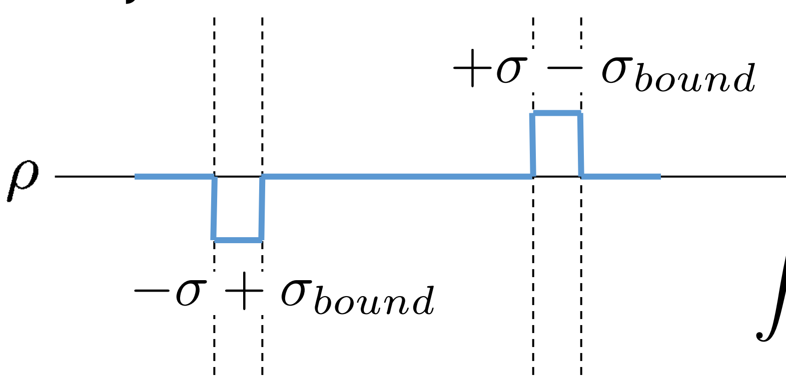


$$\int_S \epsilon_o \vec{E} \cdot d\vec{A} = \int_V \rho_{total} dV$$

$$= \int_V \left(\rho_{bound} + \rho_{free}\right) dV$$

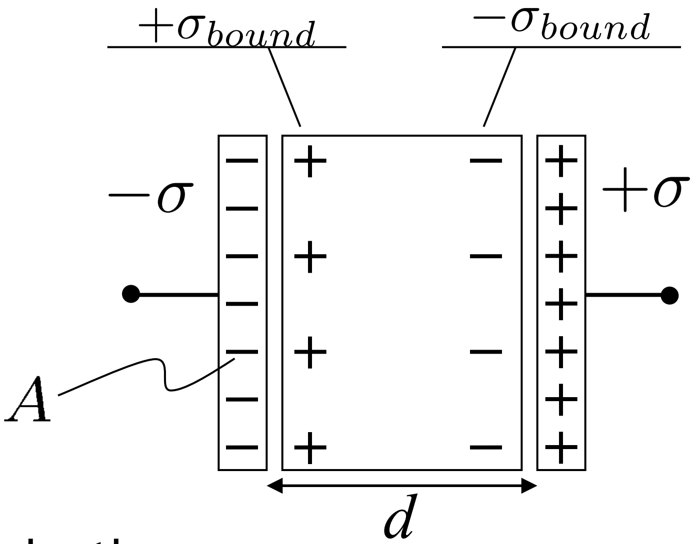
$$= \int_V \left(-\vec{\nabla} \cdot \vec{P} + \rho_{free}\right) dV$$

equivalently ...

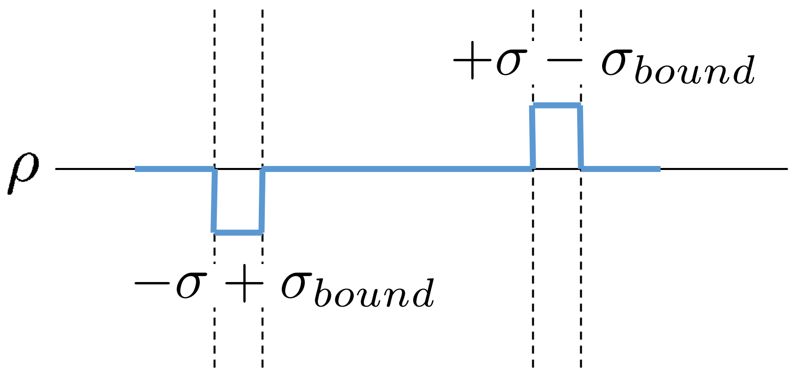


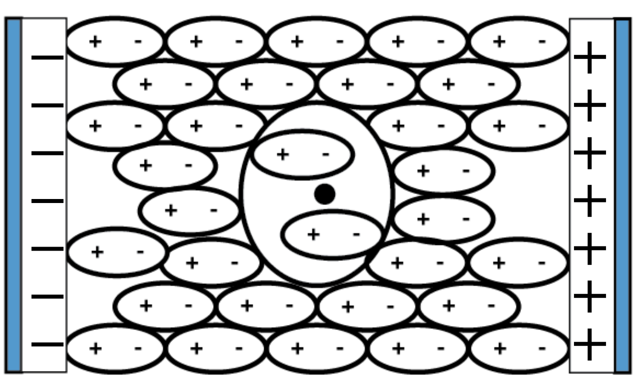
$$\int_S \left(\epsilon_o \vec{E} + \vec{P}\right) \cdot d\vec{A} = \int_V \rho_{free} dV$$

