# Massachusetts Institute of Technology <br> Department of Electrical Engineering \& Computer Science <br> 6.041/6.431: Probabilistic Systems Analysis 

(Fall 2010)

## Recitation 7 Solutions

## September 30, 2010

1. See the textbook, Problem 2.35, page 130.
2. (a)

$$
\begin{aligned}
p_{X}(1) & =\mathbf{P}(X=1, Y=1)+\mathbf{P}(X=1, Y=2)+\mathbf{P}(X=1, Y=3) \\
& =1 / 12+2 / 12+1 / 12=1 / 3
\end{aligned}
$$

(b) The solution is a sketch of the following conditional PMF:

$$
p_{Y \mid X}(y \mid 1)=\frac{p_{Y, X}(y, 1)}{p_{X}(1)}= \begin{cases}1 / 4, & \text { if } y=1 \\ 1 / 2, & \text { if } y=2 \\ 1 / 4, & \text { if } y=3 \\ 0, & \text { otherwise }\end{cases}
$$

(c) $\mathbf{E}[Y \mid X=1]=\sum_{y=1}^{3} y p_{Y \mid X}(y \mid 1)=1 \cdot \frac{1}{4}+2 \cdot \frac{1}{2}+3 \cdot \frac{1}{4}=2$
(d) Assume that $X$ and $Y$ are independent. Because $p_{X, Y}(3,1)=0$ and $p_{Y}(1)=1 / 4, p_{X}(3)$ must equal zero. This further implies $p_{X, Y}(3,2)=0$ and $p_{X, Y}(3,3)=0$. All the remaining probability mass must go to $(X, Y)=(2,2)$, making $p_{X, Y}(2,2)=5 / 12, p_{X}(2)=8 / 12$, and $p_{Y}(2)=7 / 12$. However, $p_{X, Y}(2,2) \neq p_{X}(2) \cdot p_{Y}(2)$, contradicting the assumption; thus $X$ and $Y$ are not independent.
A simpler explanation uses only two $X$ values and two $Y$ values for which all four $(X, Y)$ pairs have specified probabilities. Note that if $X$ and $Y$ are independent, then $p_{X, Y}(1,3) / p_{X, Y}(1,1)$ and $p_{X, Y}(2,3) / p_{X, Y}(2,1)$ must be equal because they must both equal $p_{Y}(3) / p_{Y}(1)$. This necessary equality does not hold, so $X$ and $Y$ are not independent.
(e) Knowing that $X$ and $Y$ are conditionally independent given $B$, we must have

$$
\frac{p_{X, Y}(1,1)}{p_{X, Y}(1,2)}=\frac{p_{X, Y}(2,1)}{p_{X, Y}(2,2)}
$$

since the $(X, Y)$ pairs in the equality are all in $B$. Thus

$$
p_{X, Y}(2,2)=\frac{p_{X, Y}(1,2) p_{X, Y}(2,1)}{p_{X, Y}(1,1)}=\frac{(2 / 12)(2 / 12)}{1 / 12}=\frac{4}{12}=\frac{1}{3}
$$

(f) Since $\mathbf{P}(B)=9 / 12=3 / 4$, we normalize to obtain $p_{X, Y \mid B}(2,2)=\frac{p_{X, Y}(2,2)}{\mathbf{P}(B)}=4 / 9$.
3. See the textbook, Problem 2.33, page 128.

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### 6.041SC Probabilistic Systems Analysis and Applied Probability

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