

Tutorial 2
February 23-24, 2006

1. Twenty five black pebbles are arranged in five rows of five pebbles each. We choose five of these pebbles at random and color them red.
 - (a) What is the probability that all the red pebbles lie in different rows?
 - (b) What is the probability that all the red pebbles lie in different rows and in different columns?

2. Consider the random variable X with PMF

$$p_X(x) = \begin{cases} \frac{x^2}{a} & \text{if } x = -3, -2, -1, 0, 1, 2, 3, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find a and $\mathbf{E}[X]$.
 - (b) What is the PMF of the random variable $Z = (X - \mathbf{E}[X])^2$?
 - (c) Using part (b) compute the variance of X .
3. Let X_1 , X_2 and X_3 be three mutually independent random variables with known all strictly positive means and variances. We define the following random variables, U , V and W :

$$\begin{aligned} U &= aX_1 + bX_2 + c \\ V &= a'X_1 + b'X_3 + c' \\ W &= a''X_2 + b''X_3 + c'' \end{aligned}$$

where $a, a', a'', b, b', b'', c, c', c''$ are all strictly positive constants.

- (a) Find the mean and variance of U and V in terms of the mean and variance of X_1 , X_2 and X_3 .
- (b) Find $E[(U - E[U])(V - E[V])]$ and $E[(U - E[U])(W - E[W])]$ in terms of the mean and variance of X_1 , X_2 and X_3 .
- (c) Two random variables Z and Y are uncorrelated if $E[(Z - E[Z])(Y - E[Y])] = 0$. Can you find $a, a', a'', b, b', b'', c, c', c''$ such that V and W are uncorrelated? If yes, give the range of possible values.
- (d) Show that independent random variables are uncorrelated.