… let’s insert an insulator sheet between capacitor plates



equivalently …





Electric displacement D

$$\vec{D} = \epsilon_o \vec{E} + \vec{P}$$

$$= \epsilon_o \vec{E} + \epsilon_o \chi_e \vec{E}$$

$$= \epsilon_o \left(1 + \chi_e\right) \vec{E}$$

$$\vec{D} = \epsilon_o \epsilon_r \vec{E} = \epsilon \vec{E}$$

Displacement Fields

$$\int_S \vec{D} \cdot d\vec{A} = \int_S \left(\epsilon_o \vec{E} + \vec{P}\right) \cdot d\vec{A} = \int_V \rho_{free} dV$$

where we define electric displacement field, D, as $\vec{D} = \epsilon_o \vec{E} + \vec{P}$

Displacement field **D** accounts for the effects of unbound (“free”) charges within materials.

$$\int_S \epsilon_o \vec{E} \cdot d\vec{A} = \int_V \rho_{total} dV$$

Electric field **E** accounts for the effects of total charges (both “bound” and “free”) within materials.

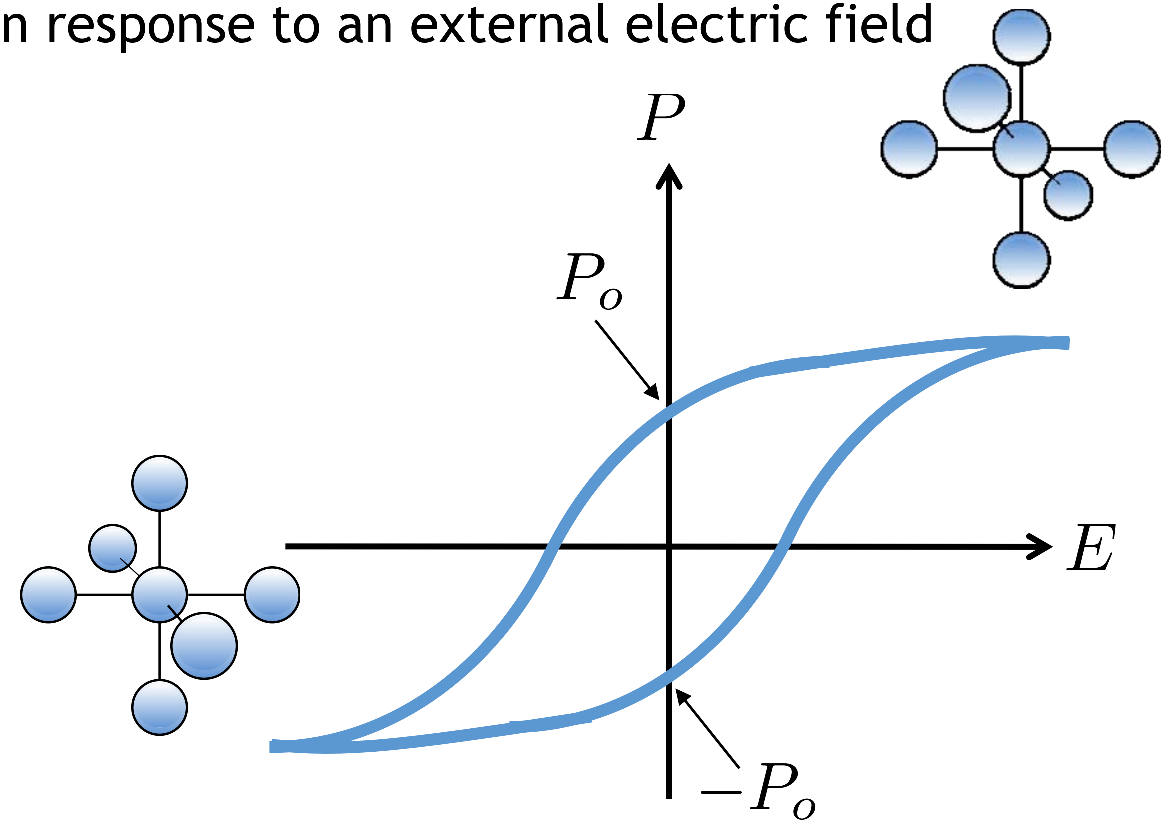
Ferroelectrics

A ferroelectric material develops a spontaneous polarization (builds up a charge) in response to an external electric field

