Tutorial 07
April 6-7, 2006

1. Suppose you are playing roulette with a biased wheel, such that your chance of winning on each spin is $p > 1/2$. Suppose that you start out with x dollars, and that on each play, you bet $2p - 1$ of what you have. Assuming that when you win a round you win what you bet, and that otherwise you lose what you bet, find your expected holdings after n rounds of play.

2. Suppose $X \sim N[0,1]$ and $Z = 0$ or 1 with equal probability. Now consider a random variable Y such that:

$$Y = \begin{cases} x & \text{if } Z = 1 \\ -x & \text{if } Z = 0 \end{cases}$$

   a) Are X, Y independent?

   b) Are Y, Z independent?

   c) Show that $Y \sim N[0,1]$.

   d) Show that $Cov(X, Y) = 0$.

3. You enter a castle with 78 floors. Your goal is to get to the treasure at the top. On each floor of the castle, there is a probability $1/2$ of bumping into someone guarding that floor. Bumping into someone results in a change in your energy (the person you bump into may be friendly and give you something to eat, or may be nasty and you may have to put up a fight, etc). Your change in energy in each such interaction is distributed as a normal random variable with mean 1 unit and standard deviation $1/2$ units, and is independent of how many people you bump into and of your change in energy in other interactions. No matter what your energy change on any given floor, you always proceed to the next floor. Let X be your total change in energy by the time you reach the treasure.

   (a) Use the total probability law to fine the PDF and transform associated with X. Is X normal?

   (b) Find the transform associated with X by viewing X as a sum of a random number of random variables.