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A McGRAW-HILL CLASSIC TEXTBOOK REISSUE

fundamentals of applied probability theory

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To this Great Nation,
which has allowed
millions of refugees,
including my parents,
to enter a land of
freedom and opportunity

PREFACE

This is a first textbook in applied probability theory, assuming a background of one year of calculus. The material represents a one-semester subject taught at M.I.T. to about 250 students per year, most of whom are in the Schools of Engineering or Management. About two-thirds of these students are undergraduates. The subject, Probabilistic Systems Analysis, serves both as a terminal course and as a prerequisite for more advanced work in areas such as communication theory, control systems, decision theory, operations research, quantitative management, statistics, and stochastic processes.

My intention is to present a physically based introduction to applied probability theory, with emphasis on the continuity of fundamentals. A prime objective is to develop in the new student an understanding of the nature, formulation, and analysis of probabilistic situations. This text stresses the sample space of representation of probabilistic processes and (especially in the problems) the need for explicit modeling of nondeterministic processes.

In the attempt to achieve these goals, several traditional details have either been omitted or relegated to an appendix. Appreciable effort has been made to avoid the segmentation and listing of detailed applications which must appear in a truly comprehensive work in this area. Intended primarily as a student text, this book is not suitable for use as a general reference.

Scope and Organization

The fundamentals of probability theory, beginning with a discussion of the algebra of events and concluding with Bayes' theorem, are presented in Chapter 1. An axiomatic development of probability theory is used and, wherever possible, concepts are interpreted in the sample space representation of the model of an *experiment* (any nondeterministic process). The assignment of probability measure in the modeling of physical situations is not necessarily tied to a relative frequency interpretation. In the last section of this chapter, the use of sample and event spaces in problems of enumeration is demonstrated.

Chapter 2 is concerned with the extension of earlier results to deal with random variables. This introductory text emphasizes the local assignment of probability in sample space. For this reason, we work primarily with probability density functions rather than cumulative distribution functions. My experience is that this approach is much more intuitive for the beginning student. Random-variable concepts are first introduced for the discrete case, where things are particularly simple, and then extended to the continuous case. Chapter 2 concludes with the topic of derived probability distributions as obtained directly in sample space.

Discrete and continuous transform techniques are introduced in Chapter 3. Several applications to sums of independent random variables are included. Contour integration methods for obtaining inverse transforms are not discussed.

Chapters 4 and 5 investigate basic random processes involving, respectively, independent and dependent trials.

Chapter 4 studies in some detail the Bernoulli and Poisson processes and the resulting families of probability mass and density functions. Because of its significance in experimentation with physical systems, the phenomenon of random incidence is introduced in the last section of Chapter 4.

Discrete-state Markov models, including both discrete-transition and continuous-transition processes, are presented in Chapter 5. The describing equations and limiting state probabilities are treated, but closed form solutions for transient behavior in the general case are not discussed. Common applications are indicated in the text and in the problems, with most examples based on relatively simple birth-and-death processes.

Chapter 6 is concerned with some of the basic limit theorems, both for the manner in which they relate probabilities to physically observable phenomena and for their use as practical approximations. Only weak statistical convergence is considered in detail. A transform development of the central limit theorem is presented.

The final chapter introduces some common issues and techniques of statistics, both classical and Bayesian. My objectives in this obviously incomplete chapter are to indicate the nature of the transition from probability theory to statistical reasoning and to assist the student in developing a critical attitude towards matters of statistical inference.

Although many other arrangements are possible, the text is most effectively employed when the chapters are studied in the given order.

Examples and Home Problems

Many of the sections which present new material to the student contain very simple illustrative examples. More structured examples, usually integrating larger amounts of material, are solved and discussed in separate sections.

For the student, the home problems constitute a vital part of the subject matter. It is important that he develop the skill to formulate and solve problems with confidence. Passive agreement with other people's solutions offers little future return. Most of the home problems following the chapters are original, written by the author and other members of the teaching staff.

These problems were written with definite objectives. In particular, wherever possible, we have left for the student a considerable share in the formulation of physical situations. Occasionally, the probability assignments directly relevant to the problems must be derived from other given information.

It did not seem feasible to sample the very many possible fields of application with other than superficial problems. The interesting aspects of each such field often involve appreciable specialized structure and nomenclature. Most of our advanced problems are based on relatively simple operational situations. From these common, easily communicated situations, it seemed possible to develop compact representative problems which are challenging and instructive.

The order of the problems at the end of each chapter, by and large, follows the order of the presentation in the chapter. Although entries below are often not the most elementary problems, relatively comprehensive coverage of the material in this text is offered by the following skeleton set of home problems:

1.03	2.04	3.05	4.05	5.05	6.02	7.04
1.08	2.07	3.08	4.09	5.06	6.03	7.06
1.09	2.11	3.09	4.12	5.10	6.04	7.08
1.12	2.17	3.10	4.13	5.12	6.07	7.15
1.13	2.26	3.12	4.17	5.13	6.08	7.16
1.21	2.27	3.13	4.18	5.14	6.13	7.19
1.24	2.28	3.21	4.22	5.16	6.17	7.20
1.30	2.30					

Some Clerical Notes

I have taken some liberties with the usual details of presentation. Figures are not numbered but they do appear directly in context. Since there are few involved mathematical developments, equations are not numbered. Whenever it appeared advantageous, equations were repeated rather than cross-referenced.

Recommended further reading, including a few detailed references and referrals for topics such as the historical development of probability theory are given in Appendix 1. Appendix 2 consists of a listing of common probability mass and density functions and their expected values, variances, and transforms. Several of these probability functions do not appear in the body of the text. A brief table of the cumulative distribution for the unit normal probability density function appears in context in Chapter 6.

The general form of the notation used in this text seems to be gaining favor at the present time. To my taste, it is one of the simplest notations which allows for relatively explicit communication. My detailed notation is most similar to one introduced by Ronald A. Howard.

Acknowledgments

My interest in applied probability theory was originally sparked by the enthusiasm and ability of two of my teachers, Professors George P. Wadsworth and Ronald A. Howard. For better or worse,

it is the interest they stimulated which led me to this book, rather than one on field theory, bicycle repair, or the larger African beetles.

Like all authors, I am indebted to a large number of earlier authors. In this case, my gratitude is especially due to Professors William Feller, Marek Fisz, and Emanuel Parzen for their excellent works.

Teaching this and related material during the past six years has been an exciting and rewarding experience, due to the intensity of our students and the interchange of ideas with my colleagues, especially Dr. Murray B. Sachs and Professor George Murray. The many teaching assistants associated with this subject contributed a great deal to its clarification. Some of the problems in this book represent their best educational (and Machiavellian) efforts.

During the preparation of this book, I had many productive discussions with Professor William Black. In return for his kindness, and also because he is a particularly close friend, I never asked him to look at the manuscript. Professor Alan V. Oppenheim and Dr. Ralph L. Miller were less fortunate friends; both read the manuscript with great care and offered many helpful suggestions. The publisher's review by Dr. John G. Truxal was most valuable.

Some award is certainly due Mrs. Richard Spargo who had the grim pleasure of typing and illustrating the entire manuscript—three times! My devoted wife, Elisabeth, checked all examples, proofread each revision of the manuscript, and provided unbounded patience and encouragement.

Finally, I express my gratitude to any kind readers who may forward to me corrections and suggestions for improvements in this text.

Alvin W. Drake

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