- The polarization does not go away when the external field is removed

- The direction of the polarization is reversible

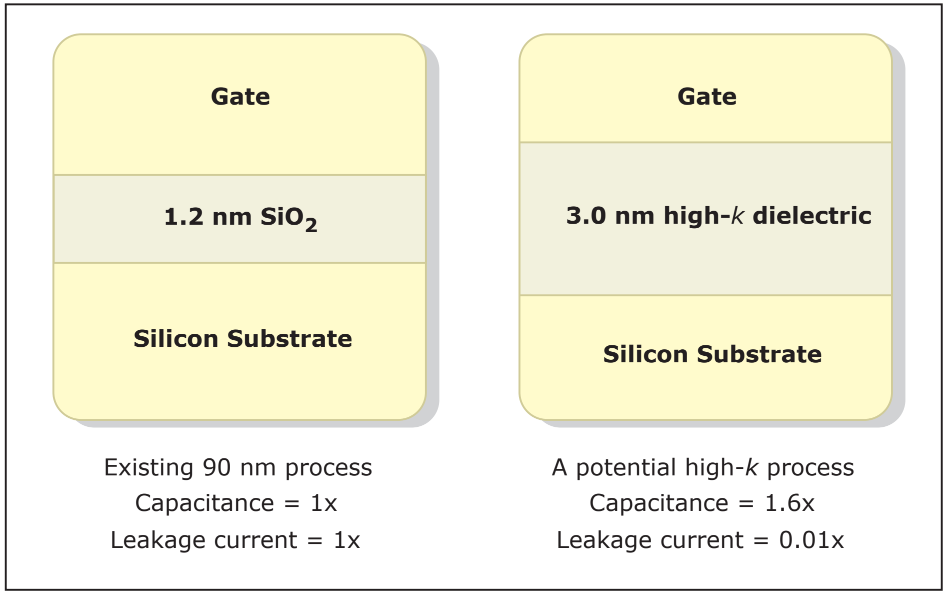
- Examples $BaTiO_3$
  $PbTiO_3$



Applications of Ferroelectric Materials

- Non-volatile FRAM (Ferroelectric Random Access Memory)

Why Worry About Dielectrics ?



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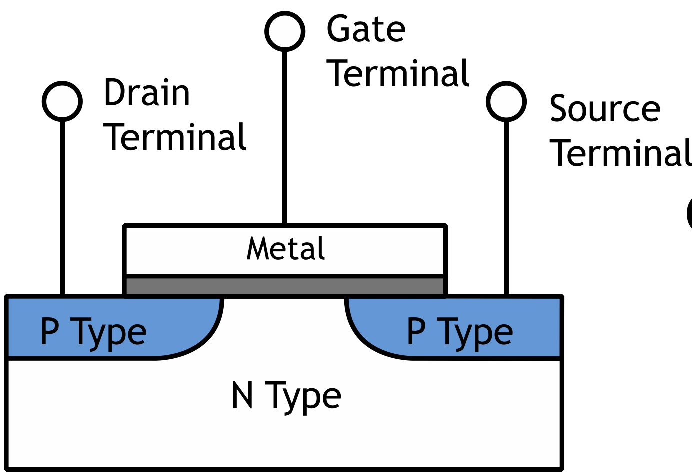


Capacitor in 90, 65nm

SiO_2 $\varepsilon/\varepsilon_o$ =3.9

Capacitor in 45nm

HfO_2 $\varepsilon/\varepsilon_o$ =25

As transistors scale, insulation within the capacitor has become leaky…

Energy Density of the Electric Field

What is the energy density stored in the capacitor ?

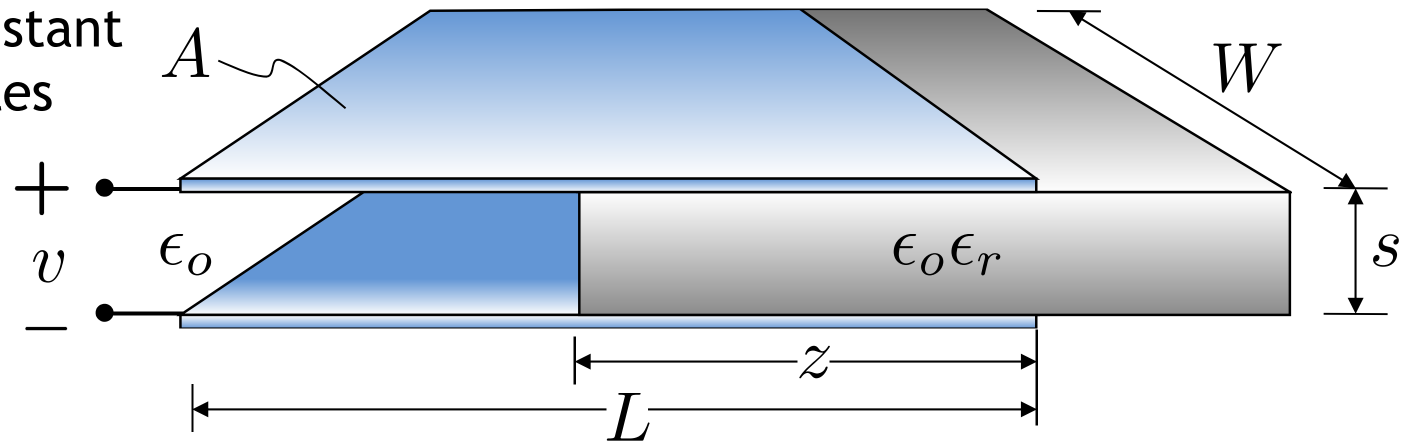
For a capacitor with large, flat plates…

$$\boxed{\begin{aligned}\frac{W_S}{V} &= \frac{\frac{1}{2}\frac{q^2}{C}}{A\cdot d} = \frac{\frac{1}{2}\frac{\epsilon_o\epsilon_r A}{d}v^2}{A\cdot d} \\ &= \frac{\frac{1}{2}\frac{\epsilon_o\epsilon_r A}{d}E^2 d^2}{d^2} = \frac{1}{2}\vec{D}\cdot\vec{E}\end{aligned}}$$

The **electric field** is not just the origin of electrostatic forces but also tells us about the stored energy !

Linear Dielectric Slab Actuator

Note: We are going to keep Q constant on the plates



$$W_S = \frac{Cv^2}{2} = \frac{Q^2}{2C} \qquad f = -\left.\frac{\partial W_S}{\partial z}\right|_Q \qquad f = \frac{Q^2}{2C^2}\frac{dC}{dz}$$

$$C(z) = \frac{\epsilon_o\epsilon_r zW}{s} + \frac{\epsilon_o(L-z)W}{s}$$

$$f = \boxed{}$$

Electromagnetic Energy Storage

Remember …

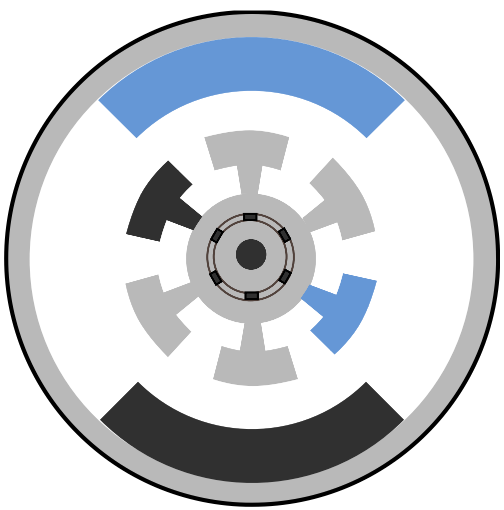
Magnetic

$$\frac{W_S}{V} = \frac{1}{2}\mu\vec{H}\cdot\vec{H}$$
$$= \frac{1}{2}\vec{B}\cdot\vec{H}$$

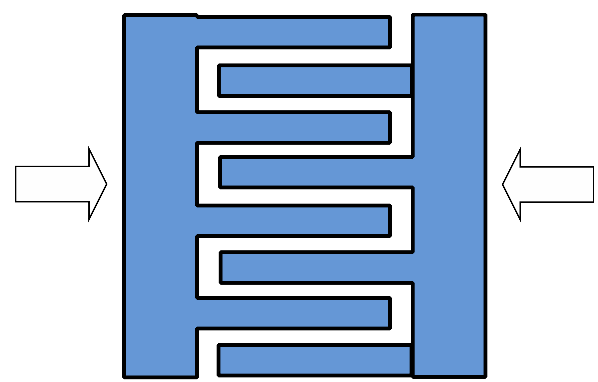
Electric

$$\frac{W_S}{V} = \frac{1}{2}\epsilon\vec{E}\cdot\vec{E}$$
$$= \frac{1}{2}\vec{D}\cdot\vec{E}$$

Magnetic machine



Electric machine



Key Takeaways

Electric displacement

$$\vec{D} = \epsilon_o \vec{E} + \vec{P}$$
$$= \epsilon_o \vec{E} + \epsilon_o \chi_e \vec{E}$$
$$= \epsilon_o \left(1 + \chi_e\right) \vec{E}$$

$$\vec{D} = \epsilon_o \epsilon_r \vec{E} = \epsilon \vec{E}$$

$$\int_S \epsilon_o \vec{E} \cdot d\vec{A} = \int_V \rho_{total} dV$$

$$\int_S \vec{D} \cdot d\vec{A} = \int_V \rho_{free} dV$$

Magnetic

$$\frac{W_S}{V} = \frac{1}{2} \mu \vec{H} \cdot \vec{H}$$
$$= \frac{1}{2} \vec{B} \cdot \vec{H}$$

Electric

$$\frac{W_S}{V} = \frac{1}{2} \epsilon \vec{E} \cdot \vec{E}$$
$$= \frac{1}{2} \vec{D} \cdot \vec{E}$$